Merging of Bursty Traffic in Weakly Stable Markovian Networks

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Outline

- Introduction
- Background
- Modeling Techniques
- Modeling Backbone traffic
- Results and Insights
- Conclusions
- Future work

Introduction

- Traffic in LAN is often "bursty" in nature
- Burst - *In data communications, a sequence of signals counted as one unit in accordance with some specific criterion or measure -* *http://business.cisco.com/glossary/*
- Effects of burstiness reflected in poor QOS for network clients.

Introduction

Why?

Limited buffering available in network.

How?

- Rapid filling of buffer space followed by slower draining as traffic calms
- Excessive packet loss in both access networks and backbone networks
- Delivery delays due to various effects, retransmissions for instance
- Large queuing delays due to full buffers and the time it takes to empty

Introduction

Did anyone notice this?

– Yes, much effort expended to determine analytical explanation of the causes of this random burstiness and its effect upon the buffering medium.

– Effect of burstiness on a single LAN well understood.

Potential research area

Little work done to analyze how the burstiness, filtered through the buffering medium, affects the backbone network performance.

Introduction

Our contribution

- Examine the effect of merging the output of several access networks with bursty traffic, onto a backbone network.
- Propose to do this analytically using dependent matrix-exponential queuing network models, developed recently.

Reasons

Exploration of merging of these models is expected to yield insight into what the merging of bursty streams of traffic does to the performance of backbone queues

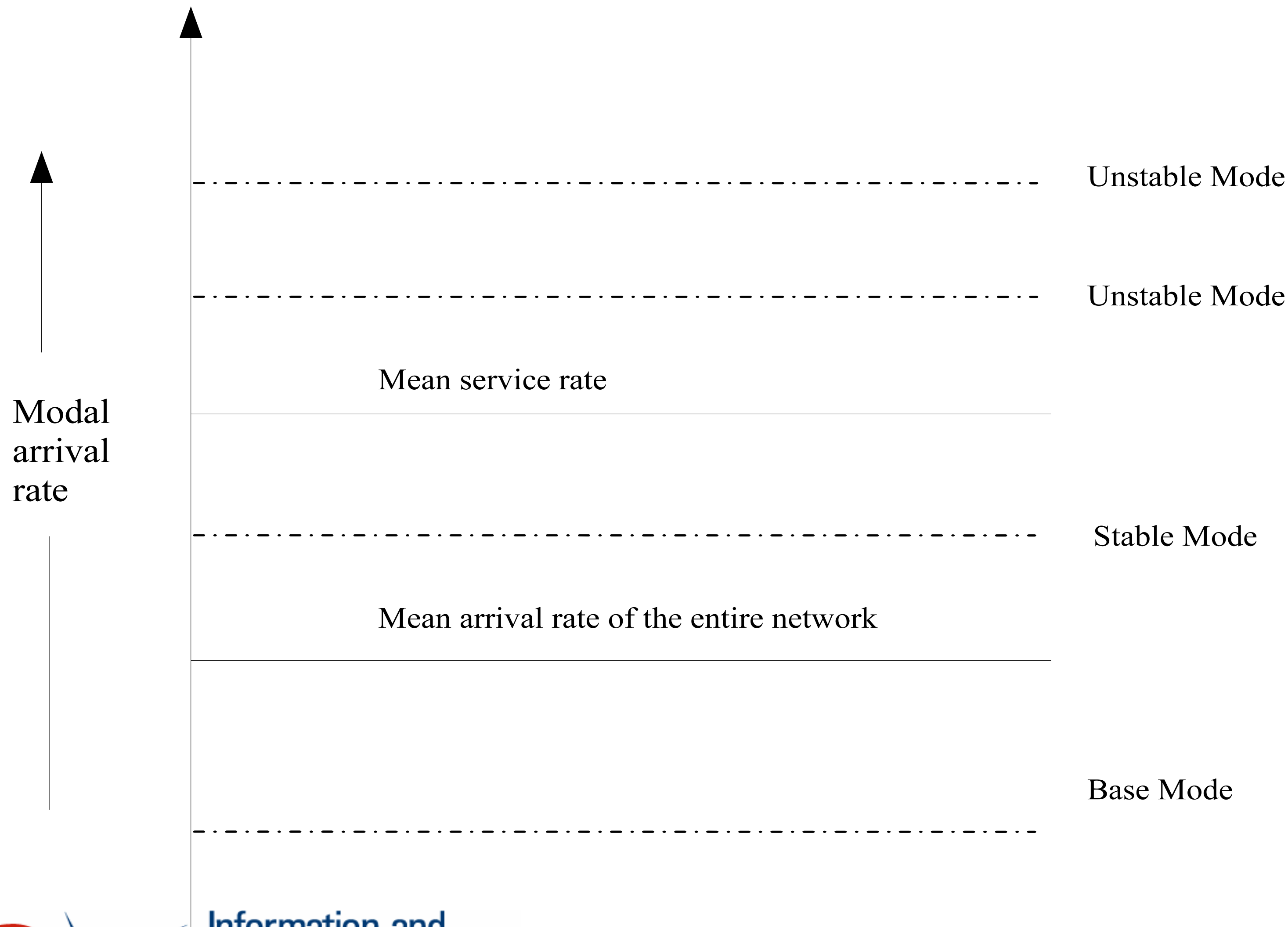
Background

- Using dependent matrix-exponential (MED) queuing models.
- Complex models are developed, involving many dimensions of state space, hence we use Kronecker operations and hat spaces.
- Exploit concepts of nearly completely decomposable (NCD) matrix exponential (ME) modes to represent traffic.
- Input traffic stream is made up of multi-time-scale strongly dependent modes of traffic, each with different average traffic rates, switched among by slower process.
- According to Jelenkovic, two kinds of modes are present – stable & unstable.

Modes

- Unstable mode:
  - Average arrival rate > Average service rate
- Stable mode:
  - Average arrival rate < Average service rate
- If one or more NCD classes of states have arrival rates > average arrival rate, then they are called *burst mode*.
- Time spent on burst modes << time spent on base mode.

Fig 1: Modes in traffic



Weak Stability

- Weakly stable state:
  - Average arrival rate < Average service rate BUT there exists one unstable mode.
- Weak stability explains buffer overflow even with large buffers.
- Weakly stable state signifies that though whole process is stable, it sometimes becomes unstable from time to time.
- To strike balance between utilization and buffer size, new models are needed.

MED – Marginals and Covariances

- ME – defined as probability distribution represented as $(\mathrm{p}, \mathrm{B}, \varepsilon')$

$$F(t) = 1 - \mathrm{p}exp(-\mathrm{B}t)\varepsilon', t \geq 0,$$

- Probability density function is defined as

$$f(t) = \frac{dF(t)}{dt} = \mathrm{p}exp(-\mathrm{B}t)\mathrm{B}\varepsilon'$$

- The covariance of a sequence of MEs

$$cov[X_n, X_{n+k}] = \mathrm{pV}(\mathrm{Y})^k\mathrm{V}\varepsilon' - (\mathrm{pV}\varepsilon')^2$$

Where,

- p is the equilibrium starting vector, such that p=pY
- ε' is the summing operator
- $\mathrm{V} = \mathrm{B}^{-1}$
- $\mathrm{Y} = \mathrm{VL}$

Kronecker product - $\otimes$

- Complex models developed involve combining many state spaces.
- Improve algebraic intuition and effectiveness.
- In order to preserve the independence of each space, the operators in the disjoint spaces are combined in the system space using the Kronecker product.
- Kronecker product of two matrices, K_1 and K_2 is

$$\mathbf{K} = \mathbf{K_1} \otimes \mathbf{K_2} = \begin{bmatrix} (\mathbf{K}_1)_{11}\mathbf{K}_2 & \dots & (\mathbf{K}_1)_{1n_2}\mathbf{K}_2 \\ \vdots & \ddots & \vdots \\ (\mathbf{K}_1)_{n_1 1}\mathbf{K}_2 & \dots & (\mathbf{K}_1)_{n_1 n_2}\mathbf{K}_2 \end{bmatrix}$$

Understanding Kronecker operations

- $\mathbf{K}_1 \otimes \mathbf{K}_2$, signifies simultaneous transitions in both spaces.
- $\mathbf{I}_1 \otimes \mathbf{K}_2$, the transitions in ‘2’ space extended to ‘1’ space.
- $\mathbf{K}_1 \otimes \mathbf{I}_2$, the transitions in ‘1’ space extended to ‘2’ space.

Hat spaces

- As dimensions increase, Kronecker representation gets more complicated.

- For a more intuitive representation, hats are introduced.

- Example: If matrix in space 1 is extended to N spaces

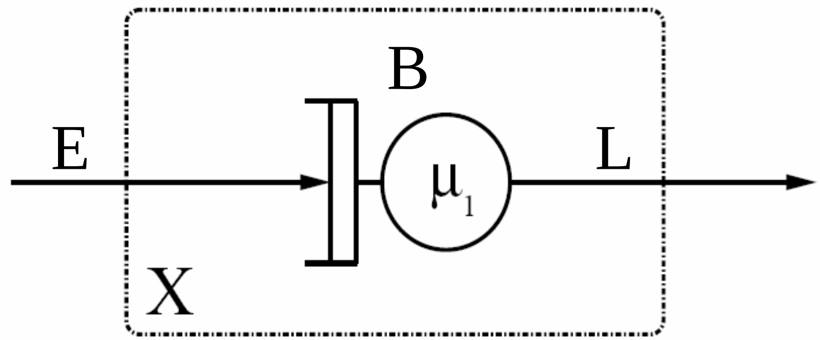
$$\widehat{\mathbf{K}}_1 = \mathbf{K}_1 \otimes \mathbf{I}_2 \otimes \mathbf{I}_3 \ldots \otimes \mathbf{I}_N$$

Modeling Techniques

Each access queue is represented by a multi-dimensional stochastic process consisting of five state spaces.

- Mode space – identifies modes the process is in.
- Arrival space – generates MED inter-arrival times within current mode.
- Burst duration space – generates MED duration of mode before change
- Queue space – identifies the number of packets in buffer queue.
- Service space – generates MED service time distribution.

Network Description



μ1 – service rate

B matrix

- Represents the changes in state which do not result in generation of an output event.
- The effect is *autogenous*, within the module.
- Usually called the progress rate matrix.

$$B_X = \begin{bmatrix} 0 & & & \\ 0 & \mu_1 & & \\ & \ddots & \ddots & \ddots \end{bmatrix}$$

Network Description

L matrix

- Represents changes in state which result simultaneously, in generation of an output event.
- The effect is *endogenous*, specific to the module affecting the world outside the module.
- Usually called the event rate matrix.

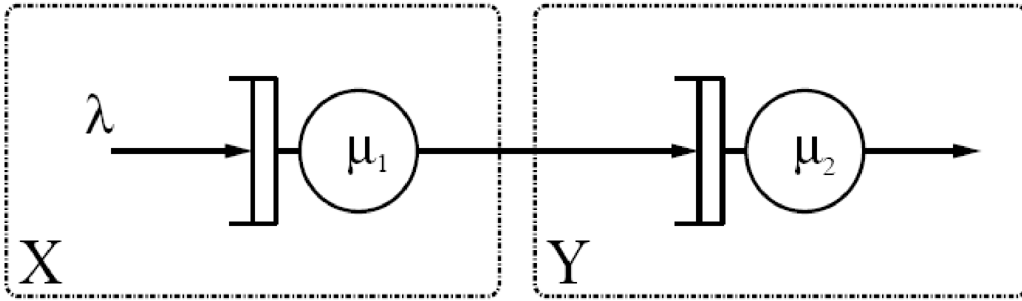
$$\mathrm{L}_X = \begin{bmatrix} 0 & & \\ \mu_1 & 0 & \\ & \ddots & \ddots \end{bmatrix}$$

E matrix

- It's a new matrix introduced and is called the event transition matrix.
- Represents changes in state resulting from input events & denoted by a probability matrix.
- The effect is *exogenous*, i.e. the effects inside the module are as a result of the events from outside the module.

$$\mathrm{E}_X = \begin{bmatrix} 0 & 1 & & \\ 0 & 0 & 1 & \\ & \ddots & \ddots & \ddots \end{bmatrix}$$

Simple Network



Simple Cascaded Network

- Effect of module X felt on module Y
- Autogenous events extend into the joint space. Hence $\hat{B}$.
- Endogenous transitions join with simultaneous exogenous events of connected module, $\hat{L}$ and $\hat{E}$.

The XY space

- Kronecker representation in <X,Y> ordered state space.

$$\mathbf{B}_{XY} = (\mathbf{B}_X \otimes \mathbf{I}_Y) + (\mathbf{I}_X \otimes \mathbf{B}_Y) - (\mathbf{L}_X \otimes \mathbf{E}_Y)$$

$$\mathbf{E}_{XY} = \mathbf{E}_X \otimes \mathbf{I}_Y$$

$$\mathbf{L}_{XY} = \mathbf{I}_X \otimes \mathbf{L}_Y$$

- Kronecker representation in <Y,X> ordered state space, for better insight

$$\mathbf{B}_{YX} = (\mathbf{B}_Y \otimes \mathbf{I}_X + \mathbf{I}_Y \otimes \mathbf{B}_X - \mathbf{E}_Y \otimes \mathbf{L}_X)$$

$$\mathbf{I}_Y \otimes \mathbf{B}_X = \begin{bmatrix} \mathbf{B}_X & & \\ 0 & \mathbf{B}_X & \\ & \ddots & \ddots \end{bmatrix} \quad \mathbf{B}_Y \otimes \mathbf{I}_X = \begin{bmatrix} 0 & & \\ -\mu_2 I_x & \mu_2 I_x & \\ & \ddots & \ddots \end{bmatrix} \quad \mathbf{E}_Y \otimes \mathbf{L}_X = \begin{bmatrix} 0 & \mathbf{L}_X & \\ & 0 & \mathbf{L}_X \\ & & \ddots & \ddots \end{bmatrix}$$

Solution for MED networks

- Computation of the infinitesimal generator matrix, Q

$$Q_{XY} = L_{XY} - B_{XY}$$

$$Q_{XY} = \begin{bmatrix} -\mathrm{B}_X & \mathrm{L}_X & & & \\ \mu_2 \mathbf{I}_X & -(\mathrm{B}_X + \mu_2 \mathbf{I}_X) & \mathrm{L}_X & & \\ 0 & \mu_2 \mathbf{I}_X & -(\mathrm{B}_X + \mu_2 \mathbf{I}_X) & \mathrm{L}_X & \\ & \ddots & \ddots & \ddots & \ddots \end{bmatrix}$$

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On expanding the Q matrix

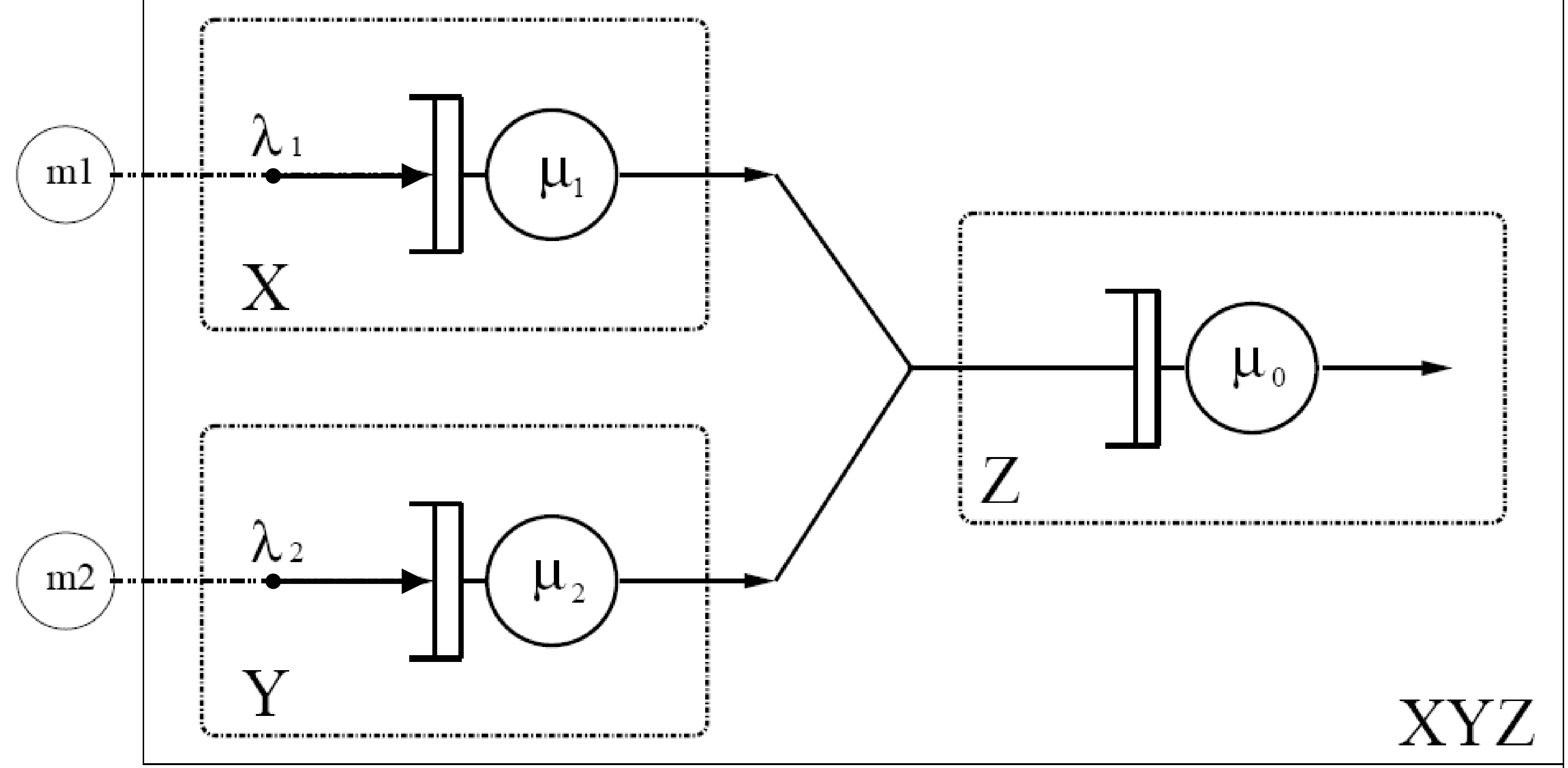
$$\mathbf{Q} = \begin{bmatrix} -\lambda & \lambda & 0 & \cdots & 0 & & & & & \\ 0 & -[\lambda+\mu_1] & \lambda & \cdots & \mu_1 & 0 & & & & \\ 0 & 0 & -[\lambda+\mu_1] & \ddots & 0 & \mu_1 & 0 & & & \\ 0 & 0 & 0 & \ddots & 0 & 0 & \ddots & \ddots & & \\ \mu_2 & 0 & 0 & 0 & -[\lambda+\mu_2] & \lambda & 0 & \cdots & 0 & \\ 0 & \mu_2 & 0 & 0 & 0 & -[\lambda+\mu_1+\mu_2] & \lambda & \cdots & \mu_1 & 0 \end{bmatrix}$$

Lossy Finite Queue

- For finite buffers, losses occur.
- This modifies the 'E' matrix.
- In contrast to the infinite queues, the last term of the E matrix is a 1 since the buffer size is finite.

$$\mathbf{B}_q = \begin{bmatrix} 0 & & & \\ 0 & \mu & & \\ 0 & 0 & \mu & \\ 0 & 0 & 0 & \mu \end{bmatrix}, \mathbf{L}_q = \begin{bmatrix} 0 & & & \\ \mu & 0 & & \\ 0 & \mu & 0 & \\ 0 & 0 & \mu & 0 \end{bmatrix}, \mathbf{E}_q = \begin{bmatrix} 0 & 1 & 0 & \\ 0 & 0 & 1 & \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Modeling backbone traffic



Network with modes

Model description

- *Insight invoking* model used.
- Two access networks and one backbone network.
- Restricted, by choice to 2 modes in the access networks when merged with similar access networks to give 4 modes in the backbone.
- *Incoming* traffic into access network is *exponential* per mode but *multi-modal* with *multi-time-scales*.
- Burst durations are *exponential*, per mode.
- *Output* traffic from *access* networks, merged to become the *input* traffic to the *backbone* network.
- *Merging* of traffic can result in the *polynomial increase* in number of modes.

Mathematical solution – Mode algebra

$\overline{\delta_0}$ - Infinitesimal rate of occurrence of shift from *base mode to any of the burst modes.*

$\overline{\delta_i}$, $i = 1,2,3$ - Rate of occurrence of the shift from *base to burst mode.*

$\underline{\delta_i}$, $i = 1,2,3$ - Rate of occurrence of the shift from *mode i to the base mode*

$$\overline{\delta_0} = \overline{\delta_1} + \overline{\delta_2} + \overline{\delta_3}$$

$$\mathbf{B}_m = \begin{bmatrix} \overline{\delta_0} & 0 & 0 & 0 \\ 0 & \underline{\delta_1} & 0 & 0 \\ 0 & 0 & \underline{\delta_2} & 0 \\ 0 & 0 & 0 & \underline{\delta_3} \end{bmatrix} \qquad \mathbf{L}_m = \begin{bmatrix} 0 & \overline{\delta_1} & \overline{\delta_2} & \overline{\delta_3} \\ \underline{\delta_1} & 0 & 0 & 0 \\ \underline{\delta_2} & 0 & 0 & 0 \\ \underline{\delta_3} & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{Q}_m = \mathbf{L}_m - \mathbf{B}_m$$

$$\pi_m * \mathbf{Q}_m = 0$$

Contd…

- Overall process rate matrix after merging

$$\mathbf{B}_{XYZ} = \widehat{\mathbf{B}}_X + \widehat{\mathbf{B}}_Y + \widehat{\mathbf{B}}_Z - (\widehat{\mathbf{L}}_X + \widehat{\mathbf{L}}_Y)\widehat{\mathbf{E}}_Z$$

- Event rate matrix for the entire network

$$\mathbf{L}_{XYZ} = \widehat{\mathbf{L}}_Z$$

- Infinitesimal generator matrix

$$\mathbf{Q}_{XYZ} = \mathbf{L}_{XYZ} - \mathbf{B}_{XYZ}$$

- Steady state probability vector is XYZ space

$$\pi_{XYZ}\mathbf{Q}_{XYZ} = 0$$

- Final state probability – arrivals are function of modes.

$$\pi_0 = \pi_m[1]\pi_{XY}(1,1) \qquad \pi_2 = \pi_m[3]\pi_{XY}(2,1)$$

$$\pi_1 = \pi_m[2]\pi_{XY}(1,2) \qquad \pi_3 = \pi_m[4]\pi_{XY}(2,2)$$

Calculation of results

Independent variables – Those that would typically be controlled by us for design purposes

- Buffer capacity of access networks
- Burst arrival rate in the access network
- Burst duration in the access network
- Base arrival rate in the access network
- Base duration in the access network
- Backbone/access network service rate

Calculation of results

Dependent variables – Those that are observed as measurements of performance.

- Network cell loss probability
- Backbone cell loss probability
- Throughput
- Output stream distributions
- Output stream auto-covariances

Calculation of results

- Mean arrival rate of the access network

$$\lambda_{mean} = \frac{burst\ arrival\ rate * burst\ duration + base\ arrival\ rate * base\ duration}{base\ duration + burst\ duration}$$

- The service rate of the access network can be found using

$$\rho = \frac{\lambda_{mean}}{\mu} \Rightarrow \mu = \frac{\lambda_{mean}}{\rho}$$

- Burst to base ratio (BBTR)

$$\begin{aligned} BBTR &= \frac{burst\ duration * burst\ arrival\ rate}{mean\ arrival\ rate * burst\ duration + base\ duration} \\ &= \frac{burst\ duration * burst\ arrival\ rate}{mean\ arrival\ rate * cycle\ time} \\ &= \frac{\lambda(0)/\overline{\delta_0}}{\lambda_{mean} * cycle\ time} \end{aligned}$$

Calculation of results

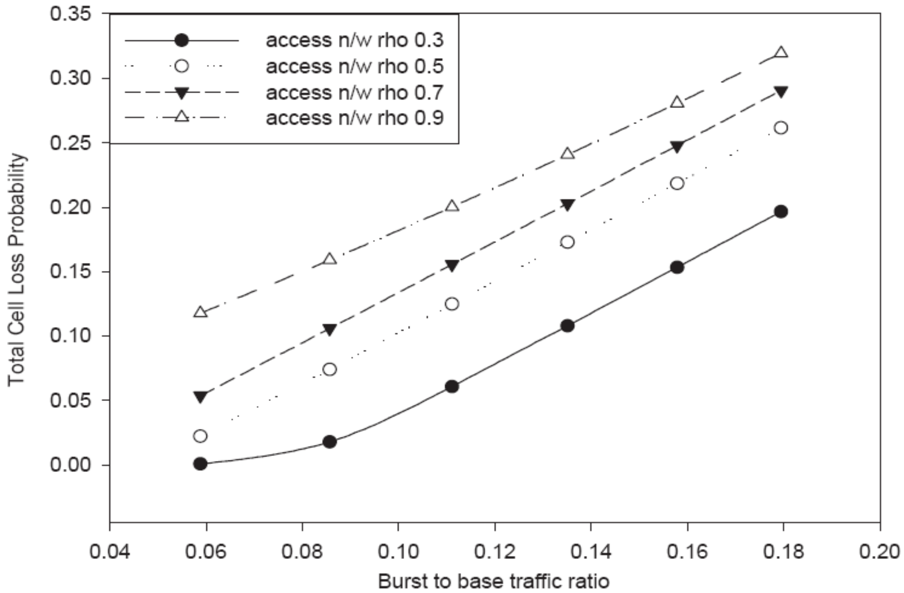
- The BBTR is the ratio of the packets in burst to the total packets
- To increase the BBTR, one of the following must be done
  - Decrease λ_{mean}
  - Decrease cycle time
  - Increase burst duration or burst arrival
- Varying base arrival rate has most significant change to ratio.
- As BBTR increases overall cell loss probability increases (CLP)
- CLP is the probability of arrival occurring while buffer is full.

$$CLP = \frac{\pi_n \mathbf{L}_a \mathbf{e}'}{\sum_i \pi_i \mathbf{L}_a \mathbf{e}'}$$

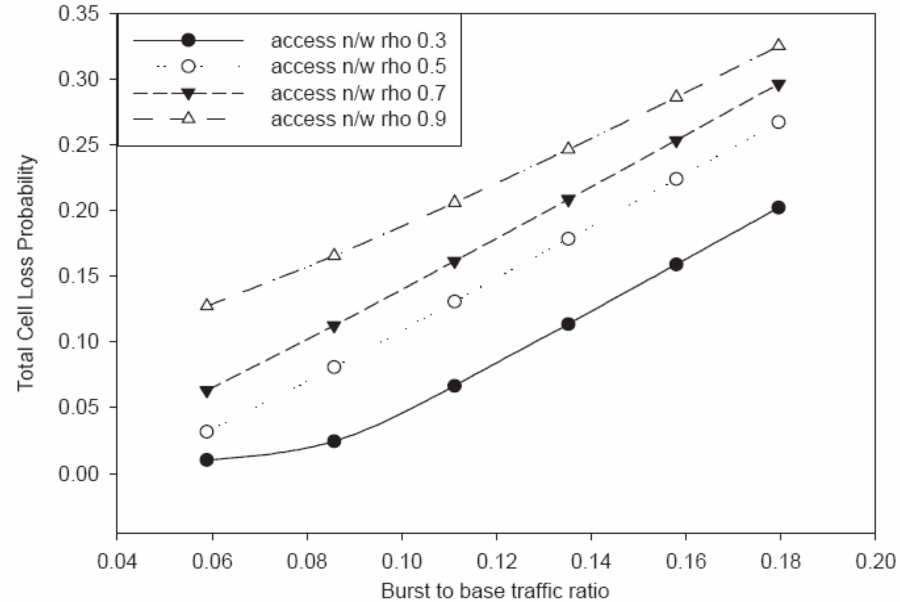
Results and insights

The analysis of the model was classified into the following cases as described below.

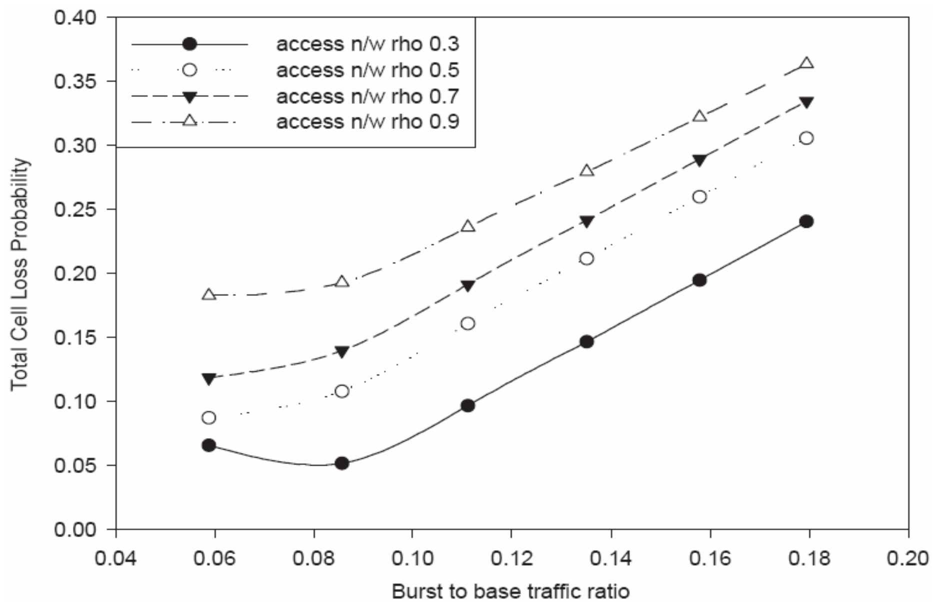
- Case 1 : Sanity check
- Case 2 : Relative contribution by backbone and access network towards changes in CLP
- Case 3 : Significance of backbone parameters in regulating the total CLP.



The service rate of the backbone (muz) is fixed to 3333.33 (pkts/sec). The rho for the access network is varied from 0.3 to 0.9. Notice the increase in the total CLP with the increase in the BBTR.



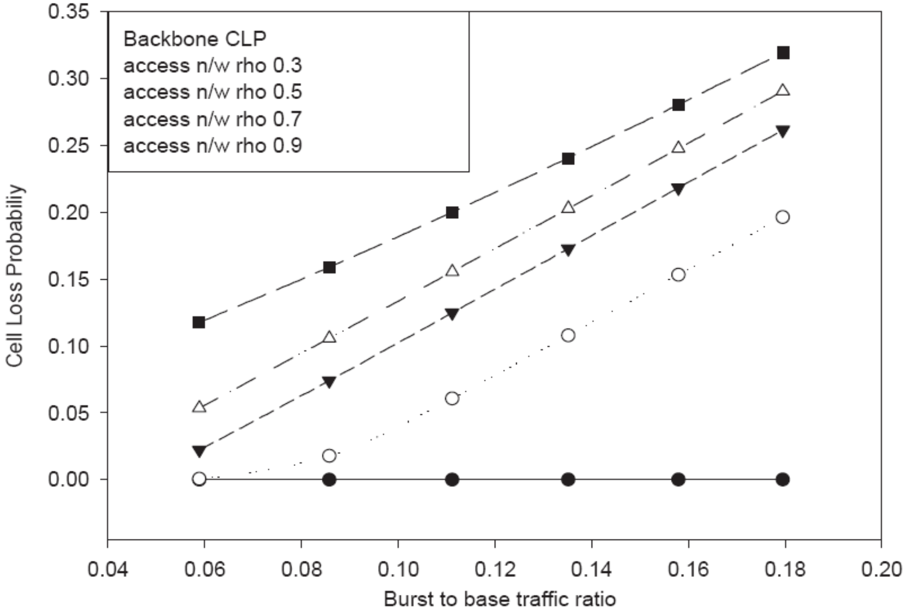
The service rate of the backbone (muz) is fixed to 1666.66 (pkts/sec). The rho for the access network is varied from 0.3 to 0.9. Notice the increase in the total CLP with the increase in the BBTR.



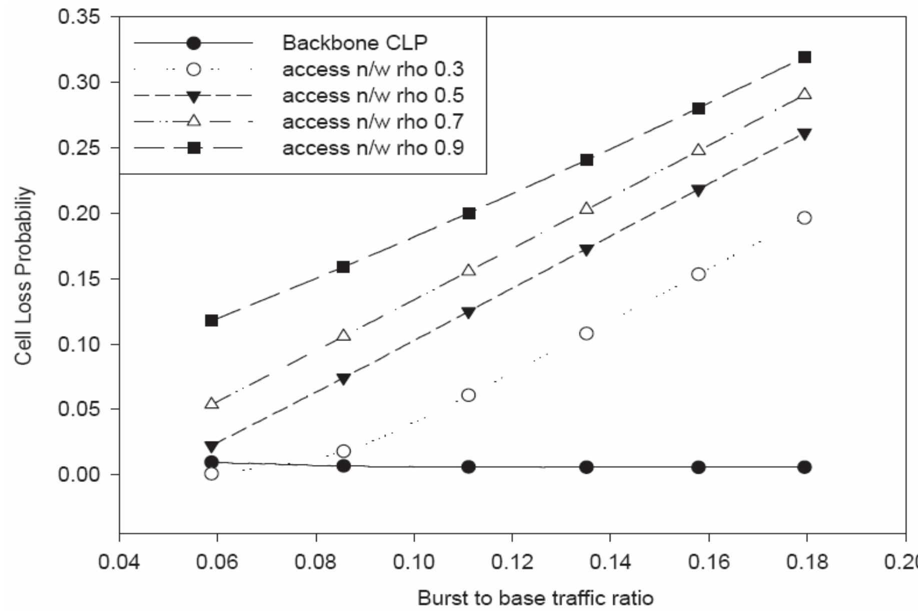
The service rate of the backbone (muz) is fixed to 1111.11 (pkts/sec). The rho for the access network is varied from 0.3 to 0.9. Notice the increase in the total CLP with the increase in the BBTR.

Case 1 Conclusions:

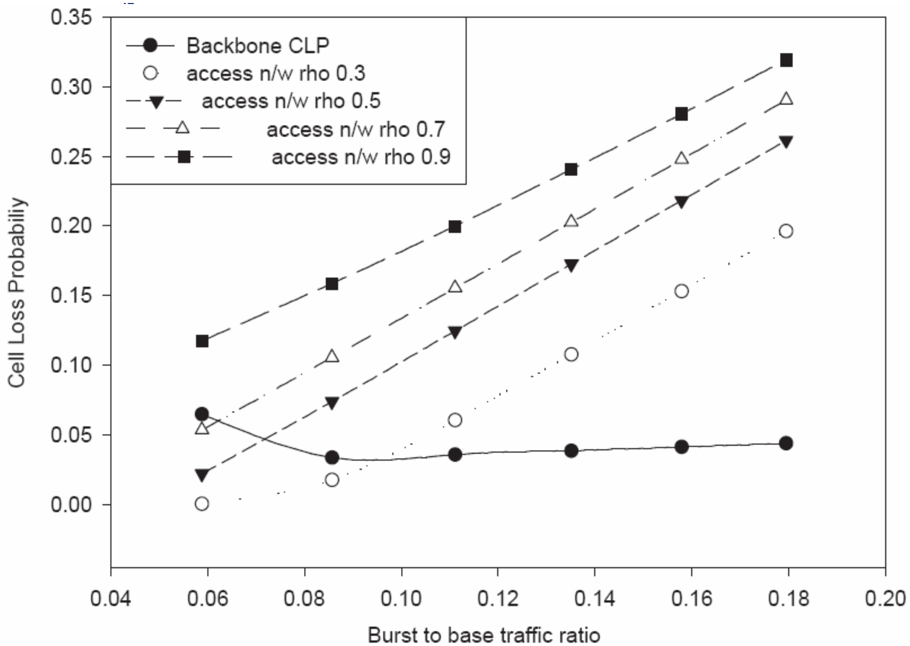
- Keeping backbone service rate fixed, increase in BBTR, increases the CLP as expected.
- Notice the high CLP in access network as the traffic ratio increases.
- As the service rate of the backbone decreases, an upward shift in the curves is noted. Indicating the generic increase of total CLP as backbone service rate decreases.



Backbone rho = 0.3. The rho for the access network is varied from 0.3 to 0.9. Access network CLP increases with BBTR. The backbone CLP reaches a constant value.



Backbone rho = 0.6. The rho for the access network is varied from 0.3 to 0.9. Access network CLP increases with BBTR. The backbone CLP reaches a constant value.

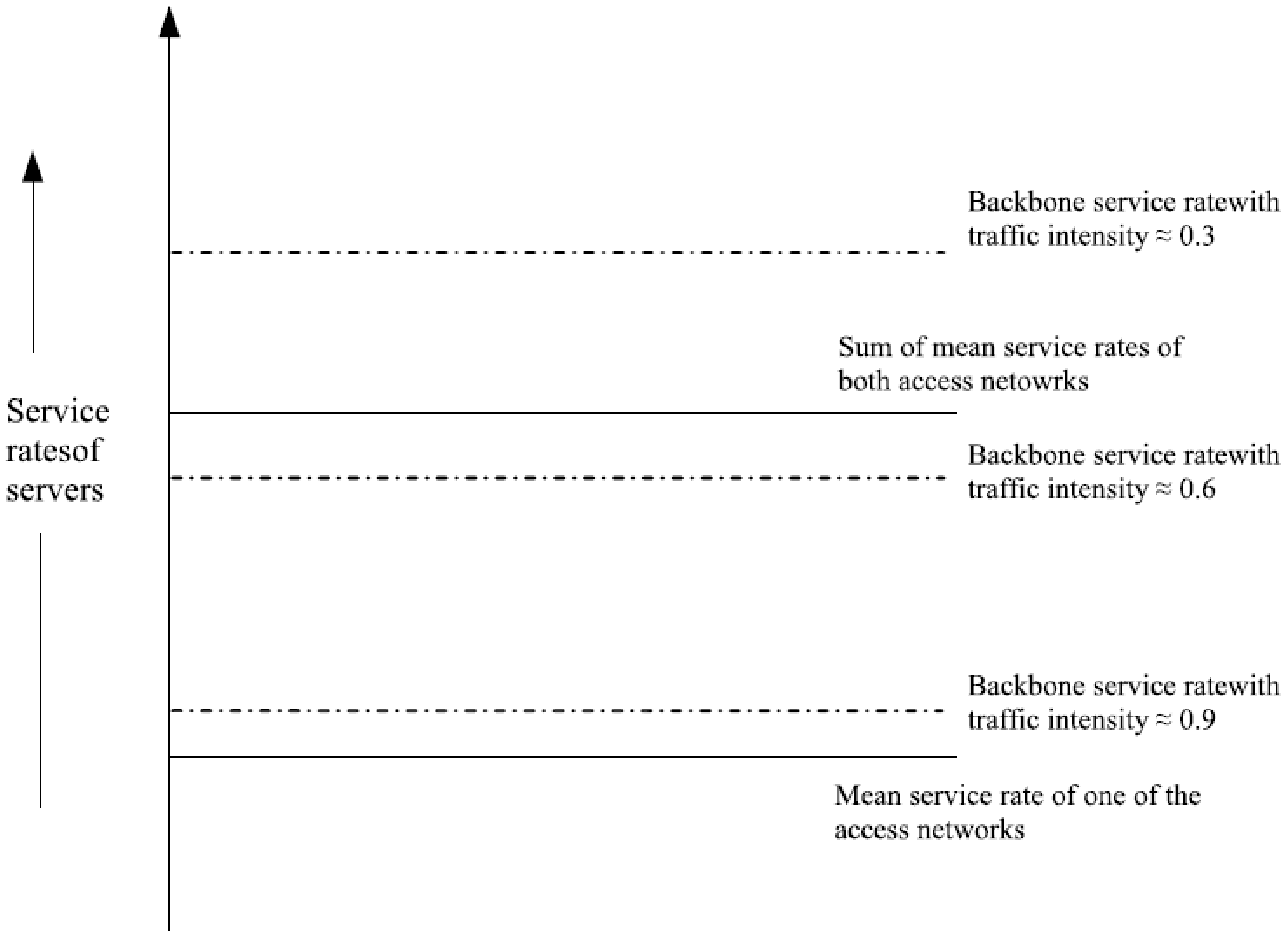


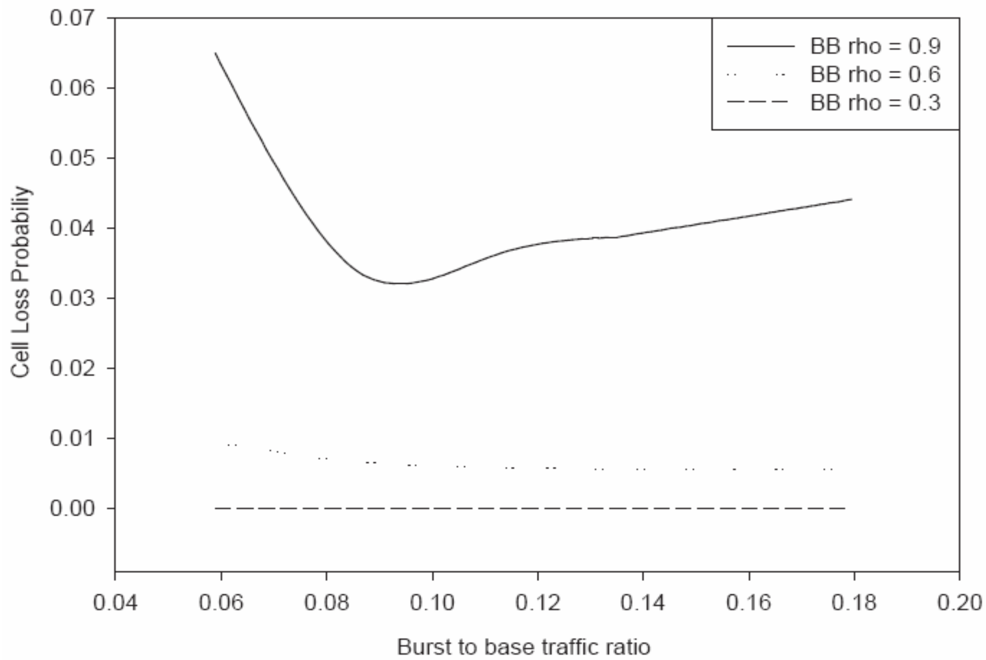
Backbone rho = 0.9. The rho for the access network is varied from 0.3 to 0.9. Access network CLP increases with BBTR. The backbone CLP reaches a constant value.

Case 2 Conclusions:

- The flat curve of the backbone CLP indicates that beyond a certain BBTR, any changes in BBTR has relatively less effect on backbone CLP unlike access network.
- As BBTR increases, total CLP increases but contribution towards the increase is predominantly from access networks.

Backbone service rate





A comparison between the backbone CLP. Notice that as the rho of the backbone increases, the CLP increases. When rho = 0.6, optimal network.

<u>Case 3 Conclusions:</u>

- Matches the expected value of the utilization of the backbone.
- CLP as a measure to determine the optimal value.

Conclusions

- Introduction of new methodology, contributing to general understanding of the dynamics of bursty traffic on larger networks, that can be used to gain more detailed insight, even with simplified models.
- Relationship between burstiness in ethernet traffic and its effect on backbone network performance has been explored using new tools for construction and analysis of MED network models.
- Introduces a much needed flexibility in analyzing networks.
- The analysis of the burstiness in the access network, filtered through the buffering medium, reaching the backbone was done successfully.

Future Work

- Future work could address the numerical efficiency issues in solving for state probabilities and other measures of performance.
- Further understanding of the modular traffic can be obtained by increasing the number of modes entering the access network.
- Modeling the access network with hyper-exponential arrival streams.
- Scope of this project can be extended based on the insights sought by the researcher.

Thank you !!!

Questions?

Concerns?

Samosas and donuts?