Usage-sensitive Pricing in Multi-service Networks

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- Pricing schemes for multi-service networks
- Influence of Pricing on PVC vs. SVC Service Preference
- Service provider's optimal pricing scheme for PVC and SVC service
- Differential Pricing for Differentiated Service

Motivation

- Communication networks are going to provide varied services
  - ATM: CBR, VBR, UBR,ABR.
    PVC and SVC
  - Internet: Guaranteed, Controlled-load, Predictive, Controlled Delay, and Committed Rate

- Efficient operation of networks is important
  - Network performance
  - Users' aggregate satisfaction
  - Utilization of resources
  - Maximum Revenue

Motivation

- Pricing can improve network efficiency
  - manage offered load and its distribution
  - clearly differentiate services
  - encourage efficient usage of resources

- Users and service providers have conflicting objectives
  - Users: get as much service for as little money as possible
  - Service provider: recover the cost and achieve some benefit

- Pricing as a management tool
  - recover the cost
    — not dealt with here
  - encourage users to act in a way that is beneficial to the network
    — focus of this work

Flat-rate vs. Usage-sensitive Pricing Scheme

- Flat-rate:
  - Independent of the transmitted traffic, the connection duration, and the allocated resources
  - Advantage: easy to implement

- Usage-sensitive:
  - a function of some combination of actual traffic transmitted, the allocated resources, call duration, and assigned priority
  - Advantage: gives users an incentive to make reasonable choices

- In this thesis we study usage-sensitive pricing schemes
  - connection-oriented networks : reserved resources pricing scheme
  - packet-oriented networks : priorities pricing scheme

Dynamic vs. Static Pricing

- Dynamic pricing scheme:
  - prices depend upon some network conditions
  - disadvantage: computational complexity, user resistance

- Static pricing scheme:
  - independent of network condition
  - advantage: simple, minimum users' involvement

- In this thesis we study static pricing schemes

Analytic Model of a Pricing Scheme

- User's Surplus function
  - represents a user's satisfaction with a service
  - surplus = utility – charges
    - utility function reflects the benefit a user receives from a service
    - in this thesis, utility is a function of user's traffic amount
- Service Provider's Surplus function
  - revenue minus cost of providing service

Pricing for PVC vs. SVC Service

- Objective:

  provide pricing incentives in order to encourage some users to choose PVC, while others choosing SVC.

- Proposed pricing function

$$\mathrm{E}\{Cost\}=\mathrm{E}\{UsageCharges\}+\mathrm{E}\{SetupCharges\}+\mathrm{E}\{BandwidthAllocationCharges\}$$

$$=\mathrm{E}\{UsageCharges\}+\mathrm{s}\cdot\mathrm{E}\{NumberOfSetups\}+a\cdot bw\cdot\mathrm{E}\{TotalConnectionTime\}$$

where:

$\mathrm{E}\{Z\}$= Expected value of Z;

s = per-connection setup charge;

a = the charge per unit time per unit bandwidth allocated to the user during one connection. We assume this is the same for every connection in one billing period;

bw = the bandwidth allocated to the user.

Pricing Function Assumptions

- Unit prices a and s are the same for every user and every connection
- Allocated bandwidth bw is the same for every user and every connection

User's Traffic Model

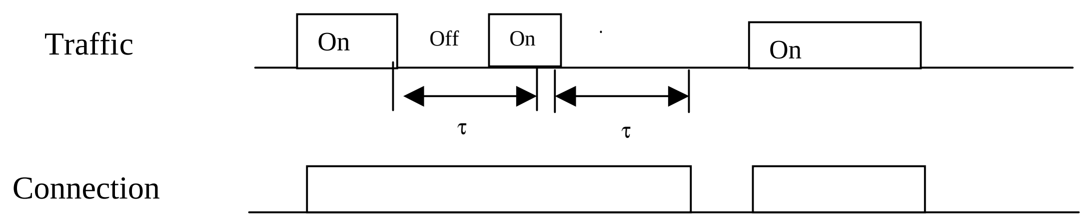
- Two-state model
  - — User traffic source either *On* or *Off*
- Traffic is independent of the prices

$$E\{N\} = \frac{T}{X+Y}$$

where: N= number of on-off cycles in a billing period;
T = the length of the billing period;
X= mean duration of *On* state;
Y= mean duration of *Off* state.

User’s Behavior Model

- User initiates connection (if necessary) when traffic source turns *On*.
- User holds connection for time τ after traffic source turns *Off*.



- User chooses best value of τ (τ^*), given pricing scheme:
  - τ small $\Rightarrow$ SVC service
  - τ large $\Rightarrow$ PVC service

Pricing Function Revisited

- The charges of service is now:

$$E\{Cost\} = E\{UsageCharges\} + s \cdot E\{NumberOfSetup\} + a \cdot bw \cdot E\{TotalConnectionTime\}$$

$$= E\{UsageCharges\} + s \cdot \text{Prob}(Off \geq \tau) \cdot E\{N\}$$
$$+ a \cdot bw \cdot [X + E\{Off < \tau\}\ \text{Prob}(Off < \tau) + \tau \cdot \text{Prob}(Off \geq \tau)] \cdot E\{N\}$$

where:

$E\{Off<\tau\}$ = the mean duration of the *Off* state given that it is less than τ;
$\text{Prob}(Off<\tau)$ = the probability that the length of *Off* state is less than τ;
$\text{Prob}(Off\geq\tau)$ = the probability that the length of *Off* state is greater than or equal to τ.

Peak Rate Bandwidth Allocation-Exponential Distribution

- Peak rate bandwidth allocation:
  - — Network allocates bandwidth = peak rate
  - — Normalize *bw* to 1

- Exponential Distribution

  Length of the "*Off*" periods is exponentially distributed.

$$p(t) = \beta \cdot \exp(-\beta t)$$

- The average cost function of one user:

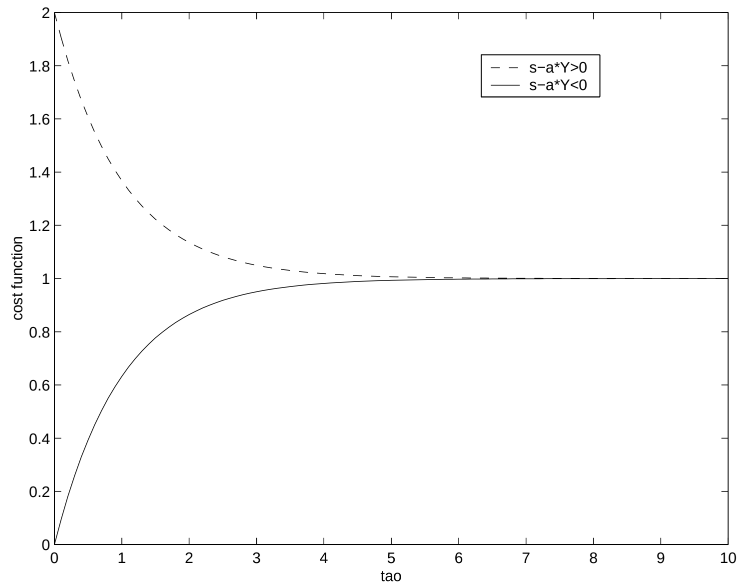
$$\mathrm{E}\,\{Cost\} = \mathrm{E}\{UsageCharges\}+\{s\cdot \exp(-\beta\tau)]\}\frac{T}{X+Y}$$

$$+a\cdot\{X+[1-\exp(-\beta\tau)]-\tau\cdot\exp(-\beta\tau)+\tau\cdot\exp(-\beta\tau)\}\frac{T}{X+Y}$$

$$=\mathrm{E}\{UsageCharges\}+[X\cdot a+\frac{a}{\beta}+(s-\frac{a}{\beta})\cdot\exp(-\beta\tau)]$$

Results

- Since exp(-βτ)>0 for the whole range of τ, the minimum points are:
  - If $s-\frac{a}{\beta}>0$, *i.e.*, $\frac{s}{a}>Y$, the minimum point occurs at $\tau=\infty$ (preference for PVC service).
  - If $s-\frac{a}{\beta}<0$, *i.e.*, $\frac{s}{a}<Y$, the minimum point occurs at $\tau=0$ (preference for SVC service).
  - If $s-\frac{a}{\beta}=0$, *i.e*, $\frac{s}{a}=Y$, the cost function is a constant with τ (no preference).



Cost function curves for exponential distribution

Peak Rate Bandwidth Allocation-Discussion

• The results of the uniform off time distribution is the same as for exponential distribution

• For specific values of *s* and *a*:

— users with *Y* less than $\frac{s}{a}$ will prefer PVC service,

$Y < \frac{s}{a}$, cost of connection setups surpass the cost of bandwidth allocation during Off state

— users with *Y* greater than $\frac{s}{a}$ will prefer SVC service

$Y > \frac{s}{a}$, cost of bandwidth allocation during Off state surpass the cost of connection setups

Service Provider's Optimal Pricing for PVC and SVC Service

- Previous work gives insight into customer behavior
- Build on this to find pricing parameters that maximize provider's net income
- Assume peak rate bandwidth allocation

Service Provider

- Surplus function represents the total income from users minus the total cost

- Can manipulate users' service demands by pricing

- More demands for SVC:
  - higher multiplexing gain, higher utilization $\Rightarrow$ less bandwidth cost
  - increasing the complexity of the system, more nodal processing and signaling capacity $\Rightarrow$ more connection setup cost

- More demands for PVC:
  - more bandwidth cost
  - less connection setup cost

- There exist a set of optimal demands that maximize the surplus

Network Model

- Fixed number of users (*N*)
- Charges over the billing period *T*
- Two service classes: SVC and PVC
- Surplus function:

Total charges for all users – total costs of provisioning services= $R - (C_b + C_c)$

where: C_b is the costs of the bandwidth resources
C_c is the costs of the processing capacity for setting up connection like signaling capacity, node processing capacity

Pricing Model

- The total user charges for service are given as:

$$a_s \cdot bw \cdot \text{connection\_time} + s_s \cdot L, \quad \text{for SVC service;}$$

$$a_p \cdot bw \cdot T + s_p, \quad \text{for PVC service.}$$

where: s_s is the unit price of one SVC connection setup,

s_p is the unit price of one PVC connection setup,

$L = \frac{T}{X+Y}$ is the number of the user's total SVC connection setups during one billing period.

a_s is the unit price of bandwidth allocated for SVC service

a_p is the unit price of the required bandwidth of PVC service

User Model

- User model
  - — Same *on-off* two-state source
    - X: mean of *On* periods
    - Y: mean of *Off* periods
  - — Each user has the same bandwidth request, and normalized to bw=1
  - — Willingness-to-pay (utility): the limit up to which user will pay for the service

$$WTP = w \cdot \text{(user's traffic)} = w \cdot T \cdot \frac{X}{X+Y} \cdot bw = w \cdot T \cdot \frac{X}{X+Y}$$

where w is the coefficient of willingness-to-pay, in the unit of monetary unit per bandwidth unit per time unit

- Surplus = WTP – charges for one connection

Optimization Problem

- Optimal pricing scheme for a given demand scenario

Maximize: $R\,(s_s, a_s, s_p, a_p) - C_b\,(s_s, a_s, s_p, a_p) - C_c\,(s_s, a_s, s_p, a_p)$

$$= \sum_{i=1}^{N_s} \{[s_s + a_s \cdot X_i]\frac{T}{X_i + Y_i}\} + N_p\{s_p + a_p \cdot T\}$$

$$- C_b\,(s_s, a_s, s_p, a_p) - C_c\,(s_s, a_s, s_p, a_p)$$

subject to: No user's cost exceeds *WTP*
where N_s is the number of SVC users for the given price set (s_s, a_s, s_p, a_p)
N_p is the number of PVC users.

- Optimal PVC pricing: minimum willingness-to-pay among PVC users

- Optimal SVC pricing scheme:set s_s and a_s to construct certain demands (N_s and N_p) and maximize charges within the willingness-to-pay

Provider Costs

- Cost of bandwidth:
  - $C_b = c \cdot T \cdot bw$, where c is the cost per bandwidth unit per time unit
  - PVC: summation of PVC users' peak rates
  - SVC: with the blocking probability less than 1%

- Cost of connection setups

$$I_s \, \Sigma_{\mathrm{Ns}} L_{\mathrm{i}} + I_p \cdot N_p$$

where: I_s is the average unit cost per SVC connection setup
I_p is the average unit cost per PVC connection setup.

Solving the Optimization Problem

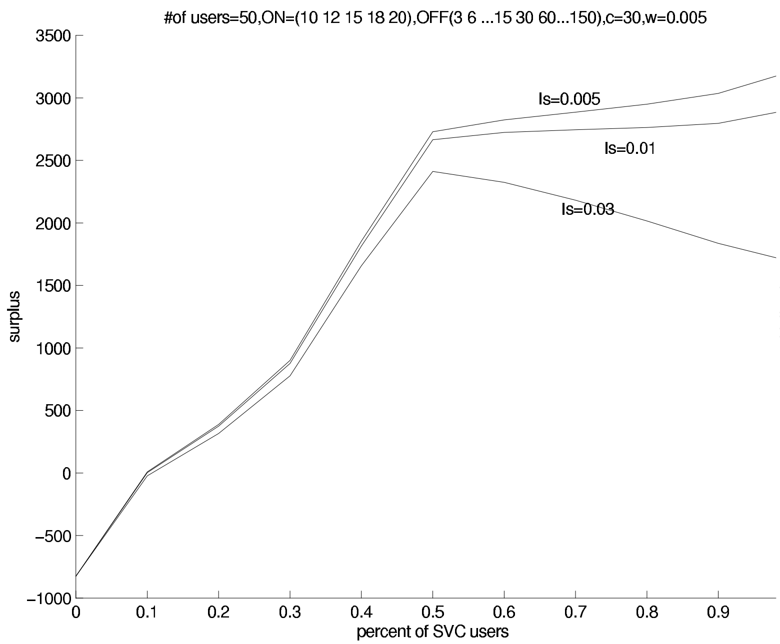
- The procedure of searching for optimal surplus:
  - For each demand scenario, find optimal prices
  - Search through all the possible demand scenarios for the one that maximizes the surplus
  - Procedural details in thesis

A “Realistic” Test Case

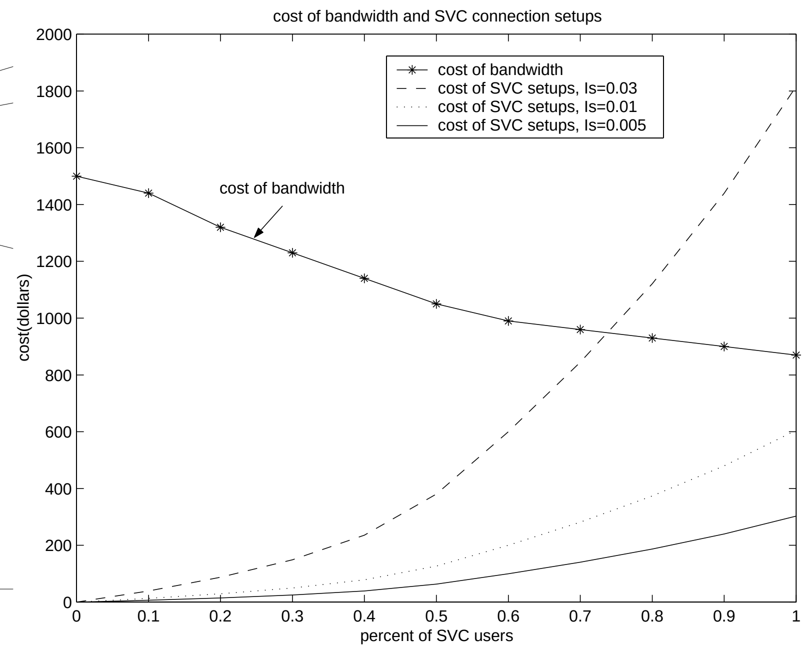
- Parameters for web-browsing application
  - *bw*: 1 Mb/s
  - *X*: 10, 12, 15, 18, 20 minutes
  - *Y*: 3, 6, 9, 12, 15, 30, 60, 90, 120, 150
  - *c*: $30 per Mb/s per month
  - *T*: 1 month
  - *w*: $0.005/Mb
  - I_p: 0
  - I_s:variable

Results of the Test Case

• Results are sensitive to SVC setup cost I_s



Test case surplus



Test case costs

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Differential Pricing for Differentiated Services

- Objectives
  - Packet-oriented network
  - pricing based on assigned priority
  - comparing a differential pricing scheme with a uniform pricing scheme
- Adopt Game theory approach
  - non-cooperative
  - user's surplus is a function of the performance of the selected service, and affected by the others' choices
  - Nash equilibrium point is the predicted outcome of a "game"
    - unilateral deviation does not help any user improve his performance

Network Model

- Single trunk network
- N users
- Service discipline

  Two priority classes, high and low, FIFO in each class

- Pricing scheme

  $P_i = p_{c(i)} \cdot$ average number of packets served in time T

  Where: $c(i)$ is the service choice made by customer i;

  $p_{c(i)}$ is the price per packet of the service class chosen by customer i,

  T is the billing interval.

  simplify to: $P_i = p_{c(i)} \cdot \lambda_i$

  Where λ_i is the arrival rate of user i' s packets.

- Uniform pricing scheme: p
- Differential pricing scheme: p_1, p_2

User’s Model

- Traffic: a Poisson process with arriving rate λ.

  The average service time for each packet is x, and

  $\overline{x^2}$ is the second moment of the average service time

- Surplus function

  $$C_i = U_i - P_i$$

  Where: U_i is the utility function

  P_i is the charges of the service

  Utility function:

  $$U_i = \lambda(A - B_i \cdot W_i)$$

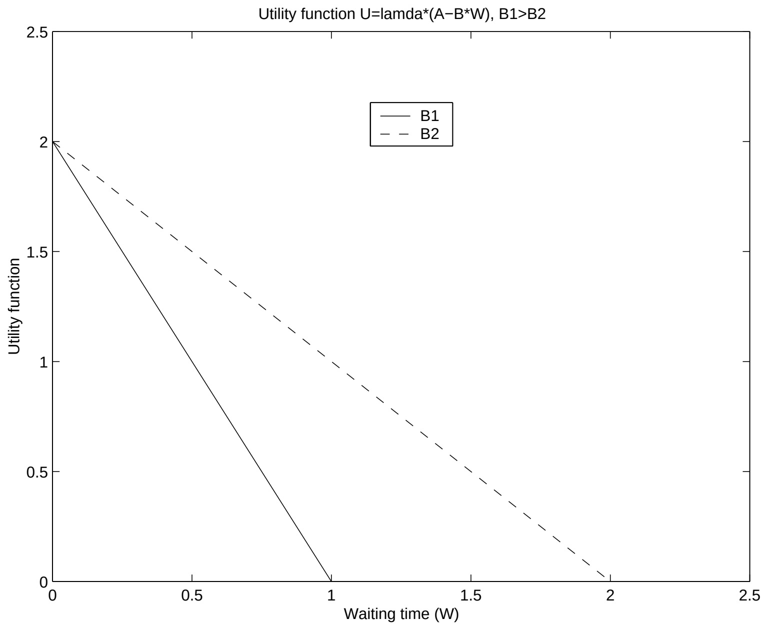
  Where: $A\lambda$ is the upper bound of the amount of money the user is willing to pay for the service;

  W_i is the waiting time experienced by user i;

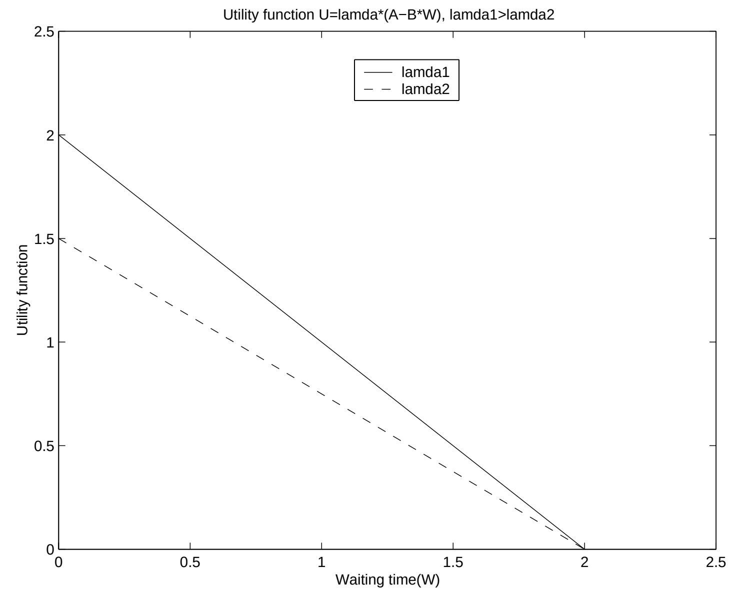
  B_i is a coefficient reflecting the effect of the delay time on user i’s benefit function.

- $C_i \geq 0$

Example Utility Functions



Same λ, different B_i



Same B , different λ

Optimal Revenue Under the Uniform Pricing Scheme

- Uniform pricing scheme

$$P = p \cdot \lambda, \text{ for all users}$$

- Optimization Problem

  Maximize: $\sum_{i}^{N} p \cdot \lambda = N \cdot p \cdot \lambda$

  Subject to: $(A - B_i \cdot W)\,\lambda - p \cdot \lambda \geq 0$, $\forall$i $=1,2,\ldots,N$

- Solution:

  $p = A - B_{\max} \cdot W$

  where: B_{max} is the maximum value of the B_i.

- Optimal revenue: $N \cdot (A - B_{\max} \cdot W) \cdot \lambda$

Optimal Pricing Under Differential Pricing Scheme

- N_1 the number of the users choosing the high priority class
  - — if $N_1 = N$, the optimal unit prices are the same as that of uniform pricing scheme.
  - — we consider the case when $N_1 < N$.

- Two-stage solution strategy
  - — find the optimal unit prices that maximize the revenue for every value of N_1 from 1 to N-1
  - — find the optimal N_1 by searching the revenues from N_1=1 to N

Conditions for Nash Equilibrium at Given N_1:

- User in high priority class:

$$\lambda(A - B_i W_1) - p_1\lambda \geq \lambda(A - B_i W_{2,+i}) - p_2\lambda \qquad i=1,\ldots N_1$$

- User in low priority class:

$$\lambda(A - B_j W_2) - p_2\lambda \geq \lambda(A - B_j W_{1,+j}) - p_1\lambda \qquad j=1,\ldots N_2$$

Where: $W_{1,+j}$ the waiting time of the high priority class when user j changes his choice from low priority to high priority and all the others remain unchanged

$W_{2,+i}$ is the average waiting time of the low priority class when user i alone changes his choice from high priority to low priority

Optimal Problem for Given N_1

Maximize: $\sum_{i=1}^{N_1} p_1\lambda + \sum_{j=1}^{N_2} p_2\lambda = N_1 p_1\lambda + N_2 p_2\lambda$

Subject to:

$p_1 \le A - B_{1\max} W_1$ $\quad (p_{1max} = A - B_{1max} W_1)$,

$p_2 \le A - B_{2\max} W_2$ $\quad (p_{2max} = A - B_{2max} W_2)$

$(p_1 - p_2) \ge B_{2\max}(W_2 - W_{1,+j})$ $\quad ((p_1 - p_2)_{min} = B_{2max}(W_2 - W_{1,+j}))$

$(p_1 - p_2) \le B_{1\min}(W_{2,+i} - W_1)$ $\quad ((p_1 - p_2)_{max} = B_{1min}(W_{2,+i} - W_1))$

Where: $B_{1\max}$ is the maximum value of B_i among the users choosing high priority class,
$B_{1\min}$ is the minimum value of B_i among the users choosing high priority class
$B_{2\max}$ is the maximum value of B_j among the users choosing low priority class.

Results: Optimal Prices: $p_{1optimal}$ and $p_{2optimal}$

- **Case 1**: if $(p_1 - p_2)_{min} \leq p_{1max} - p_{2max} \leq (p_1 - p_2)_{max}$

  $p_{1optimal} = p_{1max}$ and $p_{2optimal} = p_{2max}$.

- **Case 2**: if $p_{1max} - p_{2max} < (p_1 - p_2)_{min}$

  $p_{1optimal} = p_{1max}$ and $p_{2optimal} = p_{1max} - (p_1 - p_2)_{min}$

- **Case 3**: if $p_{1max} - p_{2max} > (p_1 - p_2)_{max}$

  $p_{1optimal} = p_{2max} + (p_1 - p_2)_{max}$ and $p_{2optimal} = p_{2max}$

Results: Differential vs. Uniform Pricing

- If N_1=N, the two pricing schemes are the same;
- If N_1<N, then:

  if $$B_{2\max} < B_{1\max}(1 - N_1\lambda x) + B_{1\min} N_1 \lambda x\{1 - \frac{1}{N[1-(N_1-1)\lambda x]}\}$$

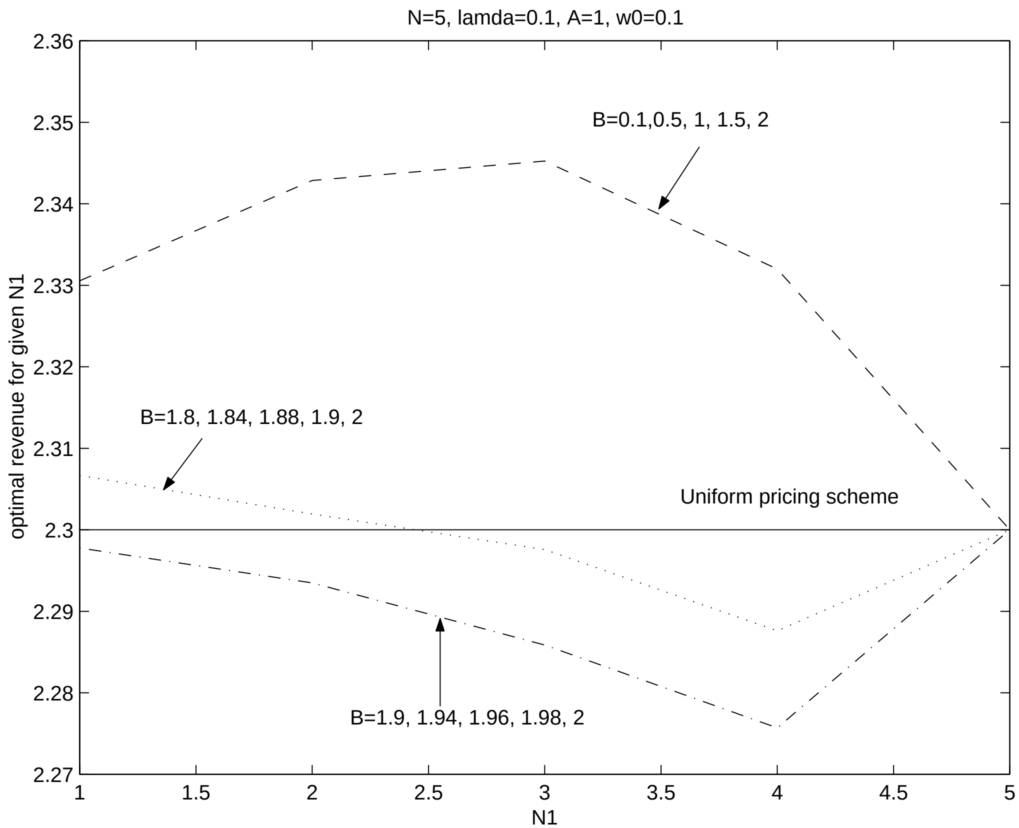
  the revenue raised by the differential pricing scheme is greater than the uniform pricing scheme. Otherwise, the revenue raised by the uniform pricing scheme is greater than the differential pricing scheme.

- Interpretation:

  If users' performance requirements are sufficiently differentiated, the differential pricing scheme will raise more revenue than the uniform pricing scheme.

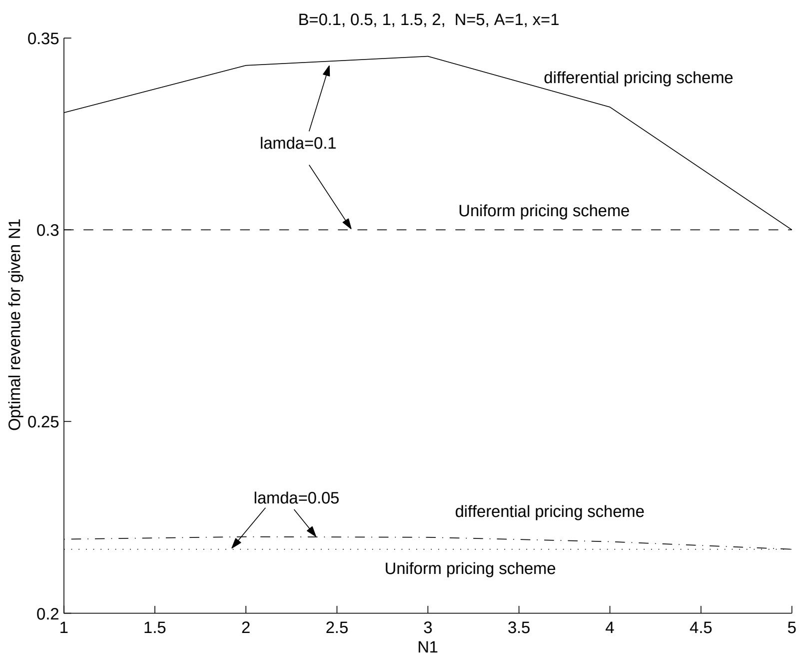
Results: Effect of User's Performance Requirements

- Large spread of performance requirements favors differential pricing

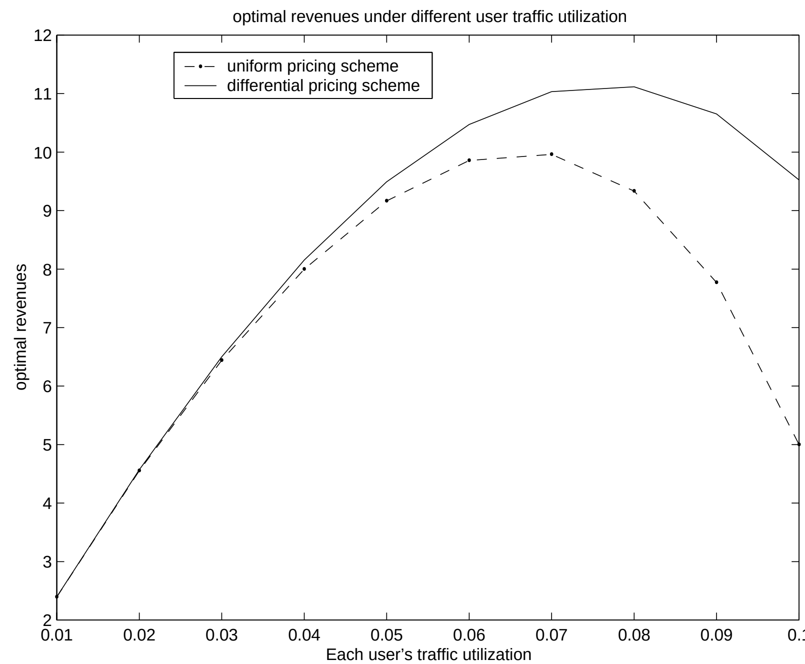


Results: Effect of User's Traffic Demands

- Large traffic demands favor differential pricing



effect of user's traffic demands



Optimal revenue vs. user's traffic demands

Summary of Contributions

- Connection-oriented networks
  - Formulated a pricing scheme and analysis model to determine the effect of pricing on SVC and PVC services
  - Formulated and solved the problem of maximizing an ATM service provider's net revenue through proper choice of SVC vs. PVC pricing parameters
    - Results are highly sensitive to SVC unit setup charges
- Packet-oriented networks
  - Formulated and solved the problem of maximizing an Internet service provider's net revenue through proper choice of priority unit prices
  - Demonstrated that differential pricing is superior to uniform pricing if the user's delay sensitivities are sufficiently different
    - Advantage of differential pricing increases with increasing traffic

Thank you for your time!

Components of a Service Charge

- Subscription part
  - reflects the fact that the user has his own connection
- Traffic part
  - depends on the number of calls, bandwidths, durations and QoS requirements
- In this thesis we concentrate on the traffic part

Peak Rate Bandwidth Allocation-Uniform Distribution

- "*Off*" periods are uniformly distributed in the range of [0, *Z*].
  - So, Y= $\frac{Z}{2}$

- User cost function is:

$$E\{cost\} = E\{Usagecharges\} + \frac{T}{X+Y}\{s[1-\frac{\tau}{Z}] + a\{X + \frac{\tau}{2}\frac{\tau}{Z} + \tau(1-\frac{\tau}{Z})]\} =$$

$$E\{UsageCharges\} + [s + aX - \frac{a\tau^2}{2Z} + (aZ - s)\frac{\tau}{Z}]\frac{T}{X+Y}$$

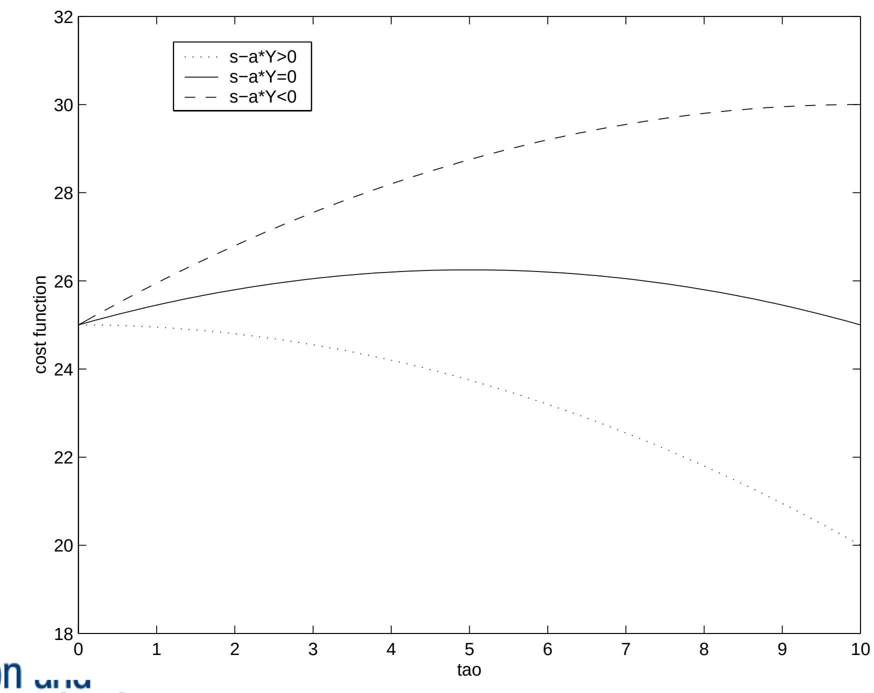
- The minimum point occurs at the edge points of the range of τ.

$$E\{Cost\}_{edge} = \begin{cases} E\{Usage\} + [s + aX]\frac{T}{X+Y}, \; for \quad \tau = 0; \\ E\{Usage\} + [s + aX + \frac{a \cdot Z}{2} - s]\frac{T}{X+Y}, \; for \quad \tau = Z. \end{cases}$$

Results

The minimum points are:

- if $s-\frac{a \cdot Z}{2} > 0$, *i.e.*, $\frac{s}{a} > Y$, the minimum point occurs at τ = Z (preference for PVC service).
- if $s-\frac{a \cdot Z}{2} < 0$, *i.e.*, $\frac{s}{a} < Y$, the minimum point occurs at τ = 0 (preference for SVC service).
- if $s-\frac{a \cdot Z}{2} = 0$, *i.e.*, $\frac{s}{a} = Y$, the cost function has the same value at the edge points of range of τ (no service preference).

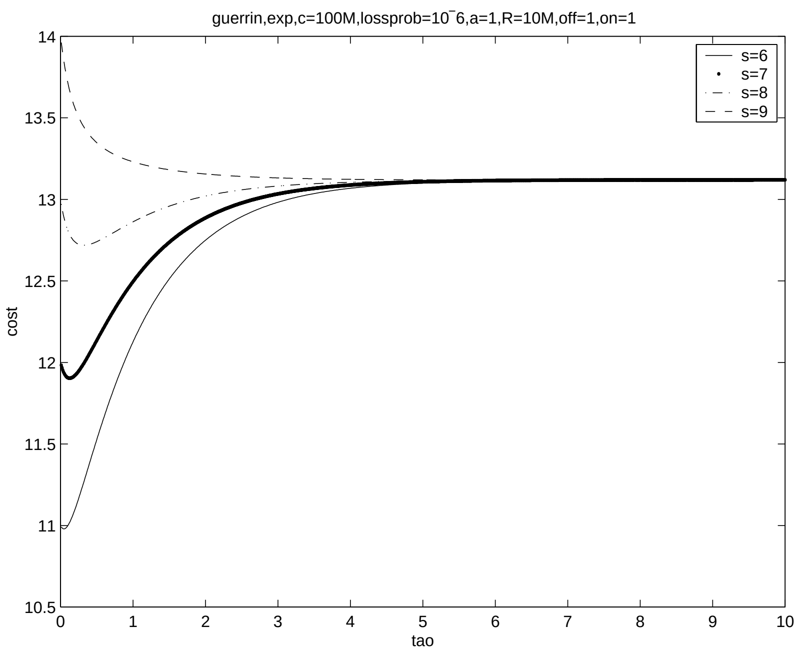


Effective Bandwidth Allocation

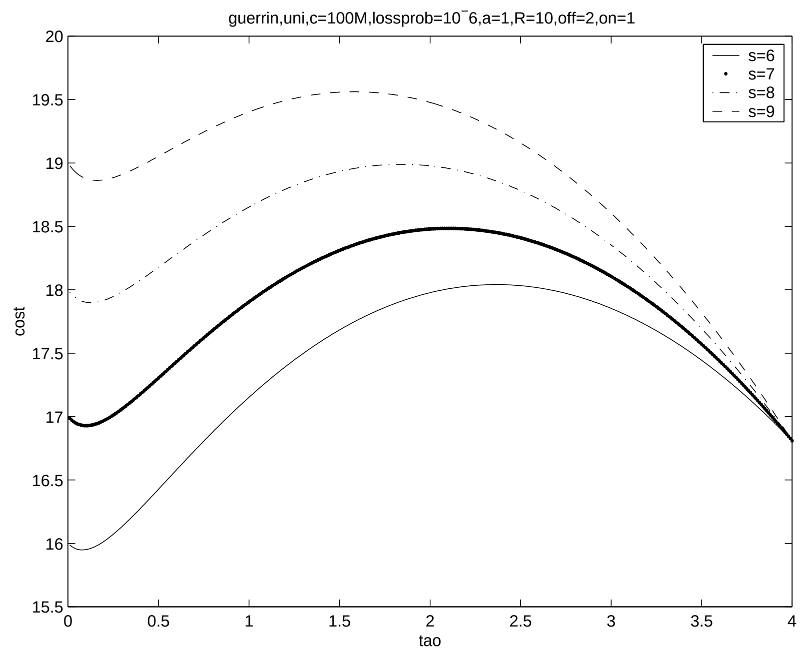
- Bandwidth allocated according to user's effective bandwidth
  - — Effective bandwidth is determined by the traffic parameters
- We used Guerrin's method for effective bandwidth
- For both exponential and uniform Off-time distributions, there are no symbolic results for the value of τ (τ^*) that minimizes the cost function
- We plot the cost functions versus τ for different values of *s* and fix other parameters

Effective Bandwidth Allocation—Results

- τ* can lie between extreme values
  - — Unlike peak rate allocation case
  - — But τ* close to 0 in these cases

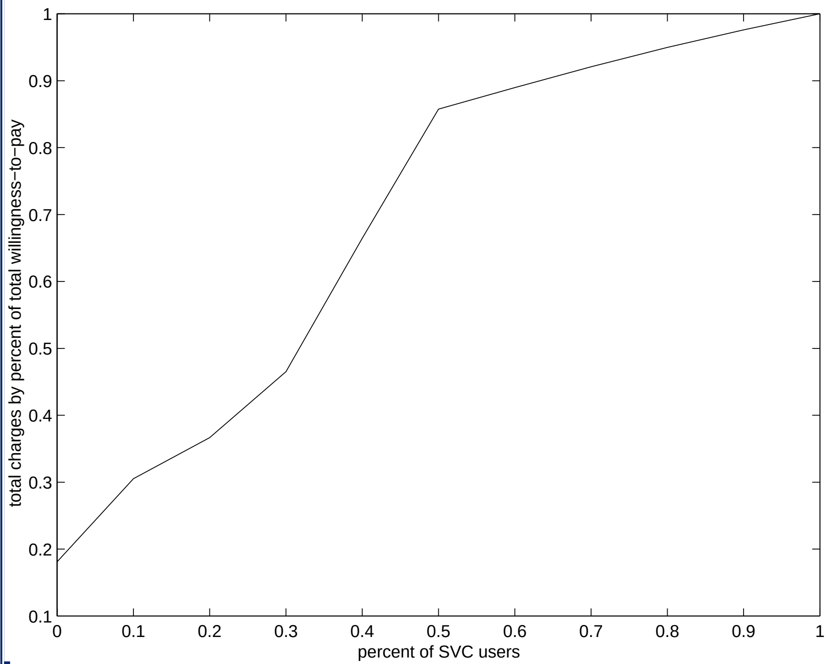


Guerin method: Cost function curves for exponential distribution

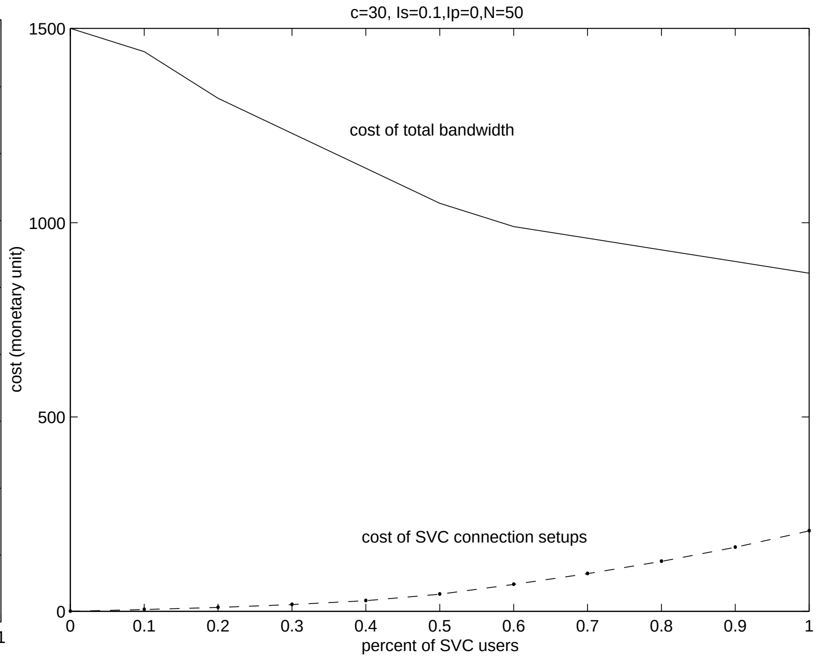


Guerin method:Cost function curves for uniform distribution

Provider Revenues and Costs for c=30, I_s =0.1



Total charges for the services



Cost of bandwidth and connection setups

Effect of Traffic Pattern for c=10, I_p=0, I_s=0.01

- Results are sensitive to spread in mean *Off* times Y_i