Optimizing Providers' Profit in Peer Networks Applying Automatic Pricing and Game Theory

PhD Defense Presentation

Sohel Khan

October 24, 2005

Outline of the Presentation

1. Introduction
2. Automatic Price Transaction Architecture
3. Providers' Profit Optimization Method
   - Select Strategic Price by Game Theory
   - Minimize Congestion Cost by Optimum Routing
   - Guarantee QoS by Traffic Engineered Network Design
4. Result
5. Conclusion
6. Appendix

Introduction: Problem Statement

- Existing Customer-Provider peer architecture and protocols do not support
  - Automatic price transaction
  - Customers' option to select any provider based on competitive service price
  - Customers' option to broadcast their budget
  - Providers' automatic mechanism to compute price and optimize Profit
    - In competitive market
    - In dynamic Internet traffic demand
- Therefore, the problem is to develop
  - Automatic price-transaction based network architecture
  - A provider's model that compute competitive price and optimize Profit
  - Demonstrate the advantages of the architecture and the model through analysis

Introduction: Proposed Solution

- This research proposes
  - A new Price transaction Architecture
    - *"Automatic Price Transaction-based One-to-Many Peer Network* Architecture"
  - A new providers' Profit optimization model
    - *"Providers Optimized Game in Internet Traffic* Model"
  - An algorithm
    - That implements the model in the architecture
- This research demonstrates
  - The validity of the model
  - Advantage of the model
    - Customer Benefit
    - Providers' Benefit
  - Providers' Profit optimization method
  - Examples of TE applications

Introduction: Research Method

This Research:

- Develops the Architecture
  - Wire-line and wireless options
  - Study only wire-line option
- Develops the Model and Algorithm
  - Determines strategically appropriate price
    - By Game Theory
  - Minimizes the network congestion sensitive cost
    - By optimum Routing technique
      - Non-linear optimization method
        - » The Gradient Projection Algorithm and the Golden Section Line Search
  - Guarantees service quality
    - By Designing Traffic Engineered Network
- Evaluates performance by
  - Mathematical Analysis
  - Simulation Study
- Studies the followings:
  - Advantages
  - Profit Optimization Strategies
  - Applications

Related Research

- Significant Internet pricing research
  - In monopoly market
  - Congestion sensitive pricing
    - Service per Customer's bid
    - Static Congestion Game
  - Game theory
    - Internet Pricing: Monopoly market
    - Congestion Issues: Monopoly market
    - Peer providers in Series
- Industry Standard Activity
  - 3GPP Wireless Price Model
  - ATIS/PTSC wireline IP Peering
  - IETF wireline VoIP Peering
- On-line Exchange Research (Bandyopadday model)
  - We extend this model (Details later)
- Price-Transaction based mechanism
  - One provider network

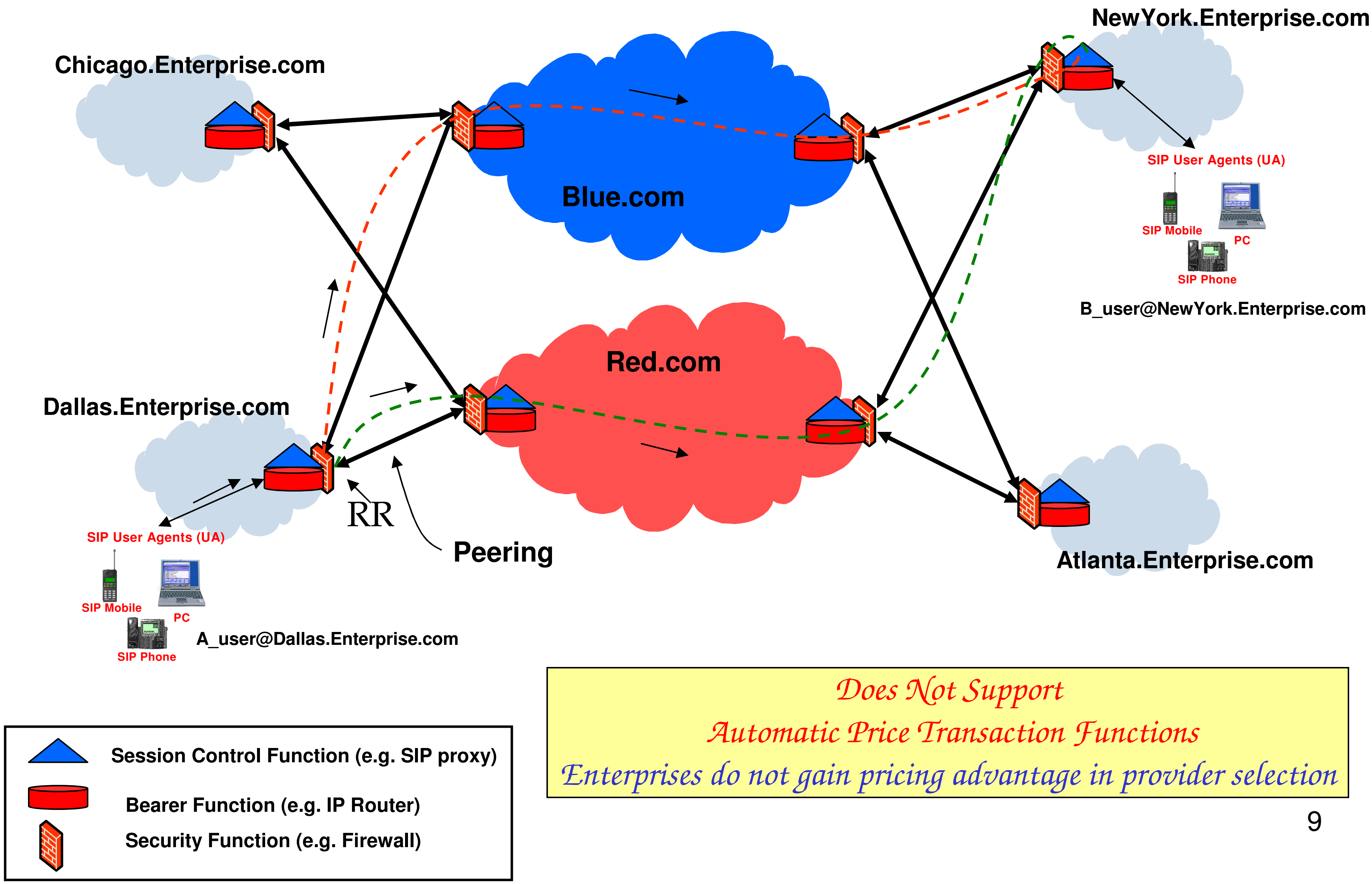
Distinguishing Characteristics of our Approach

- Automatic price transaction in one-to-many peer network
  - New idea of pricing in peer networks
  - Extends various industry standards
- Majority research are in monopoly market
  - We study Oligopoly market
- Provider's Profit optimization in oligopoly market
  - New method in internet pricing and Profit optimization
- Network Model
  - A complex network, bi-directional links, multiple paths, OD&DO call legs
- Oligopoly Model
  - Bandyopadhyay et al. model
    - Based on Bertrand Model and "Model of Sale" example
    - Symmetric market
      - All parameters are fixed
    - Commodity is not internet bandwidth
    - Two step static game of incomplete information
    - Homogeneous service
    - Uses Reinforcement Learning (RL) in simulation to determine best strategy
  - Our model
    - Extension to Bandyopadhyay et al. model
    - Asymmetric market
      - Some parameters are sensitive to the dynamic nature of Internet traffic
    - Commodity bandwidth
    - "Myopic" Markovian static game of incomplete information
    - Heterogeneous service
    - An analytical framework to determine the best strategy in dynamic internet traffic

Introduction: Contributions

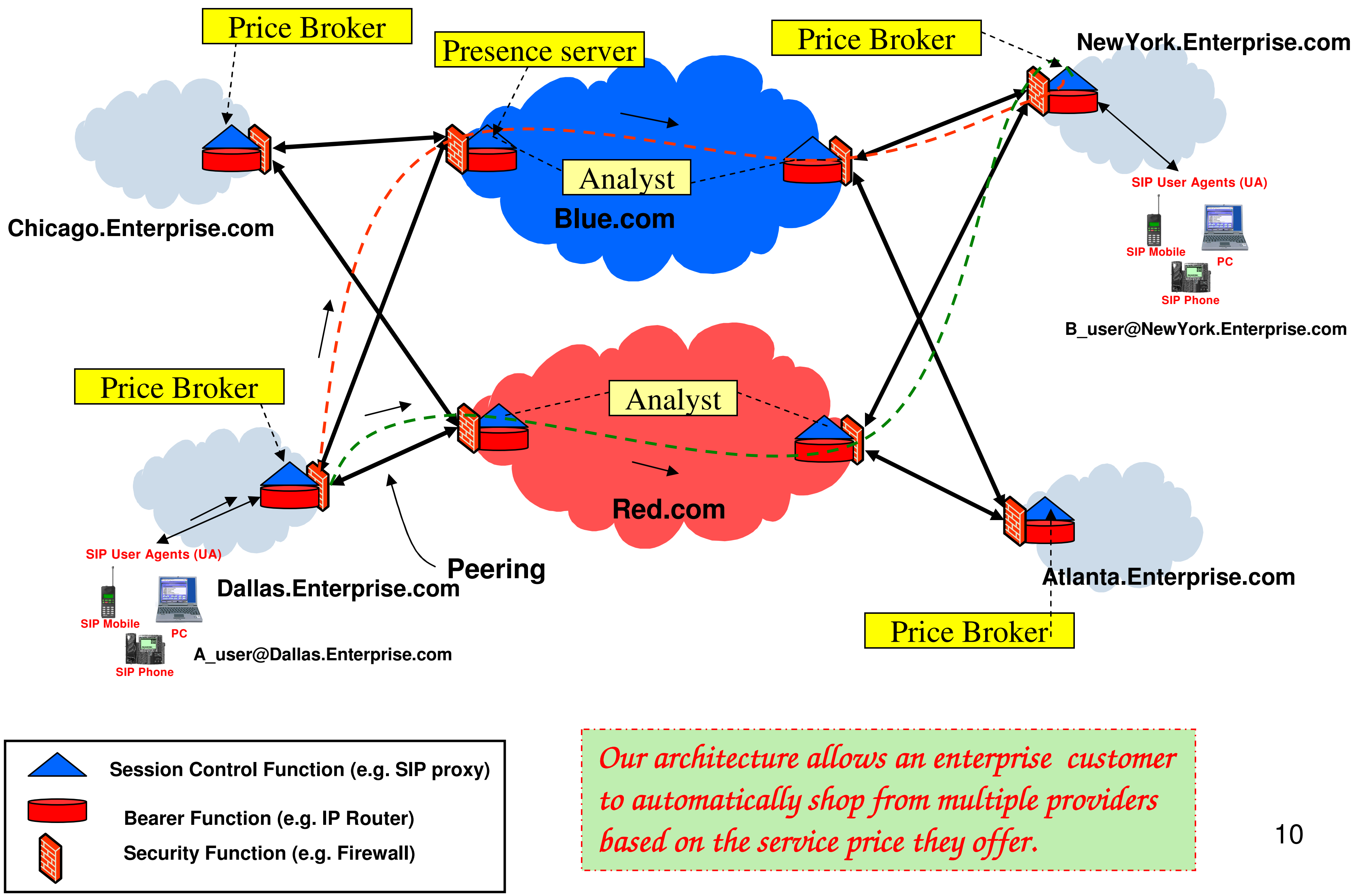
- Developed a New price transaction architecture that benefits customers and providers
  - By Automation
  - By providing options to select any provider based on competitive price
  - By allowing customer power to specify budget
  - By introducing new price transaction research in one-to-many architecture
- Developed a mathematical model for providers to
  - To compute competitive price through the best strategy
  - Optimize Profit in dynamic internet traffic demand
- Developed an algorithm and simulation model
  - To verify and study providers' game in flexible environment
- Introduced a New framework to determine Bayesian-Nash equilibrium
  - In dynamic internet traffic demand
- Demonstrated that:
  - Providers improved their Profit
    - Our approach yielded relative advantages over the existing Bertrand Oligopoly Model
  - Providers determined Best strategies (Bayesian-Nash equilibrium and Pareto-efficient outcome) using our approach
  - Providers was able to obtain fair market share of Profit and throughput
  - Providers could implement TE applications such as optimized load balancing in the network
  - Customers could enjoy market price lower than their budgets.
- Introduced new area in Internet pricing research
  - Our research is the first in Internet Oligopoly pricing research for disjoint providers
  - Existing research are for monopoly market
- Introduced pricing research in a complex network model
  - Bi-directional links, multiple paths, Origin-Destination and Destination-Origin Call Legs.

Current Managed IP Peering Architecture



Does Not Support
Automatic Price Transaction Functions
Enterprises do not gain pricing advantage in provider selection

Proposed
Automatic Price Transaction-based 1:M Peer Network Architecture

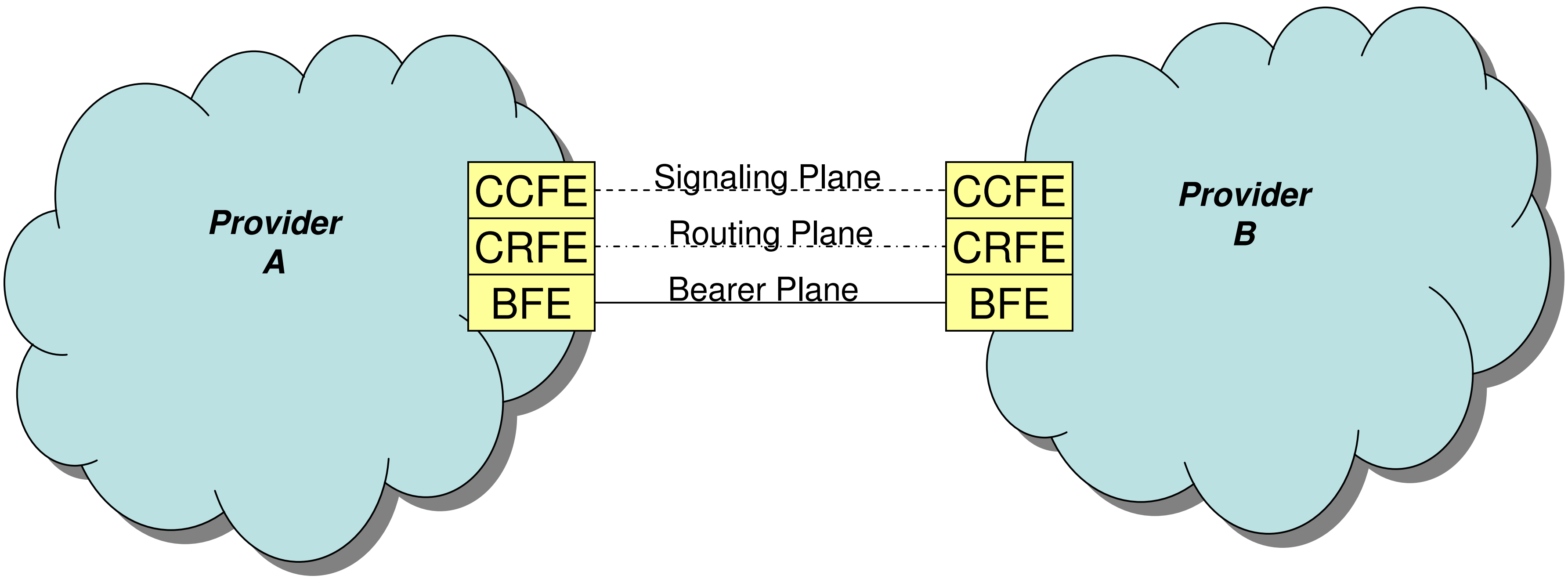


Our architecture allows an enterprise customer to automatically shop from multiple providers based on the service price they offer.

Architecture: ATIS/PTSC IP (wireline) Peering Reference Diagram

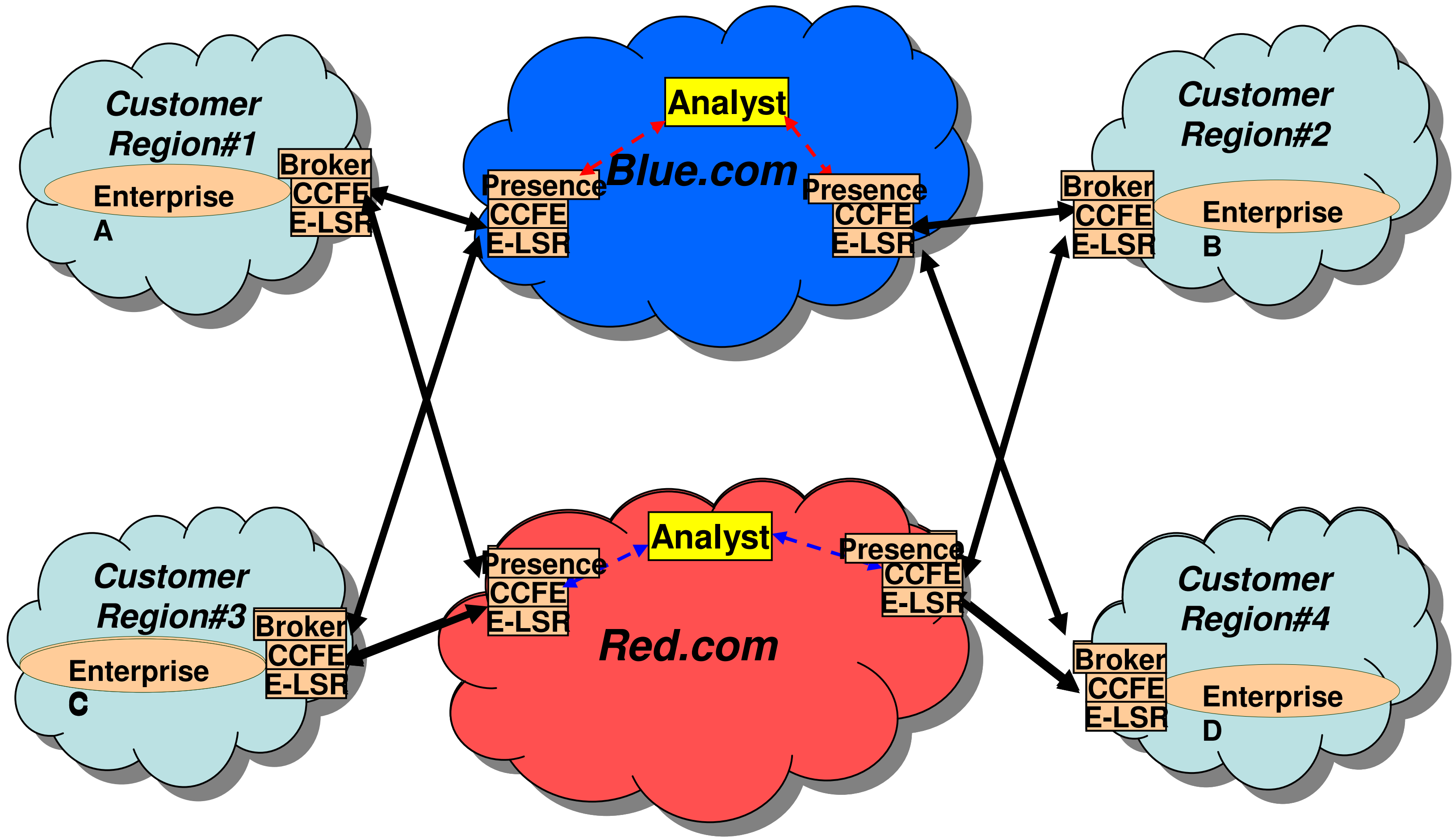
Current ATIS PTSC IP NNI Architecture specifies:

- *One-to-one peer interface*
- *No pricing architecture is supported*



CCFE: Call Control Functional Entity
CRFE: Call Routing Functional Entity
BFE: Bearer Functional Entity

Our Extension to ATIS Architecture



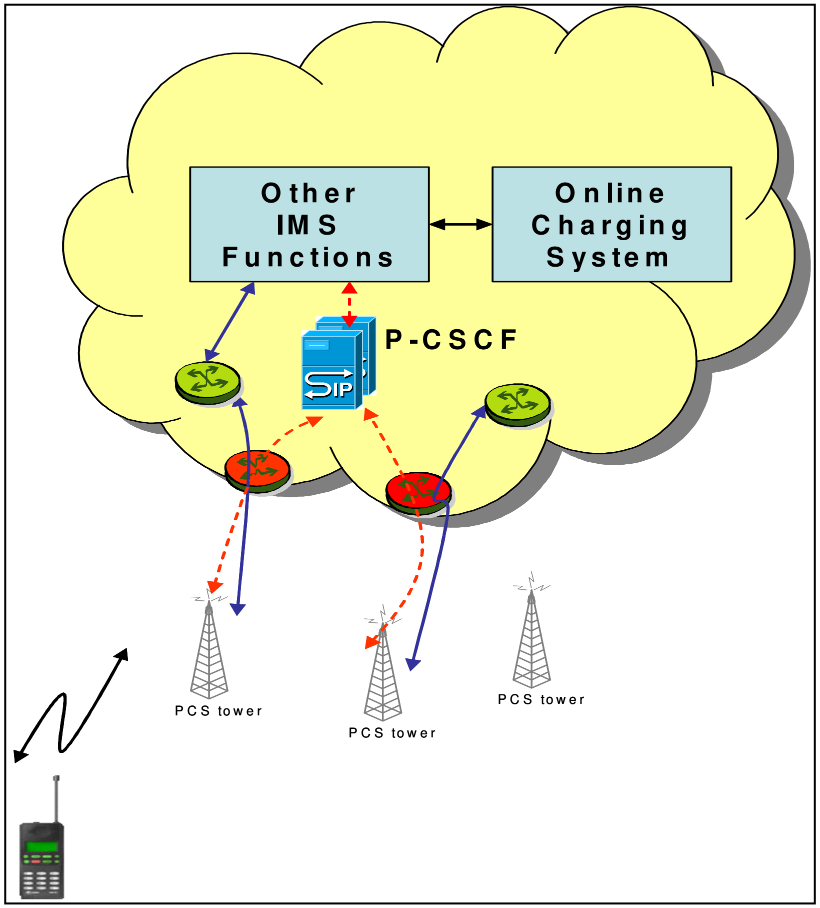
Current ATIS PTSC IP NNI Architecture specifies:

- One-to-one peer interface
- No pricing architecture is supported

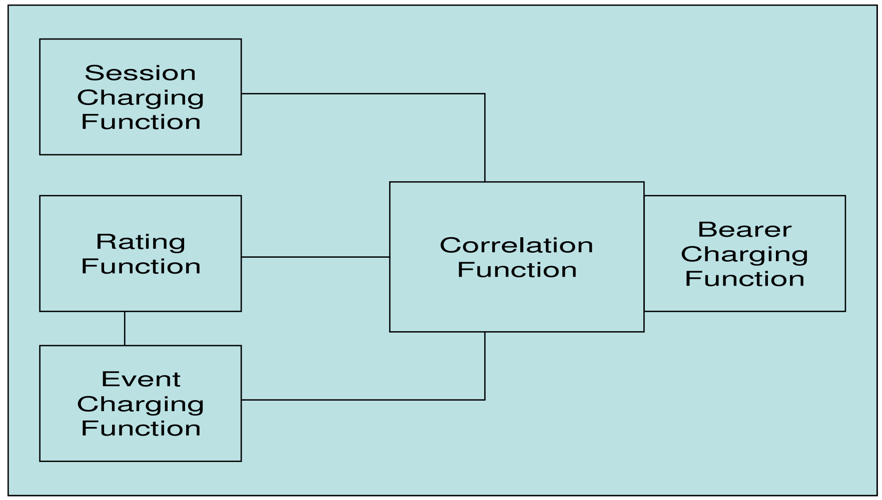
Our extension enhances ATIS Architecture to

- One-to-Many peer interface
- Automatic Price Transaction

Current 3GPP (Wireless) IMS Charging Architecture



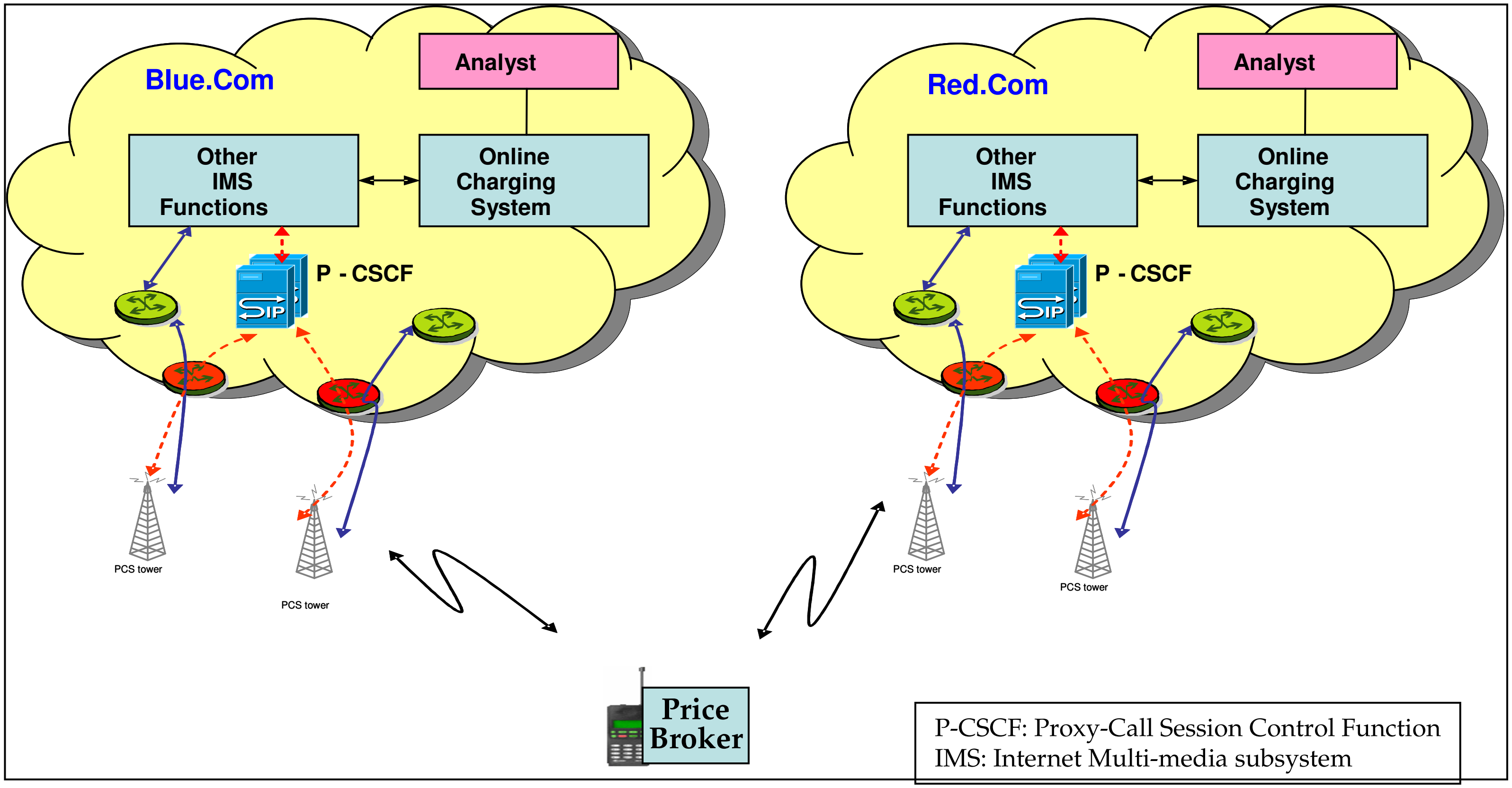
Charging Architecture



3GPP On-line Charging System (WOP)

Current 3GPP Architecture specifies
One-to-one customer-provider
Online charging architecture (work on progress)
Does not support one-to-many model

Our Extension to the 3GPP (Wireless) IMS Architecture



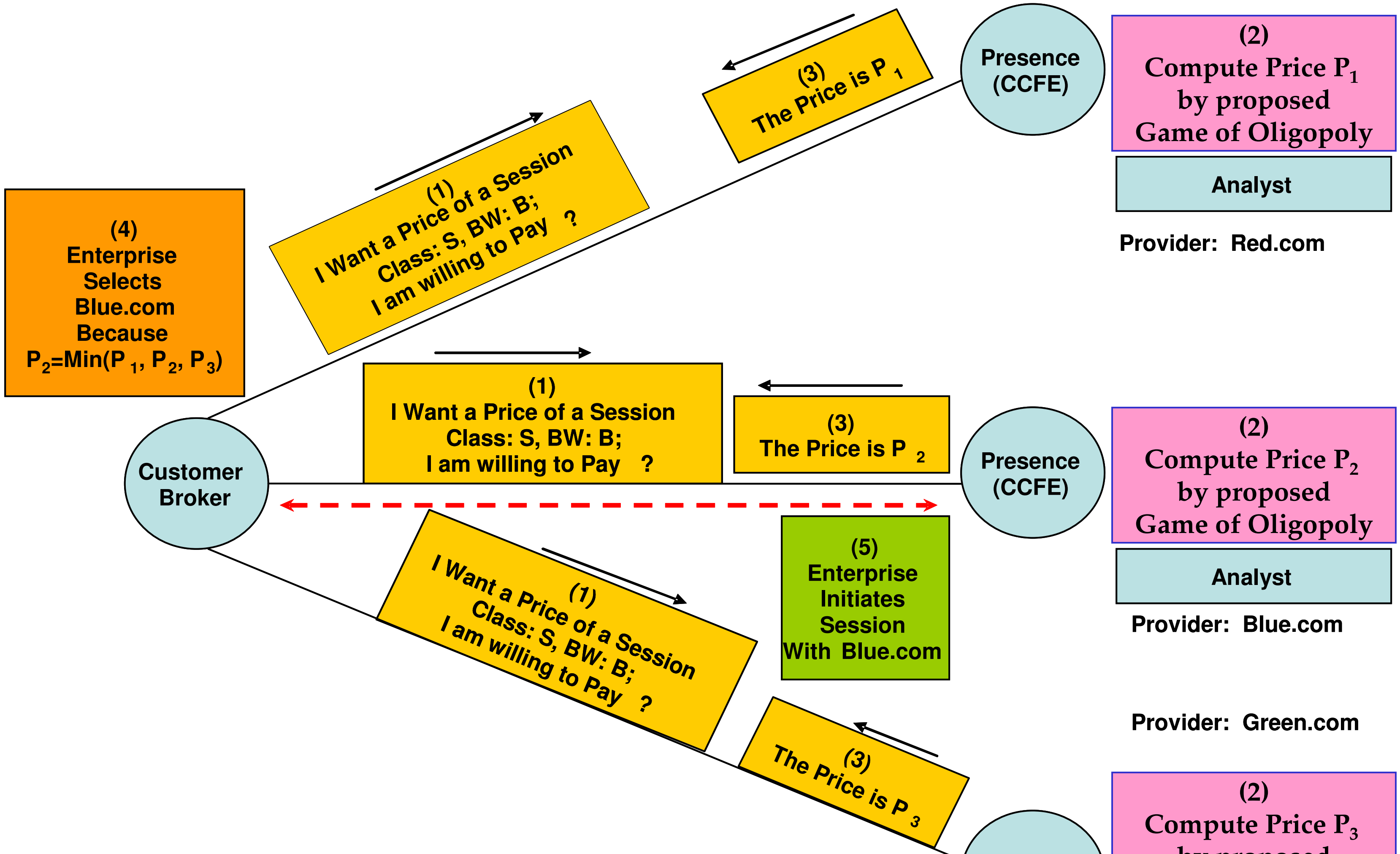
Current 3GPP Architecture supports

- *One-to-one model*
- *Does not allow price negotiation*
- *Does not allows providers to compute competitive price*

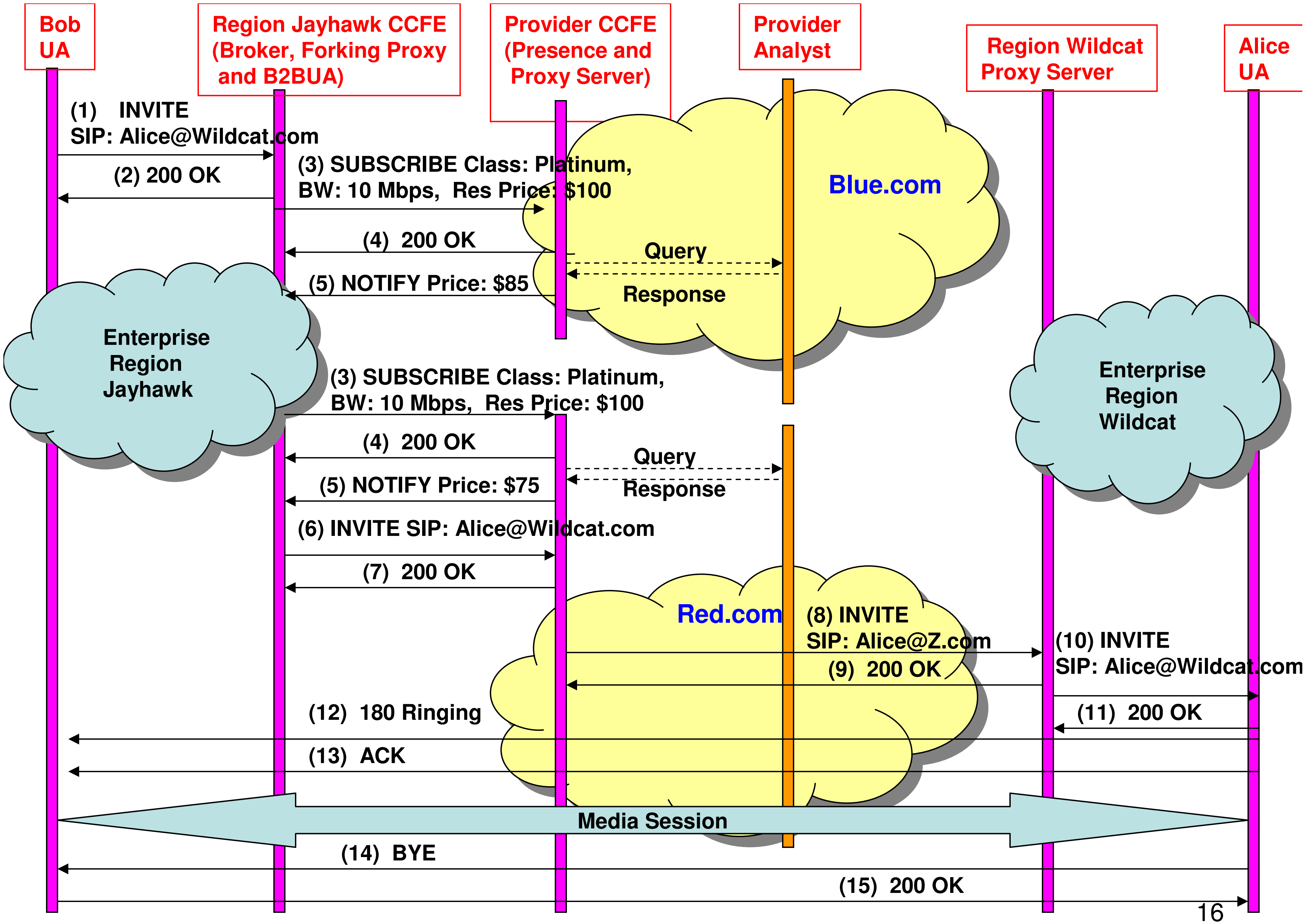
Our Extension supports

- *One-to-many model*
- *Allows automatic price negotiation*
- *Allows providers to compute competitive price*

Proposed Price Transaction Protocol

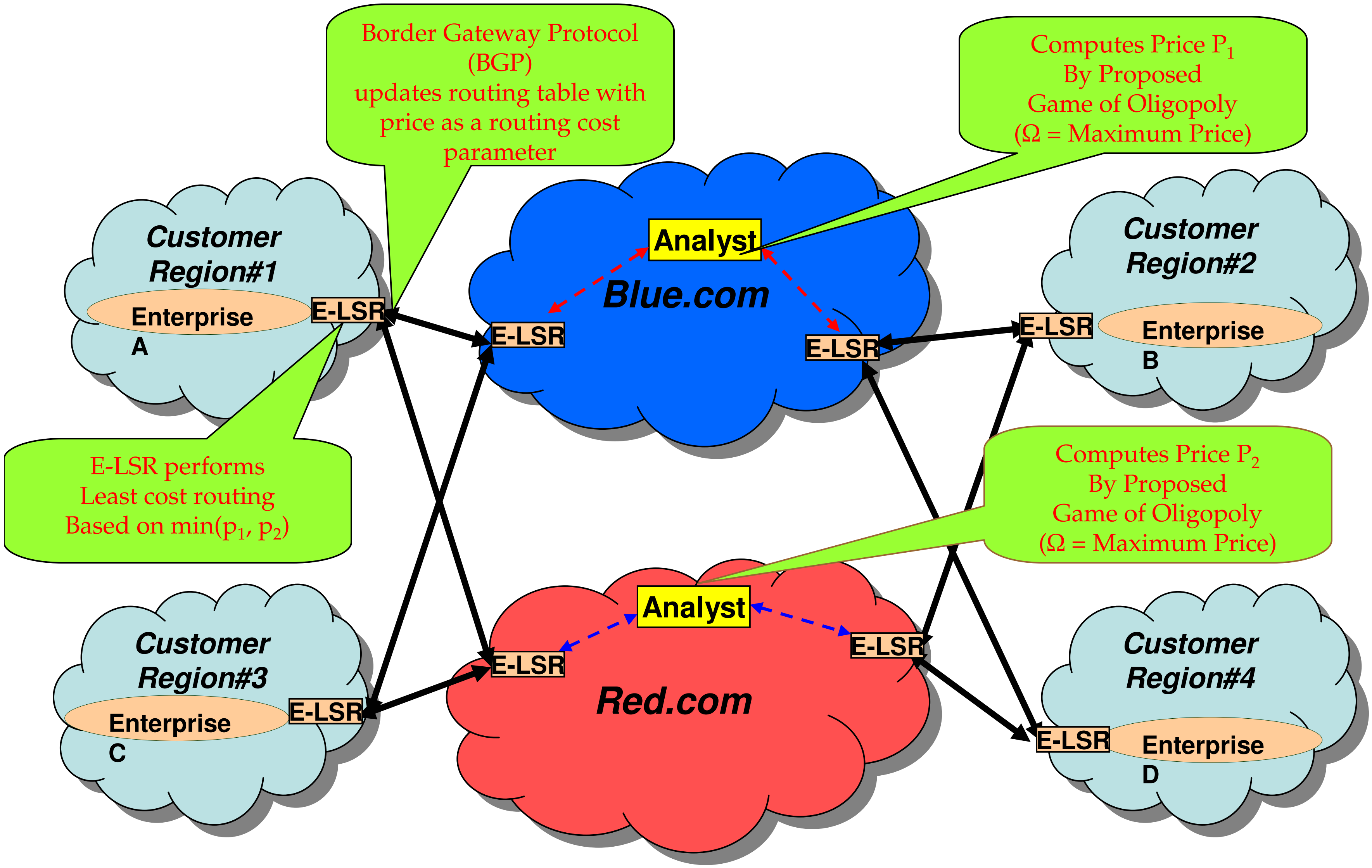


The Protocol is analogous to the
Sealed-Bid-Reverse Auction.
Customers has power to specify the highest price



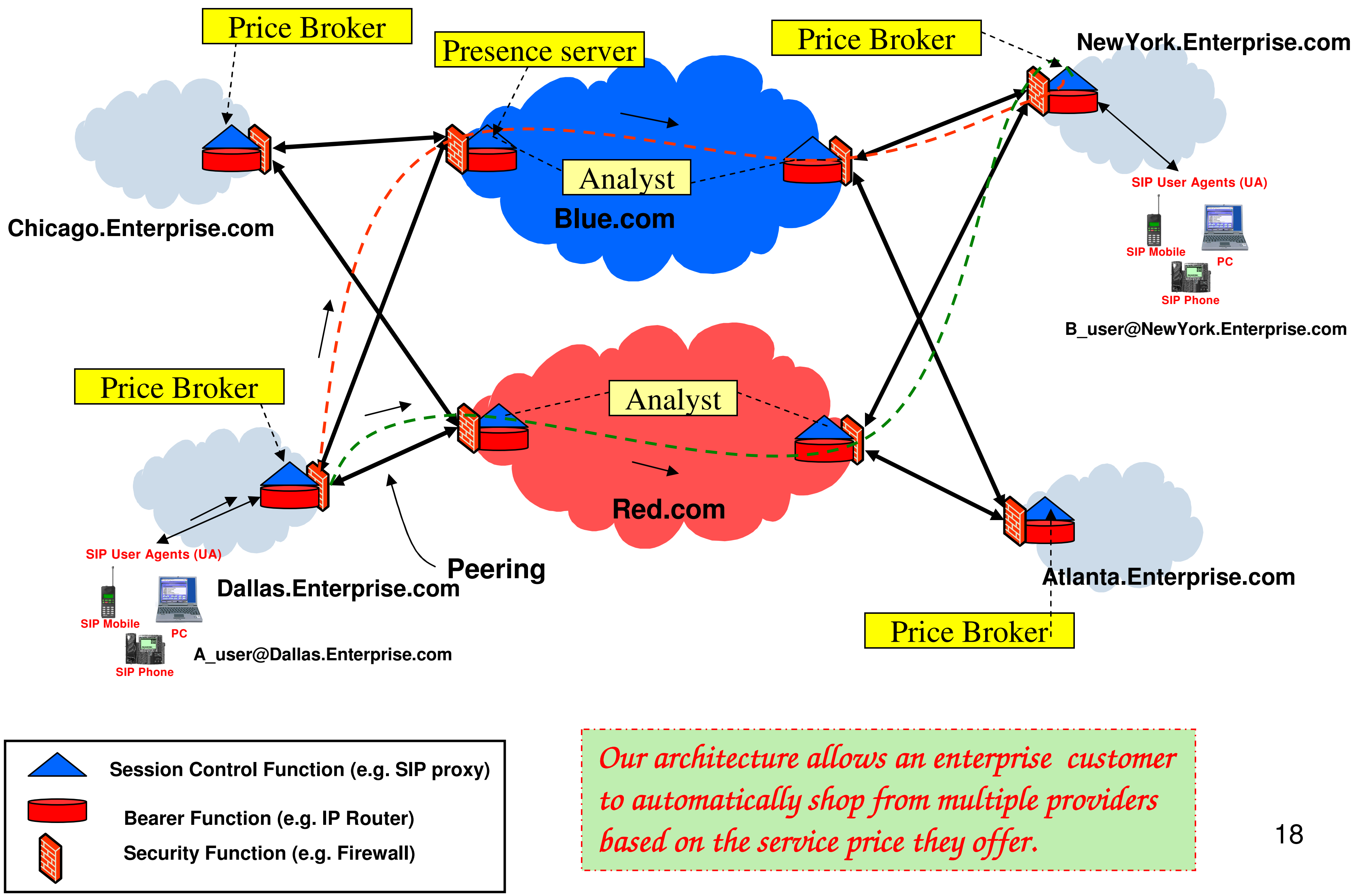
Example: Session Initiation Protocol (SIP) Call Flow (sketch)

Architecture: BGP Implementation



Many other variants of the proposed Architecture are possible
(See Table 2.1)

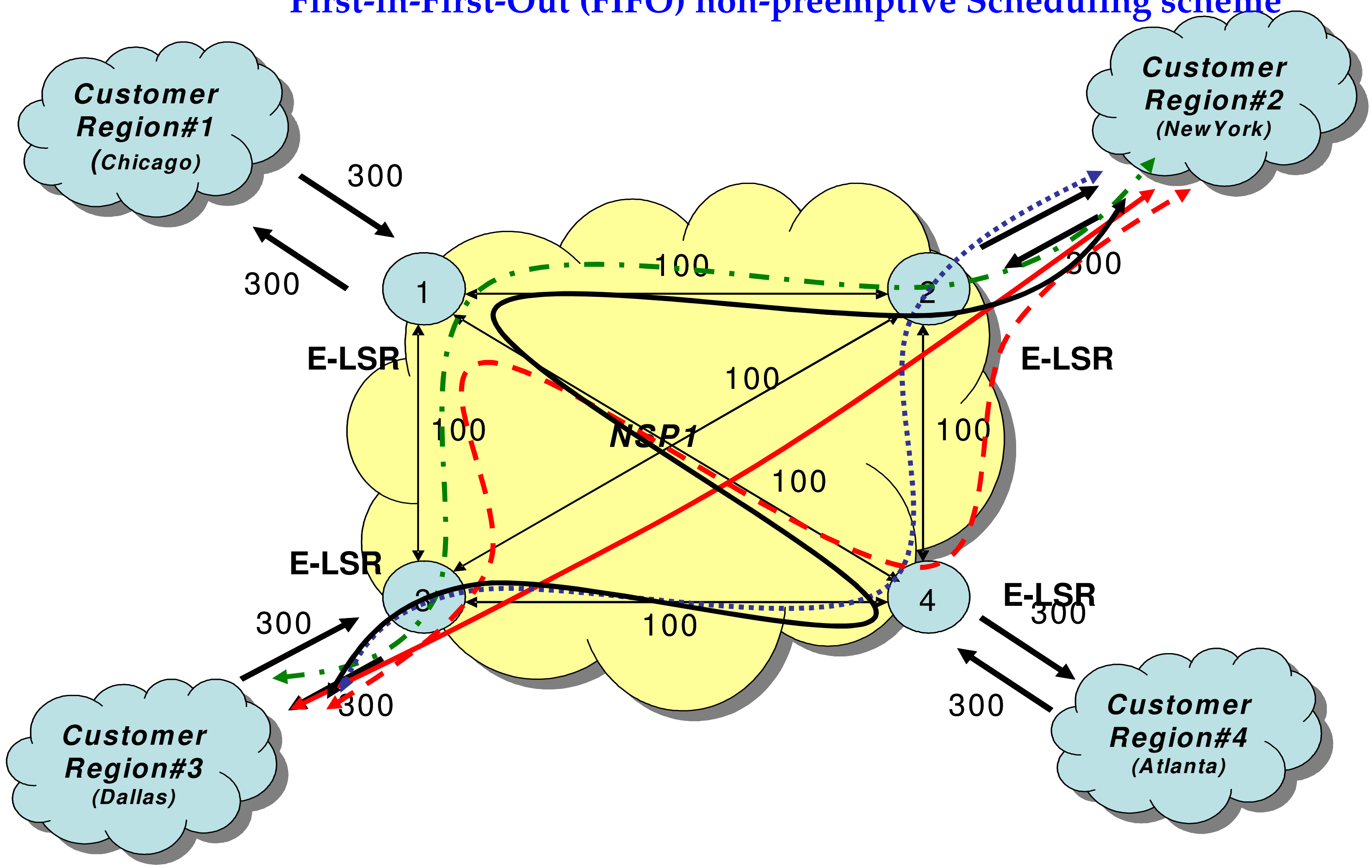
Proposed
Automatic Price Transaction-based 1:M Peer Network Architecture



Our architecture allows an enterprise customer to automatically shop from multiple providers based on the service price they offer.

Internal Network of Each Provider of our study

Nodes are fully meshed
Each O-D Pair is connected with five alternative LSPs
There are 60 LSPs in each network
Each call has two legs: O-D and D-O
- Single Integrated Queue per output link
- First-in-First-Out (FIFO) non-preemptive Scheduling scheme



Traffic Model

- Packet:
  - Arrival Pattern: Poisson Distributed
  - Mean Service Rate: Exponentially Distributed
  - Aggregate arrival distribution: Poisson
  - Aggregate mean service rate distribution: Hyper-exponential
  - Queue Theory Model
    - **M/G/1**

- For Traffic Engineering, we will use M/G/1
- For Cost Analysis, we will approximate with M/M/1

- Session: (No Queuing)
  - Arrival Pattern: Poisson Distributed
  - Mean Service Rate: Exponentially Distributed
- Assumed Traffic Mix
  - Homogeneous: Gold
  - Heterogeneous Class: Platinum, Gold, and Silver
    - The service class is differentiated by cost coefficient parameter.
    - Cost coefficient parameter depends on the type of protocol and Intelligence used
    - Example: Level of Security guarantees, addressing (IPv4 vs. IPv6), type of DSP
    - Cost coefficient parameter distinguishes Service Class (Plat, Gold, Silver)

Profit Optimization

- *Our method of* ***Providers' Profit Optimization:***
  - ***Design Traffic-Engineered Network to Guarantee QoS***
  - ***Minimize congestion sensitive cost (ωY)***
  - ***Select strategically appropriate price by Game Theory***
    - ***to maximize revenue (pY)***

$$\textit{Unit Utility at steady state}: u(p) = (\bar{p} - \bar{\omega})\bar{Y}$$

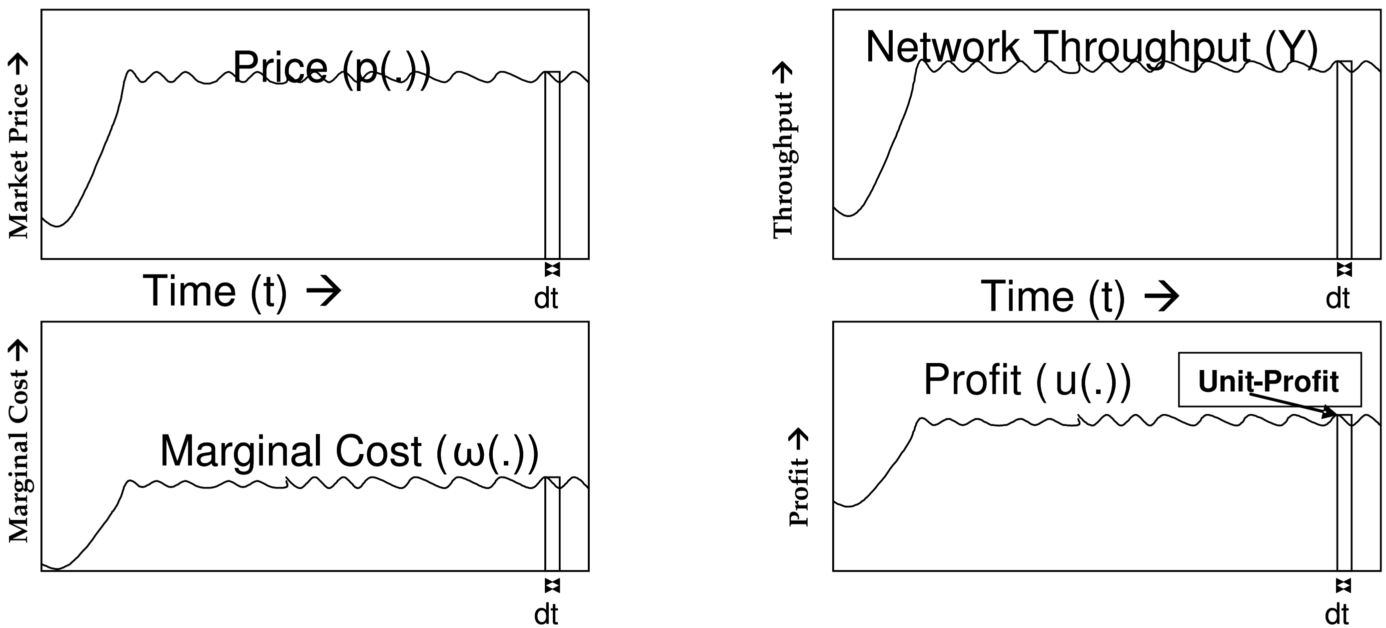
Maximize u(.)

s.t. $\begin{cases} \textbf{Network Architecture Constraint} \\ \textbf{Internet Traffic Pattern and Queue System Constraint} \\ \textbf{Game Strategy Constraint} \end{cases}$

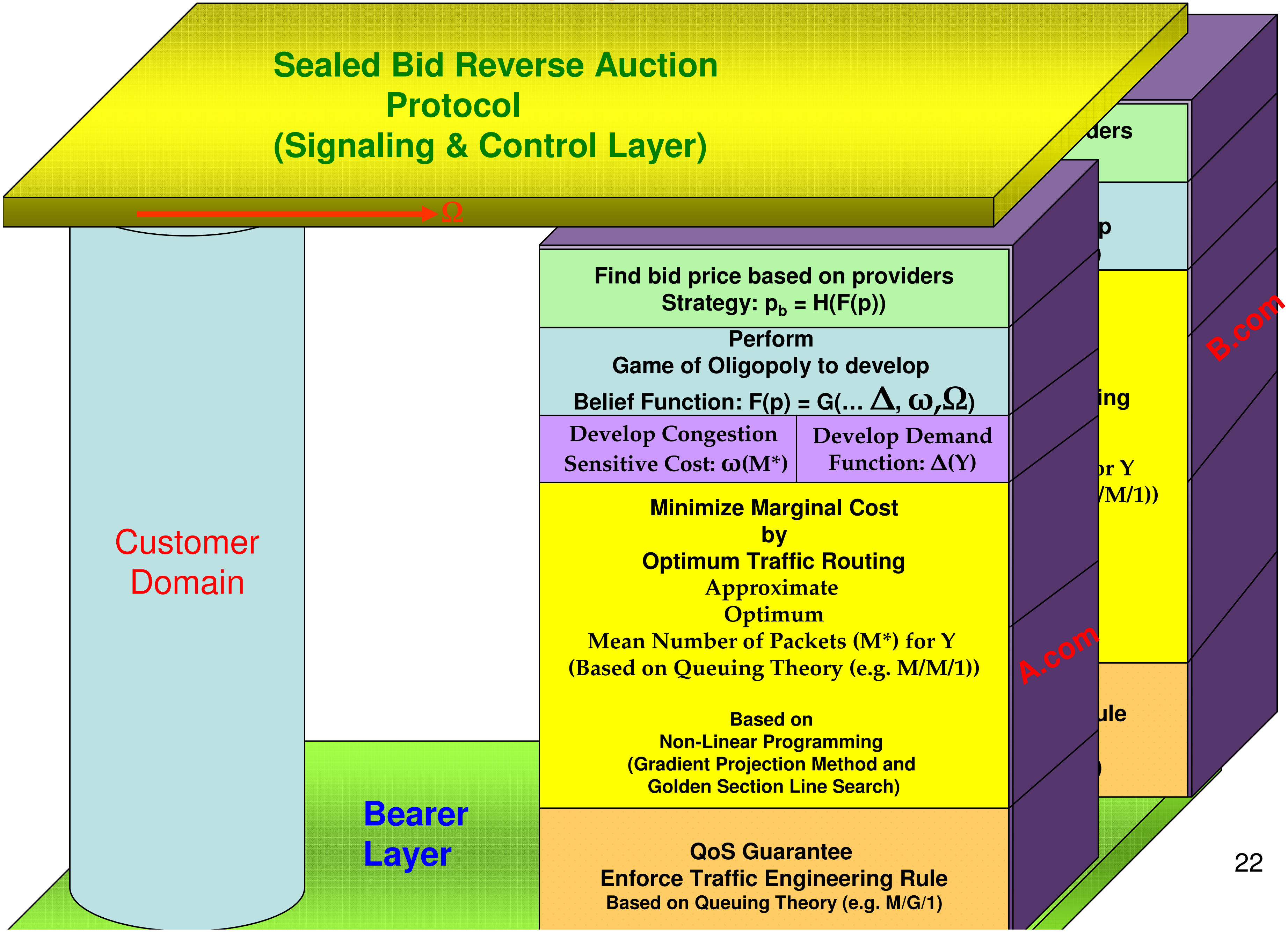
$$\textit{Maximize} \quad u(p)$$
$$= Max\,(p - \omega)Y$$
$$Max\ pY + Max\,(-\omega Y) \Rightarrow Max\,(p - \omega)Y$$
$$(\textit{Maximize } pY + \textit{Minimize } \omega Y) \Rightarrow \textit{Maximize } u(p)$$

$$profit_n = \sum_{\forall k} (p_{s,t,k} - \omega_{n,s,t,k}) d_{n,k} y_{n,s,k}$$

p: call unit price
ω: call marginal cost
d: call duration
y: call bandwidth



Algorithm


Sealed Bid Reverse Auction
Protocol
(Signaling & Control Layer)
Ω
Customer
Domain
Bearer
Layer
Find bid price based on providers
Strategy: $p_b = H(F(p))$
Perform
Game of Oligopoly to develop
Belief Function: $F(p) = G(\ldots \Delta, \omega, \Omega)$
Develop Congestion
Sensitive Cost: $\omega(M^*)$
Develop Demand
Function: $\Delta(Y)$
Minimize Marginal Cost
by
Optimum Traffic Routing
Approximate
Optimum
Mean Number of Packets (M^*) for Y
(Based on Queuing Theory (e.g. M/M/1))
Based on
Non-Linear Programming
(Gradient Projection Method and
Golden Section Line Search)
QoS Guarantee
Enforce Traffic Engineering Rule
Based on Queuing Theory (e.g. M/G/1)
A.com
B.com

QoS Guarantee

- We develop TE Rules to guarantee mean delay less than 1 msec.
  - Homogeneous services
    - Link Load (Green) < 90%
  - Heterogeneous Services
    - Link Load (Blue) < 20%
    - Link Load (Green) < 30%
    - Link Load (Red) < 40%
- Based on M/G/1 System

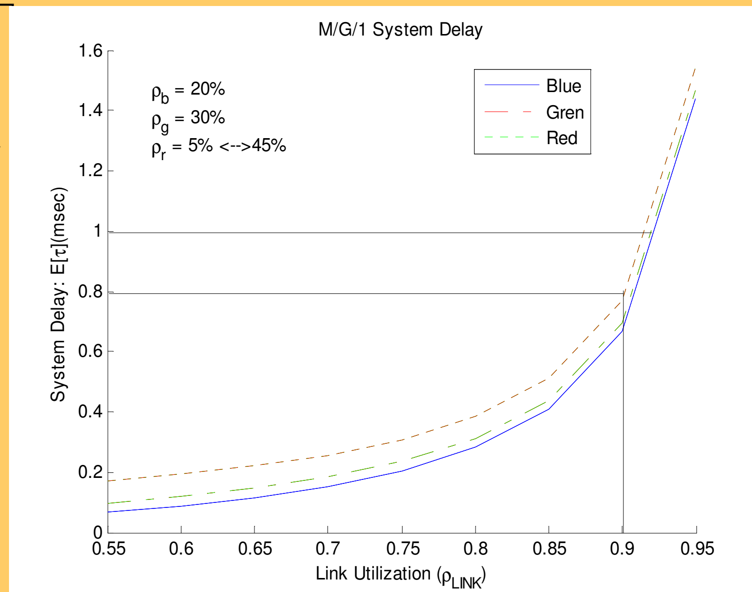
$$E[\tau_b] = \frac{E[L_b]}{C}, \quad E[\tau_g] = \frac{E[L_g]}{C}, \quad E[\tau_r] = \frac{E[L_r]}{C}$$

$$E[\tau_b^2] = 2.\left(\frac{E[L_b]}{C}\right)^2, \quad E[\tau_g^2] = 2.\left(\frac{E[L_g]}{C}\right)^2, \quad E[\tau_r^2] = 2.\left(\frac{E[L_r]}{C}\right)^2$$

$$E[\hat{\tau}] = \frac{\lambda_b}{\lambda} E[\tau_b] + \frac{\lambda_g}{\lambda} E[\tau_g] + \frac{\lambda_v}{\lambda} E[\tau_r]$$

$$E[\hat{\tau}^2] = \frac{\lambda_b}{\lambda} E[\tau_b{}^2] + \frac{\lambda_g}{\lambda} E[\tau_g{}^2] + \frac{\lambda_v}{\lambda} E[\tau_r{}^2]$$

$$E[T_s] = \frac{E[L_s]}{C} + \frac{\lambda E[\hat{\tau}^2]}{2(1 - \lambda E[\hat{\tau}])}$$



Service Cost Function

- Assumption: Following four influences on the service cost:
  - Congestion in the network
    - Degrades the service quality
      - causes the delay in packet transmission.
    - The degradation of service is detrimental to the revenue
    - Providers have to pay to the Enterprise for jitter (Expense
    - An indicator of network congestion
      - Mean packet count (M) in the queue system
  - Protocol used to provide a service
    - Service cost coefficient (δ_s)
  - Amount of service (commodity)
    - Throughput (Y)
  - Providers' fixed cost (θ)

$$Cost_{n,s,t}(Y_{n,t}) = g(Y_{n,t}) = \delta_s \hat{M}_{n,t} Y_{n,t} + \theta_n Y_{n,t}$$

$$f(Y_{n,t}) \rightarrow M_{n,t}$$

Minimize Cost

A Cost Function assumption

- Service cost is a functions of network congestion
- Mean packet count in network queue system is a congestion indicator

$$Minimize \quad \omega_{n,t}$$

$$= Minimize \quad \{\delta_s (Y_{n,t} \frac{\partial \hat{M}_{n,t}}{\partial Y_{n,t}} + \hat{M}_{n,t}) + \theta_n\}$$

$$Minimize\ \hat{M} \Leftarrow optimize\ network\ traffic\ routes\ applying\ nonlinear\ \ program$$

$$Minimize \quad \left(Y_{n,t} \frac{\partial \hat{M}_{n,t}}{\partial Y_{n,t}} \right) \Leftarrow \left\langle \begin{array}{l} route\ optimization\ loadbalances\ and \\ reduces\ change\ in\ \hat{M}_{n,t}\ \ in\ low\ load \end{array} \right.$$

Minimize Marginal Cost

- Minimize Congestion Cost by Optimum Routing Method
  - Minimizing Mean Packet Count
- Mean Packet Count (M/M/1 Model): $\hat{M} = \sum E[packets] = \sum_{l} \frac{\sum_{j:l\in j} x_j}{C_l - \sum_{j:l\in j} x_j}$
- Non-Linear Program:

$$Minimize: \hat{M} = \sum_{l} \frac{\sum_{j:l\in j} x_j}{C_l - \sum_{j:l\in j} x_j}$$

$$Subject\ to: \sum_{j:l\in J} x_j \le \rho_{TE} C_l$$

$$\sum_{j\in J(w)} x_j = r_w$$

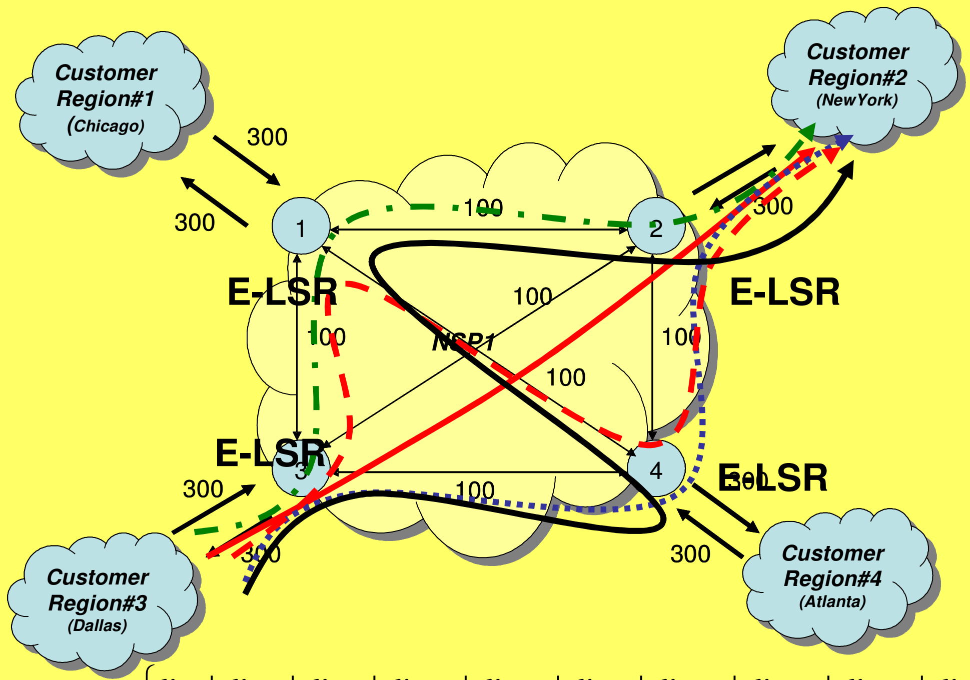
$$x_j \ge 0$$

We implement Gradient Projection and Golden Section line search to satisfy Karush-Kuhn-Tucker condition

In each game instance (each request for bid), this optimization is performed

(See dissertation for details, we provide highlight in next three slides)

Non-linear Program: Constraints



$$[\mathbf{H}] = \begin{cases} x_{12}+x_{132}+x_{142}+x_{1342}+x_{1432}-r_{12}=0 \\ x_{21}+x_{231}+x_{241}+x_{2431}+x_{2341}-r_{21}=0 \\ x_{13}+x_{123}+x_{143}+x_{1243}+x_{1423}-r_{13}=0 \\ x_{31}+x_{321}+x_{341}+x_{3421}+x_{3241}-r_{31}=0 \\ x_{14}+x_{124}+x_{134}+x_{1324}+x_{1234}-r_{14}=0 \\ x_{41}+x_{421}+x_{431}+x_{4231}+x_{4321}-r_{41}=0 \\ x_{42}+x_{412}+x_{432}+x_{4132}+x_{4312}-r_{42}=0 \\ x_{24}+x_{214}+x_{234}+x_{2314}+x_{2134}-r_{24}=0 \\ x_{23}+x_{213}+x_{243}+x_{2143}+x_{2413}-r_{23}=0 \\ x_{32}+x_{312}+x_{342}+x_{3412}+x_{3142}-r_{32}=0 \\ x_{34}+x_{314}+x_{324}+x_{3124}+x_{3214}-r_{34}=0 \\ x_{43}+x_{413}+x_{423}+x_{4213}+x_{4123}-r_{43}=0 \end{cases}$$

$$[\mathbf{G}_{Inequality}] = \begin{cases} x_{12}+x_{123}+x_{124}+x_{1243}+x_{1234}+x_{312}+x_{3124}+x_{3412}+x_{4123}+x_{412}+x_{4312}-\rho_{TE}C_{12}\le 0 \\ x_{21}+x_{321}+x_{421}+x_{3421}+x_{4321}+x_{213}+x_{4213}+x_{2143}+x_{3214}+x_{214}+x_{2134}-\rho_{TE}C_{21}\le 0 \\ x_{13}+x_{134}+x_{213}+x_{413}+x_{132}+x_{1342}+x_{1324}+x_{4132}+x_{2134}+x_{2413}+x_{4213}-\rho_{TE}C_{13}\le 0 \\ x_{31}+x_{431}+x_{312}+x_{314}+x_{231}+x_{2431}+x_{4231}+x_{2314}+x_{4312}+x_{3142}+x_{3124}-\rho_{TE}C_{31}\le 0 \\ x_{14}+x_{142}+x_{214}+x_{314}+x_{143}+x_{1432}+x_{1423}+x_{2314}+x_{3214}+x_{3142}+x_{2143}-\rho_{TE}C_{14}\le 0 \\ x_{41}+x_{241}+x_{412}+x_{413}+x_{341}+x_{2341}+x_{3241}+x_{4132}+x_{4123}+x_{2413}+x_{3412}-\rho_{TE}C_{41}\le 0 \\ x_{42}+x_{142}+x_{342}+x_{421}+x_{423}+x_{1342}+x_{1423}+x_{3142}+x_{3421}+x_{4213}+x_{4231}-\rho_{TE}C_{42}\le 0 \\ x_{24}+x_{241}+x_{243}+x_{124}+x_{324}+x_{2431}+x_{3241}+x_{2413}+x_{1243}+x_{3124}+x_{1324}-\rho_{TE}C_{24}\le 0 \\ x_{23}+x_{231}+x_{234}+x_{123}+x_{423}+x_{1234}+x_{1423}+x_{2314}+x_{2341}+x_{4123}+x_{4231}-\rho_{TE}C_{23}\le 0 \\ x_{32}+x_{132}+x_{432}+x_{321}+x_{324}+x_{4321}+x_{3241}+x_{4132}+x_{1432}+x_{3214}+x_{1324}-\rho_{TE}C_{32}\le 0 \\ x_{34}+x_{134}+x_{234}+x_{341}+x_{342}+x_{1342}+x_{1234}+x_{2341}+x_{2134}+x_{3421}+x_{3412}-\rho_{TE}C_{34}\le 0, \\ x_{43}+x_{431}+x_{432}+x_{143}+x_{243}+x_{2431}+x_{4321}+x_{1432}+x_{4312}+x_{1243}+x_{2143}-\rho_{TE}C_{43}\le 0 \end{cases}$$

x_p: BW of an LSP
Each O-D pair has Five LSPs.
Total: 60 LSPs in each provider network
C_l: Capacity of each link
ρ_{TE}: TE load

$$[\mathbf{G}_{non\_neg.}] = \{-\mathrm{x}_j \le 0,\ j \in J$$

Non-Linear Program: Initial Feasible Point

- Gradient Projection Method requires an initial feasible vector (X_0)
- Determine: New Session Route Vector (NV)
  - Minimum Hop Routing
    - Step 1
      - Select the shortest path (one hop route)
    - If fails Step 1, Step 2
      - Select either of the two hop route with equal probability
    - If fails Step 2, Step 3
      - Select either of the three hop route with equal probability
- Anticipated Route Vector = (Current Route Vector) + NV

- Initial feasible vector (X_0) $\leftarrow$ Anticipated Route Vector

Non-Linear Program: Gradient Projection Snapshot

Inequality Constraints:

$$\begin{bmatrix} {}_{(12\times 60)}\mathbf{G}_{Inequaltiy} \\ {}_{(60\times 60)}\mathbf{G}_{Non-negative} \end{bmatrix}\left[{}_{(60\times 1)}\mathbf{Y}\right]-\begin{bmatrix} {}_{(12\times 1)}\rho_{TE}\mathbf{C} \\ {}_{(60\times 1)}\mathbf{0} \end{bmatrix} \le \left[{}_{72\times 1}\mathbf{0}\right]$$

Equality Constraints:

$$\left[{}_{12\times 60}\mathbf{H}_{LSP}\right]\left[{}_{60\times 1}\mathbf{Y}\right]-\left[{}_{12\times 1}\mathbf{R}\right]=\left[{}_{12\times 1}\mathbf{0}\right]$$

$$\Rightarrow \left[{}_{12\times 1}\mathbf{H}\right]=\left[{}_{12\times 1}\mathbf{0}\right]$$

Working Matrix: $[\mathbf{W}]=\begin{bmatrix} \mathbf{G}_{\text{Active}} \\ \mathbf{H} \end{bmatrix}$

This working matrix is the foundation of the working surface (A_q)
Direction of movement ($\mathbf{d}$) is found as follows:

$$\mathbf{P}=\mathbf{I}-\mathbf{A}_\mathbf{q}^\mathbf{T}(\mathbf{A}_\mathbf{q}\mathbf{A}_\mathbf{q}^\mathbf{T})^{-1}\mathbf{A}_\mathbf{q}$$

$$\mathbf{d}=-\mathbf{P}\nabla f(\mathbf{x})^T$$

Find Maximum distance (α_{Max}):

$$g_{inactive}(\mathbf{x})+\alpha_{Max}\, g_{inactive}(\mathbf{x})\mathbf{d}=\mathbf{b}$$

Use Golden Section Line search to find optimum point in each feasible segment:

$$Minimize \quad f(x_k+\alpha_k \mathrm{d}_k)$$

$$s.t. \quad [A]\le[b]$$

Minimum is achieved at $d_k = 0$ and $\lambda \ge 0$ such that the following FONC is satisfied

$$\nabla f(\mathbf{x_k})+\lambda_k^T\mathbf{A_q}=\mathbf{0}$$

Non-Linear Program: Output

- Output of Non-linear program
  - Optimized Mean Packet Count

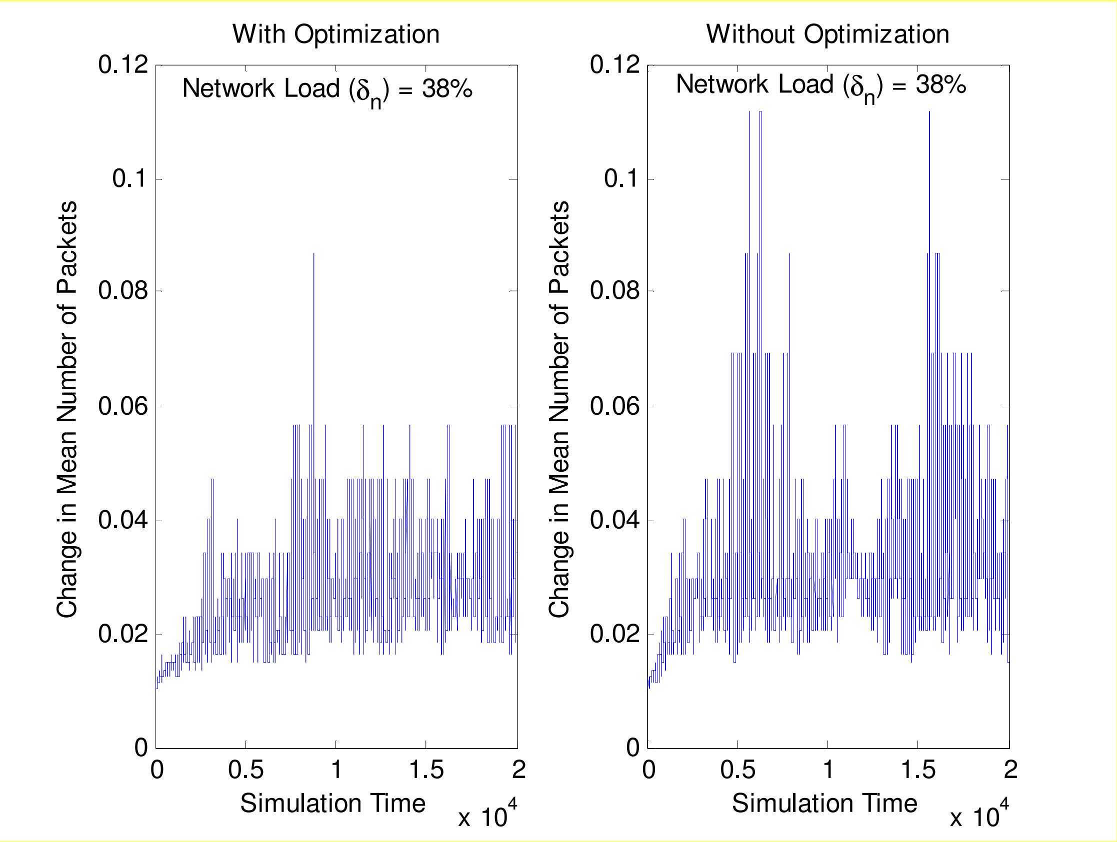
$$\boxed{\hat{M}^{*}}$$

  - Optimum Routes
  - Fair Load Distribution Inside the Network

-→Minimization of Cost

Minimizing Change in Congestion

$$\text{Minimize} \quad \left(\frac{\partial \hat{M}_{n,t}}{\partial Y_{n,t}}\right) \Leftarrow \left\langle \begin{array}{l} \text{route optimization loadbalances and} \\ \text{reduces change in } \hat{M}_{n,t} \text{ in low load} \end{array} \right.$$



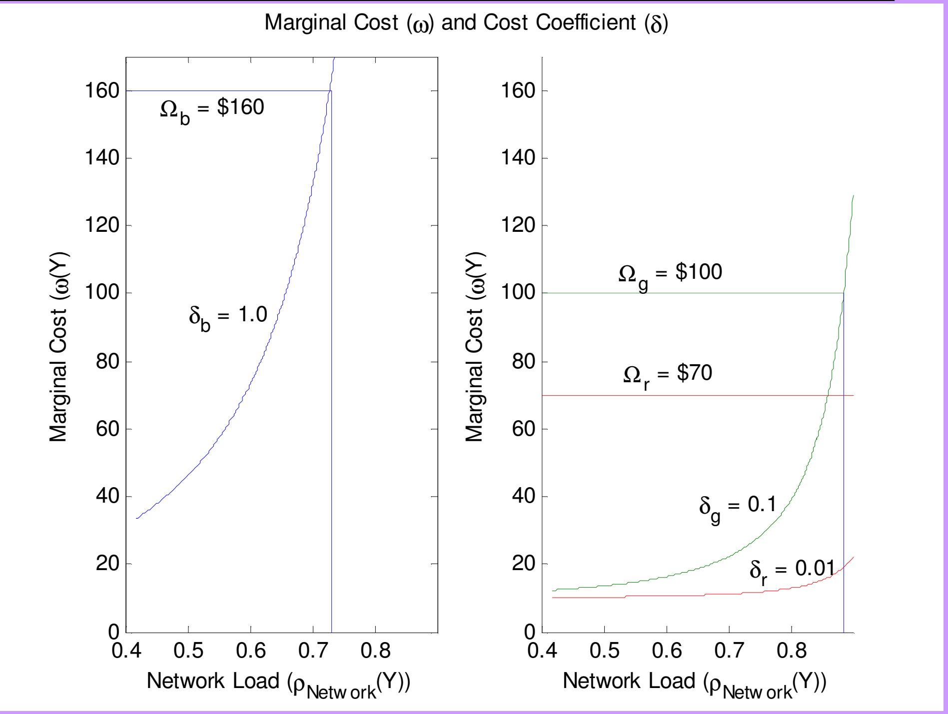
Congestion Sensitive Optimized Marginal Cost

We develop the Marginal Cost Function from the Optimized Mean Packet Count (M*):

$$\omega_{ns,t}(\hat{M}^*_{n,t}) = \delta_s (Y_{n,t} \frac{\partial \hat{M}^*_{n,t}}{\partial Y_{n,t}} + \hat{M}^*_{n,t}) + \theta_n$$

$$\frac{\partial \hat{M}^*_{n,t+1}}{\partial Y_{n,t+1}} \approx \frac{\hat{M}^*_{n,t+1} - \hat{M}^*_{n,t}}{(b_{OD} + b_{DO})}$$

Simulation



Three classes (Blue, Green, Red)
Cost coefficient parameter depends on security levels (High, Medium, Low)

- Cost coefficient parameter distinguishes Service Class

Mean Packet Count: M/M/1

	Blue	Green	Red
Provider 1	$1.00(Y_{1,t} \frac{\partial \hat{M}^*_{1,t}}{\partial Y_{1,t}} + \hat{M}^*_{1,t}) + 10$	$0.10(Y_{1,t} \frac{\partial \hat{M}^*_{1,t}}{\partial Y_{1,t}} + \hat{M}^*_{1,t}) + 10$	$0.01(Y_{1,t} \frac{\partial \hat{M}^*_{1,t}}{\partial Y_{1,t}} + \hat{M}^*_{1,t}) + 10$
Provider 2	$1.00(Y_{2,t} \frac{\partial \hat{M}^*_{2,t}}{\partial Y_{2,t}} + \hat{M}^*_{2,t}) + 10$	$0.10(Y_{2,t} \frac{\partial \hat{M}^*_{2,t}}{\partial Y_{2,t}} + \hat{M}^*_{2,t}) + 10$	$0.01(Y_{2,t} \frac{\partial \hat{M}^*_{2,t}}{\partial Y_{2,t}} + \hat{M}^*_{2,t}) + 10$

Game Theory Model

- Our Model is
  - Based on Bertrand Oligopoly Model
  - A Myopic Markovian-Bayesian Static Game of Incomplete Information
- Our models extends
  - Bandyopadhyay et al. On-Line-Exchange Model

- Bandyopadhyay et al. On-Line-Exchange Model
  - Based on Bertrand Model and "Model of Sale" example
  - Symmetric market
    - All parameters are fixed
  - Commodity is not Internet bandwidth
  - Two step static game of incomplete information
  - Homogeneous service
  - Uses Reinforcement Learning (RL) in simulation to determine best strategy

- Our Model:
  - Extension to Bandyopadhyay et al. model
  - Asymmetric market
    - Demand and cost are functions of the dynamic nature of Internet traffic
  - Commodity is internet bandwidth
  - "Myopic" Markovian static game of incomplete information
  - Heterogeneous service
  - An analytical framework to determine the best strategy in dynamic internet traffic

Oligopoly Model Selection

- Oligopoly
  - A small number of providers collectively influence
    - Market condition such as price, capacity
  - A single provider alone cannot completely control the market
- Two well-established fundamental models of Oligopoly
  - Bertrand Model
    - Strategic Variable: Price
  - Cournot Model
    - Strategic Variable: Capacity (quantity)

Oligopoly Model

- In the Internet, providers strategically interact
  - Long term:
    - Adds more capacity, i.e. "bandwidth wars"
  - Short term:
    - Price adjustment in fixed capacity, i.e., "price wars"
- Our Model is based on Bertrand Oligopoly Model
  - Short term
    - Session arrival and departure in a relatively short time period
  - Capacity does not change during the game
  - Providers adjust price to win over customers
  - Customers subscribe to the service from the lowest priced provider.

Game Model Selection

- Game Theory
  - The mathematical theory pertaining to the strategic interaction of decision makers
- There are four fundamental classes of game

Game Class	Equilibrium
Static Game of Complete Information	Nash Equilibrium
Dynamic Game of Complete Information	Subgame-perfect Nash equilibrium
Static Game of Incomplete Information	Bayesian Nash equilibrium
Dynamic Game of Incomplete Information	Perfect Bayesian Equilibrium

- Complete Information:
  - Providers' payoff or strategies are common knowledge
- Incomplete Information:
  - At least one player is unware of the payoffs or strategies of other providers
- Static Game
  - Players simultaneously interacts (chooses actions) without the knowledge of past
- Dynamic Game
  - Players repeatedly interacts based on the knowledge of game history (e.g., payoff)

Game Model

- Our model is Myopic Markovian-Bayesian Game of Incomplete Information
  - Each provider is a rational player
  - Each provider's payoff is private information.
  - All providers simultaneously select bid price without past knowledge of payoffs
  - "Myopic Markovian"
    - Each session is an instance of the game
    - Game uses one step nearsighted information
  - The game is also known as Bayesian Static Game of Incomplete Information
    - Developed based on Bayes' Conditional Probability Rule

Game Parameters

- Our Game Parameters
  - Strategic Players : A few Internet Service Providers
  - Strategic variable : Bid Price (p_{bid})
  - Commodity: the bandwidth of services in the Internet
  - Services: Homogeneous/Heterogeneous (Plat., Gold, Silv.) Services
  - Capacity: Peer capacity in bw (Fixed)
  - Demand: Sensitive to Internet traffic throughput (Variable)
  - Marginal Cost: Sensitive to network congestion (Variable)
  - Customer’s limited Budget: Reservation price (Fixed)
  - Payoff: Profit

Bayesian Static Game of Incomplete Information

- Static Bayesian Game of two Providers (A.com, B.com)
  - In Static Bayesian game, a provider's strategy is to maximize its' expected Profit
  - G = {$Action_A$, $Action_B$; $Type_A$, $Type_B$; $Belief_A$(), $Belief_B$(); $Payoff_A$(), $Payoff_B$()}
    - Action = Bid price (p_{bid})
    - Type = Provider's marginal cost (ω)
    - Payoff = Expected Profit ($E(u(.)$)
    - **$Belief_A(.) = Prob_A(Type_B \mid Type_A)$**
      - A.com's belief or uncertainty of B's Type given that A.com knows own type
      - It is a conditional probability function
      - It is also referred to as the Mixed Strategy Profile
  - A.com develops a set of feasible strategies from the belief function:

$$strategy\ h_{Aj} : Action_A \longleftarrow h_{Aj}(., Belief_A(.))$$

Game Model: Belief functions and Strategies

- The Belief function is the main entity of this Game
- Belief Function: $F_A(p)$:
  - is the Rejection probability of A.com for A's bid price p.
    - A.com's belief of B.com's winning probability for A.com's bid price p_A

$$F_A(p_A^{bid}) = \mathrm{Prob}(p_B \le p_A^{bid})$$

- Strategy space h is the set of functions over F(p)
  - Strategy is identified by the rejection probability γ
- A Strategy, h_{Aj} = "95% probability of having the bid rejected"

$$p_{n,s,t}^{bid} : F_{n,s,t}(p_{n,s,t}^{bid}) = prob(p \le p_{n,s,t}^{bid}) = 0.95$$

Belief Function ($\mathcal{F}(p)$)

- Belief function
  - It is a cumulative distribution function F(p)
- $F_A(p)$
  - A.com's belief of B.com's winning probability for A.com's bid price p_A

$$F_A(p_A^{bid}) = \text{Prob}(p_B \le p_A^{bid})$$

  - A.com's probability of having its p_A bid rejected
    - **The Rejection probability of A.com**

- A.com's rejection probability = 90%

$$F_A(p_A^{bid}) = \text{Prob}(p_B \le p_A^{bid}) = 0.90$$

  - A.com believes that B.com will select bid-prices at most p_A with 90%probability
  - A.com's winning probability = 10%

Strategy

- Strategy space h is the set of functions over F(p)
  - The strategy space is constructed from the Type and Action space
  - A.com's set of strategies h_{Aj} is the set of all possible functions with domain (input) $\boldsymbol{Type_A}$ and range (output) $\boldsymbol{Action_A}$.

$$strategy\ h_{Aj} : Action_A \longleftarrow h_{Aj}(Belief_A(\ldots, Type_A))$$

- A Strategy, h_{Aj} = "95% probability of having the bid rejected"

$$p^{bid}_{n,s,t} : F_{n,s,t}(p^{bid}_{n,s,t}) = prob(p \leq p^{bid}_{n,s,t}) = 0.95$$

- Strategy is identified by the rejection probability γ

$$F(p_{my\ bid}) = Prob(p_{others\ bid} \leq p_{my\ bid}) = \gamma$$

Game Model: Belief Parameters

- $F(p) = \text{Game}(N, \Gamma, \Delta(Y), \Omega_s, \omega(M^*))$
  - N: Number of providers in the market
  - Γ: Market Capacity
  - Δ(Y): Market Demand (function of throughput)
  - Ω_s: Customer Reservation Price, function of service type (s)
  - ω(M*)): Marginal Cost (function of mean packet count, M)

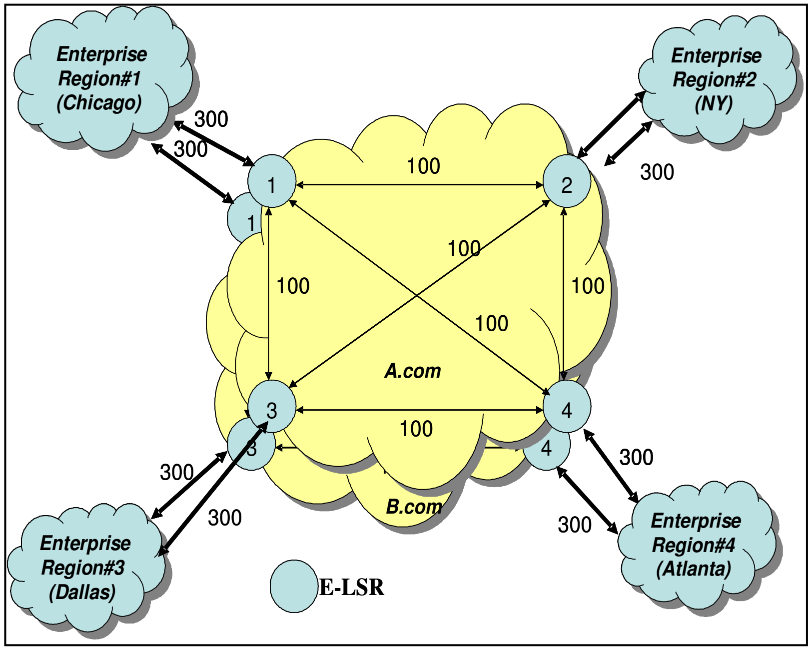
$$\Gamma = \sum_{n=1}^{N} K_n \rho_{TE} = \rho_{TE} \sum_{n=1}^{N} K_n$$

$$\Delta(Y_{n,t}) = \begin{cases} \rho_{TE}(N-1)K + \varepsilon & NY_{n,t} \le \rho_{TE} K, \varepsilon > 0 \\ NY_{n,t} & \rho_{TE} K < NY_{n,t} \le \Delta_{Max} \end{cases}$$

$$\omega_{n,s,t}(\hat{M}^*_{n,t}) = \delta_s \left(Y_{n,t} \frac{\partial \hat{M}^*_{n,t}}{\partial Y_{n,t}} + \hat{M}^*_{n,t}\right) + \theta_n$$

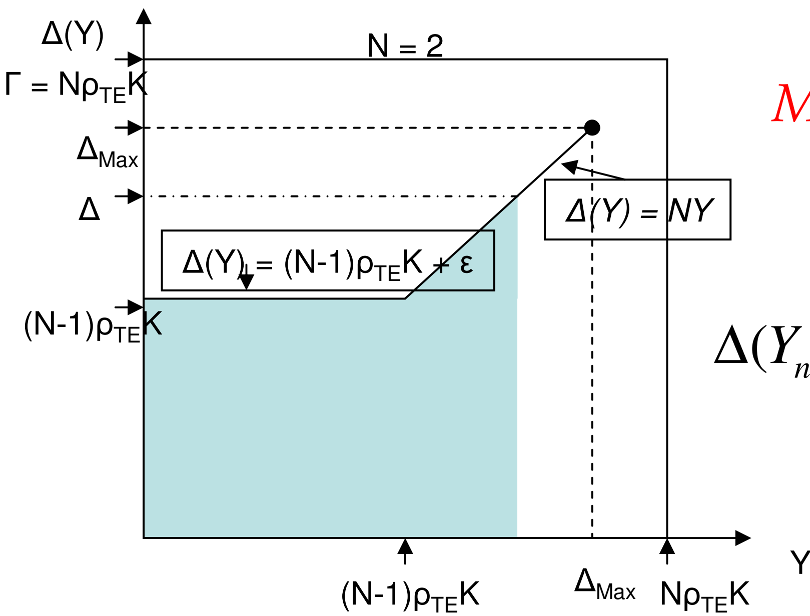
$$\frac{\partial \hat{M}^*_{n,t}}{\partial Y_{n,t}} = \frac{C}{(C - \frac{1}{12} Y_{n,t})^2} \qquad Analysis$$

$$\frac{\partial \hat{M}^*_{n,t+1}}{\partial Y_{n,t+1}} \approx \frac{\hat{M}^*_{n,t+1} - \hat{M}^*_{n,t}}{(b_{OD} + b_{DO})} \qquad Simulation$$



Market Capacity (Γ): Aggregate Traffic Engineered access bandwidth capacities of all providers in a market

$$\Gamma = \sum_{n=1}^{N} K_n \rho_{TE} = \rho_{TE} \sum_{n=1}^{N} K_n$$



Market Demand (Δ):

$$\Delta(Y_{n,t}) = \begin{cases} \rho_{TE}(N-1)K + \varepsilon & NY_{n,t} \le \rho_{TE}K, \varepsilon > 0 \\ NY_{n,t} & \rho_{TE}K < NY_{n,t} \le \Delta_{Max} \end{cases}$$

Market Demand

- Max Market Demand (Δ_{Max})
  - Aggregate Bandwidth in active session by all the customers from all the providers at a certain instant of game (t)
  - An NSP cannot meet the demand (Δ) of the whole market
    - $\rho_{TE}K < \Delta$
  - Maximum Market Demand is less than Market Capacity
    - $\Delta_{Max} < \Gamma$
  - Market Demand is greater than N-1 providers' aggregate capacity
    - $\rho_{TE}(N-1)K < \Delta <= \Delta_{Max}$
- Proposed Market Demand is a function of traffic served (Network output/production)
  - Network is loss-less (no packet drop occurs in the network)

$$\Delta(Y_{n,t}) = \begin{cases} \rho_{TE}(N-1)K + \varepsilon & NY_{n,t} \le \rho_{TE}K, \varepsilon > 0 \\ NY_{n,t} & \rho_{TE}K < NY_{n,t} \le \Delta_{Max} \end{cases}$$

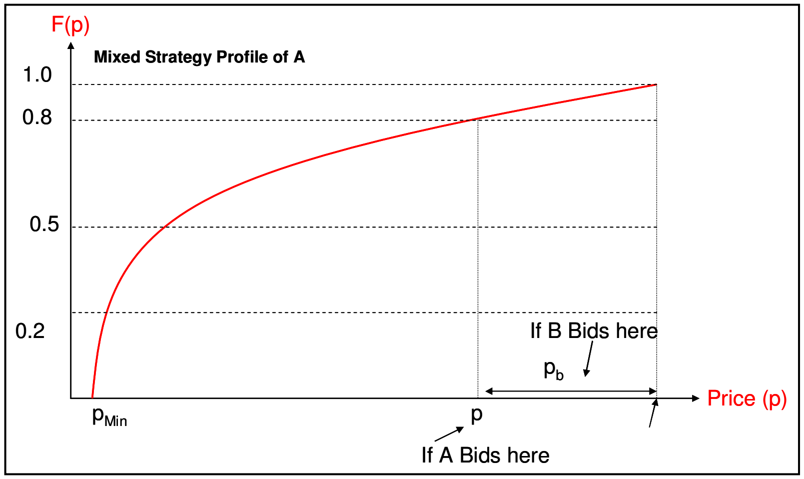
Y_t : Sum of output (production) traffic bandwidth in all the egress ports of an NSP at a certain Instant of the game (t)

Reservation Price of the Institution

- Reservation price (Ω) is the price that a customer is willing to pay in the Reverse Auction
  - It can be considered as customer's budget.
- We do not study the method of determining Ω.
- We assume
  - Enterprises (customers) are rational
    - Reservation price is selected during the business agreement
    - Enterprises do not violate the agreement
      - Do not change the reservation price during the game
  - for Homogeneous services, Ω is a same fixed value for all providers
  - For Heterogeneous services, Ω_s depends on the type of service
- Enterprises may adopt their own strategies to determine Ω.
  - This will require another larger research
    - For example, Enterprise selects reservation price by considering monopoly market (assume that all providers constitute a Super-provider)

Deriving Belief Function

A.com's price lower than B.com price



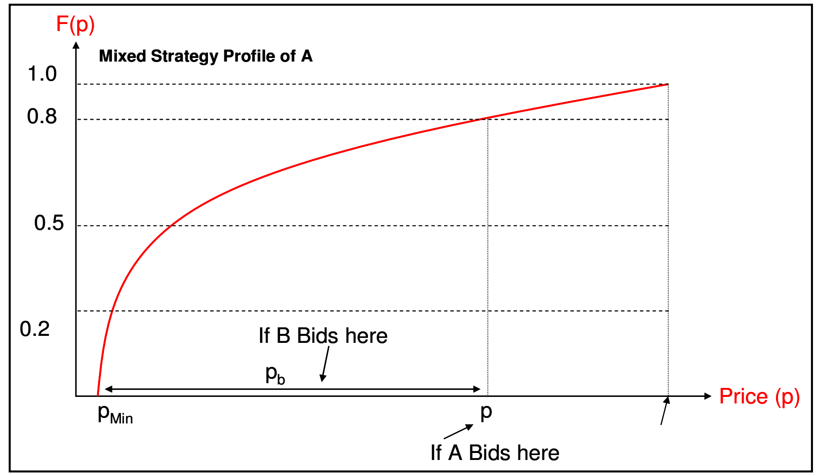
This event occurs: 1-F(p)=prob(p_b > p)

$$u_L(p) = (p - \omega(.))\overline{Y} = (p - \omega(.))\rho K$$

If p = p_{Min}

$$u_L(p_{Min}) = (p_{Min} - \omega(.))\rho K$$

A.com's price higher than B.com price



This event occurs: F(p)=prob(p_b <= p)

$$u_H(p) = (p - \omega(.))(\Delta(.) - \rho K)$$

If p = Ω

$$u_H(\Omega) = (\Omega - \omega(.))(\Delta(.) - \rho K)$$

Expected Unit Profit = $\overline{u}(p) = u_L(p)(1 - F(p)) + u_H(p)F(p)$

Game Model: Belief Function Equations

The derived Belief function for 2 providers is as follows:

$$F(p) = \frac{(p - \omega(M^{*}_{n,t}))\rho_{TE}K - (\Omega_s - \omega(M^{*}_{n,t}))(\Delta(Y_{n,t}) - \rho_{TE}K)}{(p - \omega(M^{*}_{n,t}))(2\rho_{TE}K - \Delta(Y_{n,t}))}$$

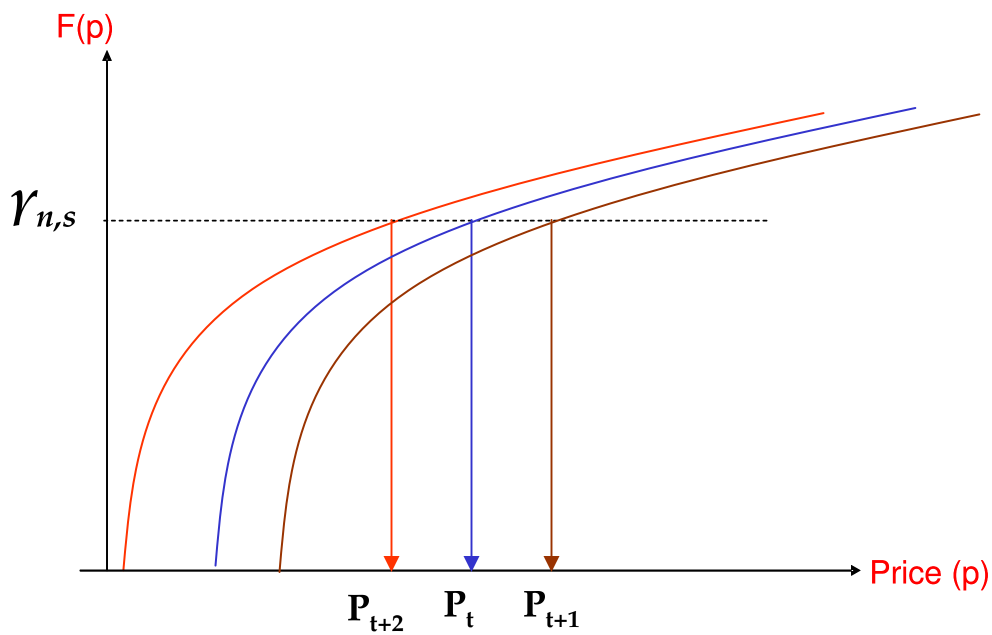
The derived Belief function for N providers is as follows:

$$F_{n,s,t}(p_{n,s,t}) = \left[\frac{(p_{n,s,t} - \omega_{n,s,t}(M^{*}_{n,t}))\rho_{TE}K - (\Omega_s - \omega_{n,s,t}(M^{*}_{n,t})(\Delta(Y^{*}_{n,t}) - (N-1)\rho_{TE}K))}{(p_{n,s,t} - \omega_{n,s,t}(M^{*}_{n,t}))(N\rho_{TE}K - \Delta(Y^{*}_{n,t}))} \right]^{\frac{1}{N-1}}$$

Dissertation presents the derivation of the belief function and associated parameters

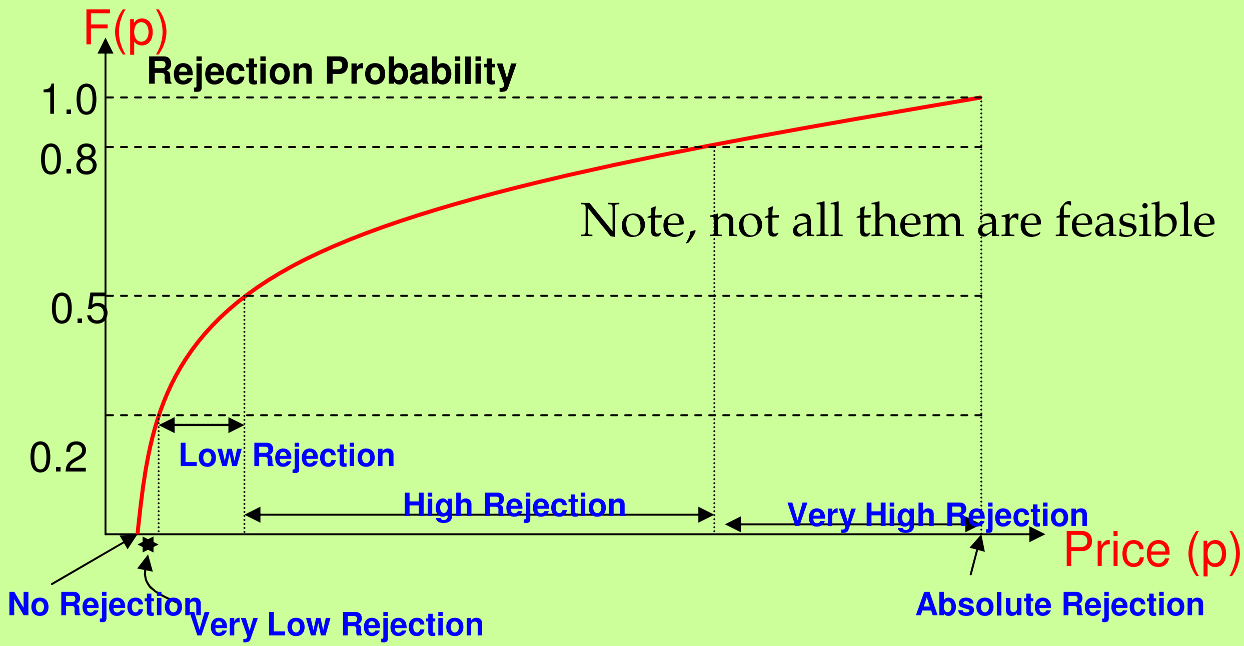
Game Model: Properties of the Belief Function

- Belief function shifts left or right on the p axis (x-axis)
  - due to the change in the network production and Network congestion
  - as a function of Mean packet count in the network
  - causes a bid price of a service to change
- Each service class has a distinct Belief function
- For each call, each provider has a distinct Belief function



Feasible Strategies

A Provider finds a bid price of a service from F(p) using h(.)



Strategy	Feasible strategies
Very Low Rejection	$p^{bid}_{n,s,t} : F_{n,s,t}(p^{bid}_{n,s,t}) = prob(p \le p^{bid}_{n,s,t}) = \gamma = 0.05$
Low Rejection	$p^{bid}_{n,s,t} : F_{n,s,t}(p^{bid}_{n,s,t}) = prob(p \le p^{bid}_{n,s,t}) = \gamma = 0.35$
Rejection Neutral	$p^{bid}_{n,s,t} = Mean(F_{n,s,t}(p))$
High Rejection	$p^{bid}_{n,s,t} : F_{n,s,t}(p^{bid}_{n,s,t}) = prob(p \le p^{bid}_{n,s,t}) = \gamma = 0.65$
Very High Rejection	$p^{bid}_{n,s,t} : F_{n,s,t}(p^{bid}_{n,s,t}) = prob(p \le p^{bid}_{n,s,t}) = \gamma = 0.95$

Bid Price

Derived Bid Price for Rejection Neutral Strategy:

$$p_{n,s,t}^{Neutral} = \frac{(\Delta(Y_{n,t}^*) - \rho_{TE} K)(\Omega_s - \omega_{n,s,t}(M_{n,t}^*))}{(2\rho_{TE} K - \Delta(Y_{n,t}^*))} \left[\begin{array}{l} \ln\left(\dfrac{\Omega_s - \omega_{n,s,t}(M_{n,t}^*)}{p_{Min,n,s,t} - \omega_{n,s,t}(M_{n,t}^*)} \right) \\ + \omega_{n,s,t}(M_{n,t}^*) \left(\dfrac{1}{p_{Min,n,s,t} - \omega_{n,s,t}(M_{n,t}^*)} - \dfrac{1}{\Omega_s - \omega_{n,s,t}(M_{n,t}^*)} \right) \end{array} \right]$$

Derived Bid Price for any Strategy ($\gamma_{n,s}$):

$$p_{n,s,t}^{\gamma} = \omega_{n,s,t}(M_{n,t}^*) + \left[\frac{1}{\left(p_{Min,n,s,t} - \omega_{n,s,t}(M_{n,t}^*)\right)} - \frac{\gamma_{n,s}}{\left(\dfrac{(\Delta(Y_{n,t}^*) - \rho_{TE} K)(\Omega_s - \omega_{n,s,t}(M_{n,t}^*))}{(2\rho_{TE} K - \Delta(Y_{n,t}^*))} \right)} \right]^{-1}$$

Dissertation presents the derivation of these functions

Market Price

We determine Analytical Market Price from the Bid Price

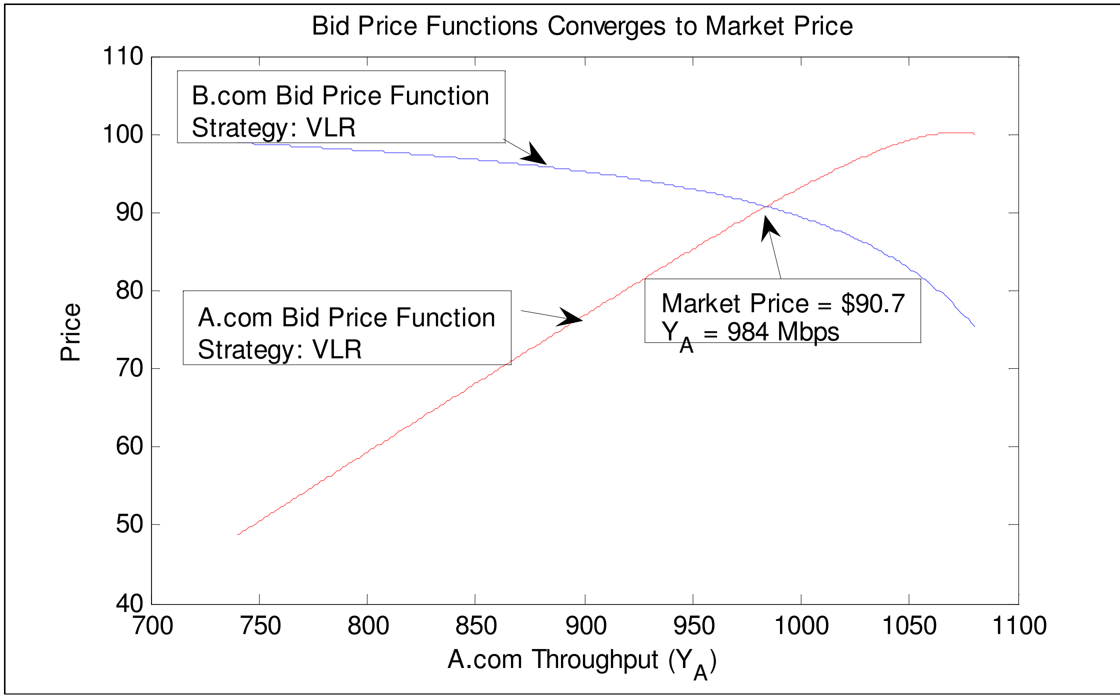
When Two Providers use an Identical Strategy Set:

$$p^{*}_{Market,s,t} = p^{\gamma}_{n,s,t}(Y_{n,t})$$

When Two Providers do not use an Identical Strategy Set:

- **Market price can be found by solving bid price equations of both providers**
- **Bid price equations are hyperbolic function**
  - **Solving by algebraic method is seemingly difficult**
  - **We apply Numerical Analysis in MATLAB to solve bid price equations**

Finding Market Price by Numerical Analysis



- ***Bid prices converge to market price***
  - ***At a steady state market***

$$p^{*}_{Market,s,t} = p^{bid}_{A,s,t}(Y^{*}_{A,t}) = p^{bid}_{B,s,t}(Y^{*}_{B,t})$$

$$Y_A^* \neq Y_B^*, \quad \Delta^* = Y_A^* + Y_B^*$$

$$p_{A,s,t}^{\gamma_{A,s}^{j}} = \omega_{A,s,t}(Y^*_{A,t}) + \left[\frac{1}{\left(p_{Min,A,s,t} - \omega_{A,g,t}(Y^*_{A,t})\right)} - \frac{\gamma^{j}_{A,s}}{\left(\frac{(\Delta(Y^*_{A,t}) - \rho_{TE}K)(\Omega_s - \omega_{A,s,t}(Y^*_{A,t}))}{(2\rho_{TE}K - \Delta(Y^*_{A,t}))}\right)} \right]^{-1}$$

$$p_{B,s,t}^{\gamma_{B,s}^{k}} = \omega_{B,s,t}(\Delta^* - Y^*_{A,t}) + \left[\frac{1}{\left(p_{Min,B,s,t} - \omega_{B,s,t}(\Delta^* - Y^*_{A,t})\right)} - \frac{\gamma^{k}_{B,s}}{\left(\frac{(\Delta(\Delta^* - Y^*_{A,t}) - \rho_{TE}K)(\Omega_g - \omega_{B,s,t}(\Delta^* - Y^*_{A,t}))}{(2\rho_{TE}K - \Delta(\Delta^* - Y^*_{A,t}))}\right)} \right]^{-1}$$

Profit

Homogeneous Service (All strategies):

$$u_n^*(.) = (p_{n,g,t}^* - \omega_{n,g,t}^*)Y^*$$

We study homogeneous service based market mainly by math. equations

Heterogeneous Service (Identical Strategy Set):

$$u_n^*(.) = (p_{n,b,t}^* - \omega_{n,b,t}^*)(\frac{2}{9})Y_{n,t}^* + (p_{n,g,t}^* - \omega_{n,g,t}^*)(\frac{3}{9})Y_{n,t}^* + (p_{n,r,t}^* - \omega_{n,r,t}^*)(\frac{4}{9})Y_{n,t}^*$$

Heterogeneous Service (Non-Identical Strategy Set):

Throughput of each service is unknown

$$Y_{n,t} = Y_{n,b,t} + Y_{n,g,t} + Y_{n,r,t}$$

One equation three unknowns

Unique Profit cannot be determined by math.

We study heterogeneous service based market mainly by simulation

Results

We demonstrate

- Validation
- Advantages:
  - Customer's benefit
    - Is market price less than customers' budget (reservation price)?
  - Provider's benefit
    - Is market price above marginal cost?
    - Does providers' obtain positive Profit?
    - Can providers optimize in fair market share Profit?
- Profit Maximizing Strategies
  - Best Strategies (Bayesian-Nash and Pareto-Efficient)
- TE Application

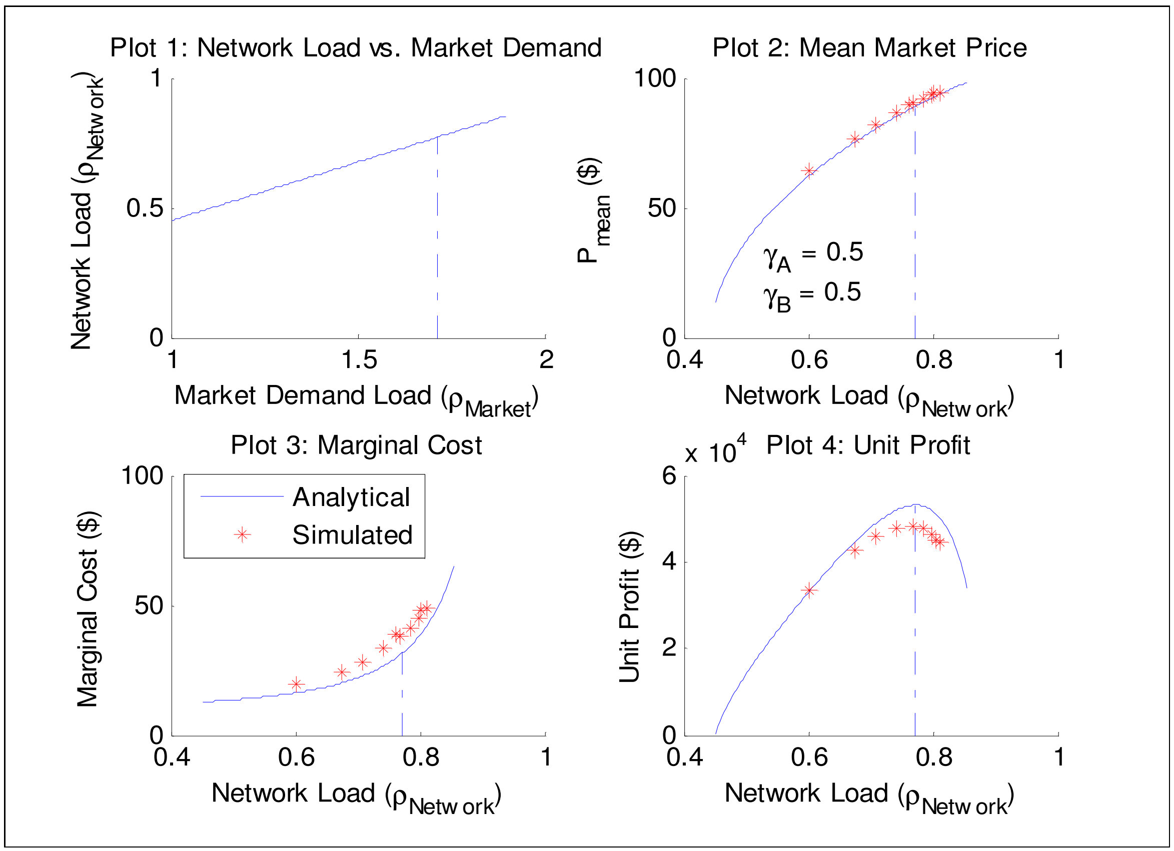
Homogeneous Service Market: $\{h_A, h_B\} = \{RN, RN\}$

Unit Profit Curve:

- *Monotonous*
- *Bound*
- *Concave:*

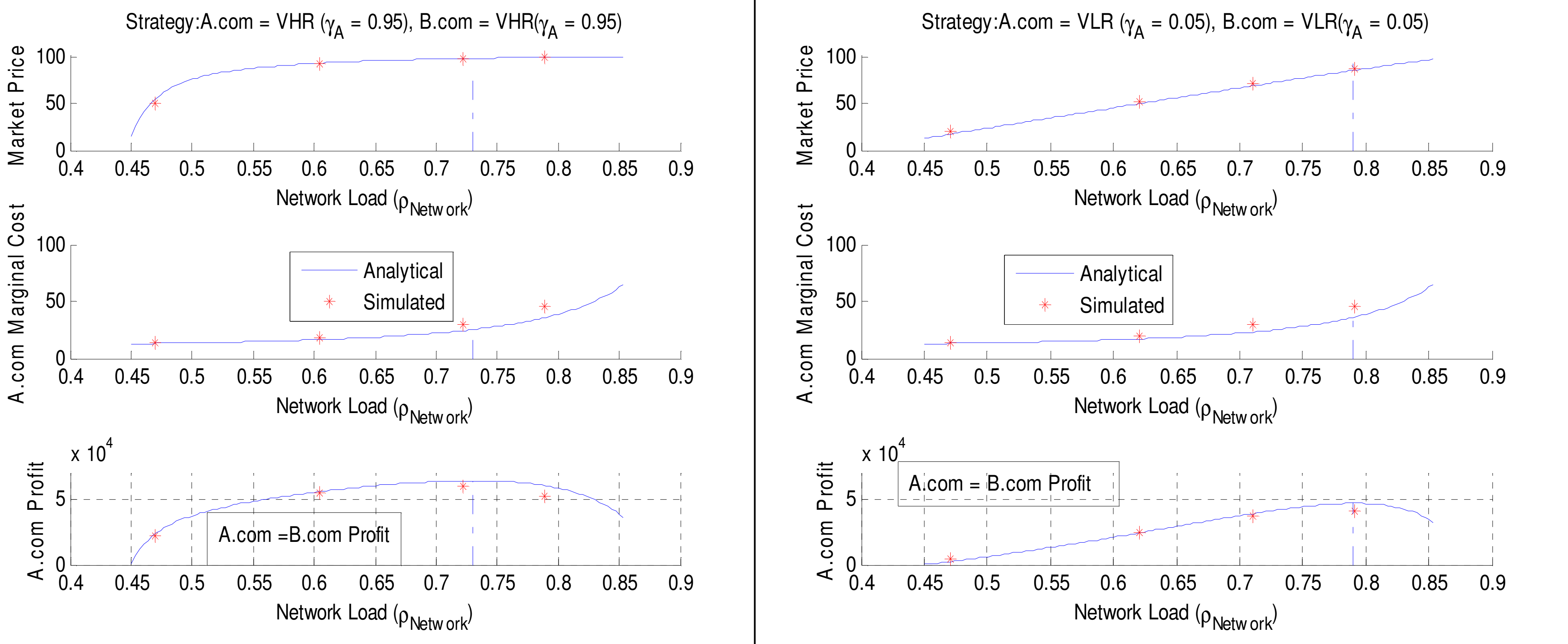
$$u(\psi\rho_{n,1} + (1-\psi)\rho_{n,2}) \geq \psi u(\rho_{n,1}) + (1-\psi)u(\rho_{n,2}), \quad \psi \in [0,1]$$

- *Simulation validates Analysis*
- *Advantages:*
  - *Market Price less than Reservation Price*
  - *Market Price more than Marginal Cost*
  - *Optimizes in Positive Profit in Fair share of*
    - *Market demand and throughput*
    - *Optimum load is around 0.7704*



Homogeneous Service Market (Identical Strategies)

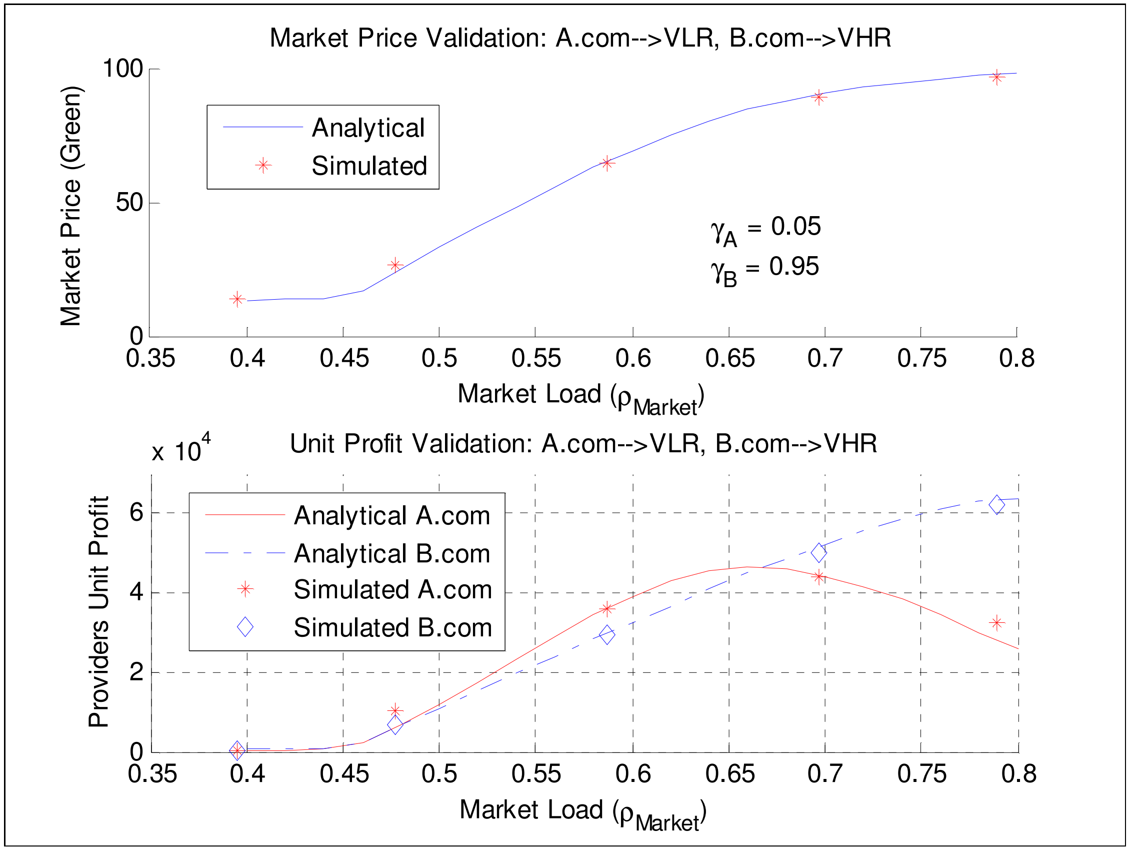
- Simulation Validates Analysis
- Advantages:
  - Market Price less than Reservation Price
  - Market Price more than Marginal Cost
  - Optimizes in Positive Profit in Fair share of
    - Market demand and throughput
    - Optimum Load is around 0.74 to 0.77
- {VHR,VHR} yields higher Profit, VHR strategy dominates



Homogeneous Service Market (Non-Identical Strategies)

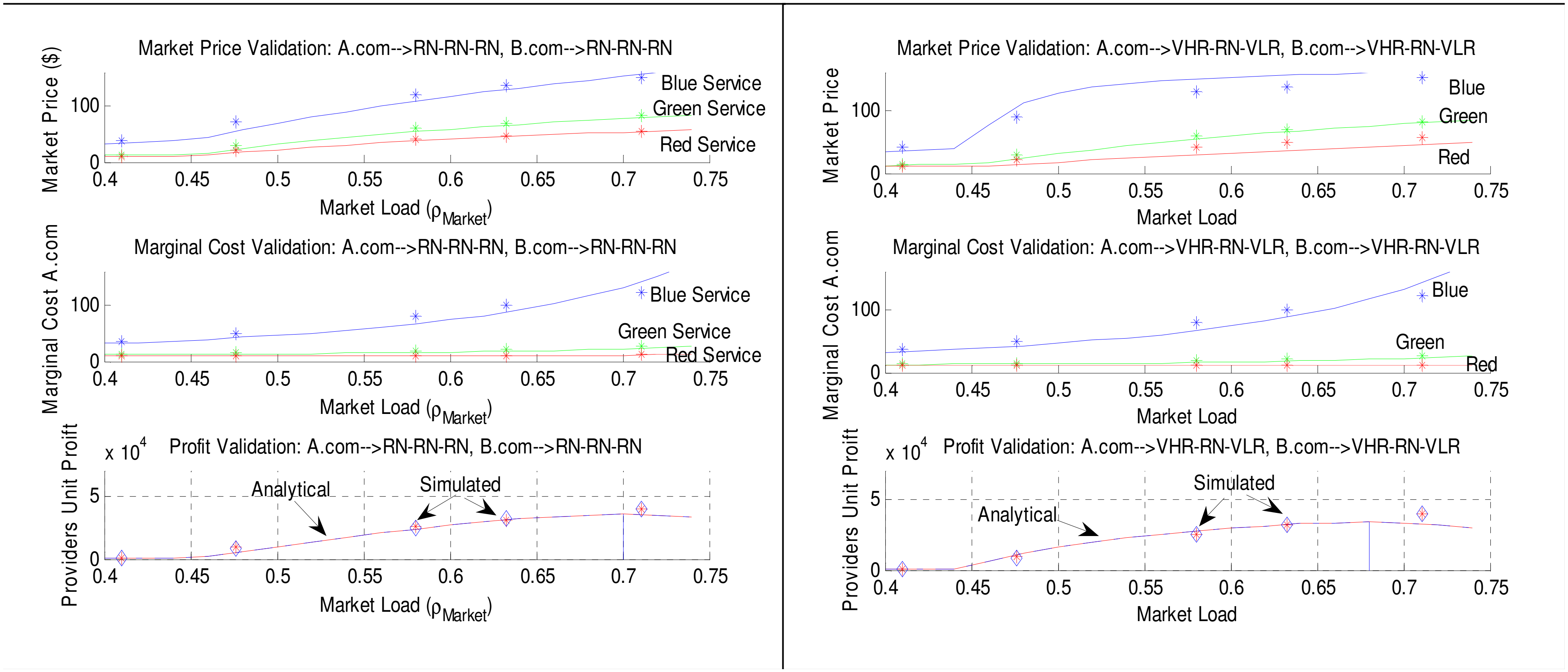
- ***Lower rejection strategy***
  - ***causes to operate in lower optimum load***
- ***Higher rejection strategy***
  - ***causes to operate in higher optimum load***
- ***Higher rejection strategy yields higher Profit***
  - ***Higher rejection strategy is dominant***

- ***Simulation Validates Analysis***
- ***Advantages:***
  - ***Market Price less than Reservation Price***
  - ***Market Price more than Marginal Cost***
  - ***Optimizes in Positive Profit***



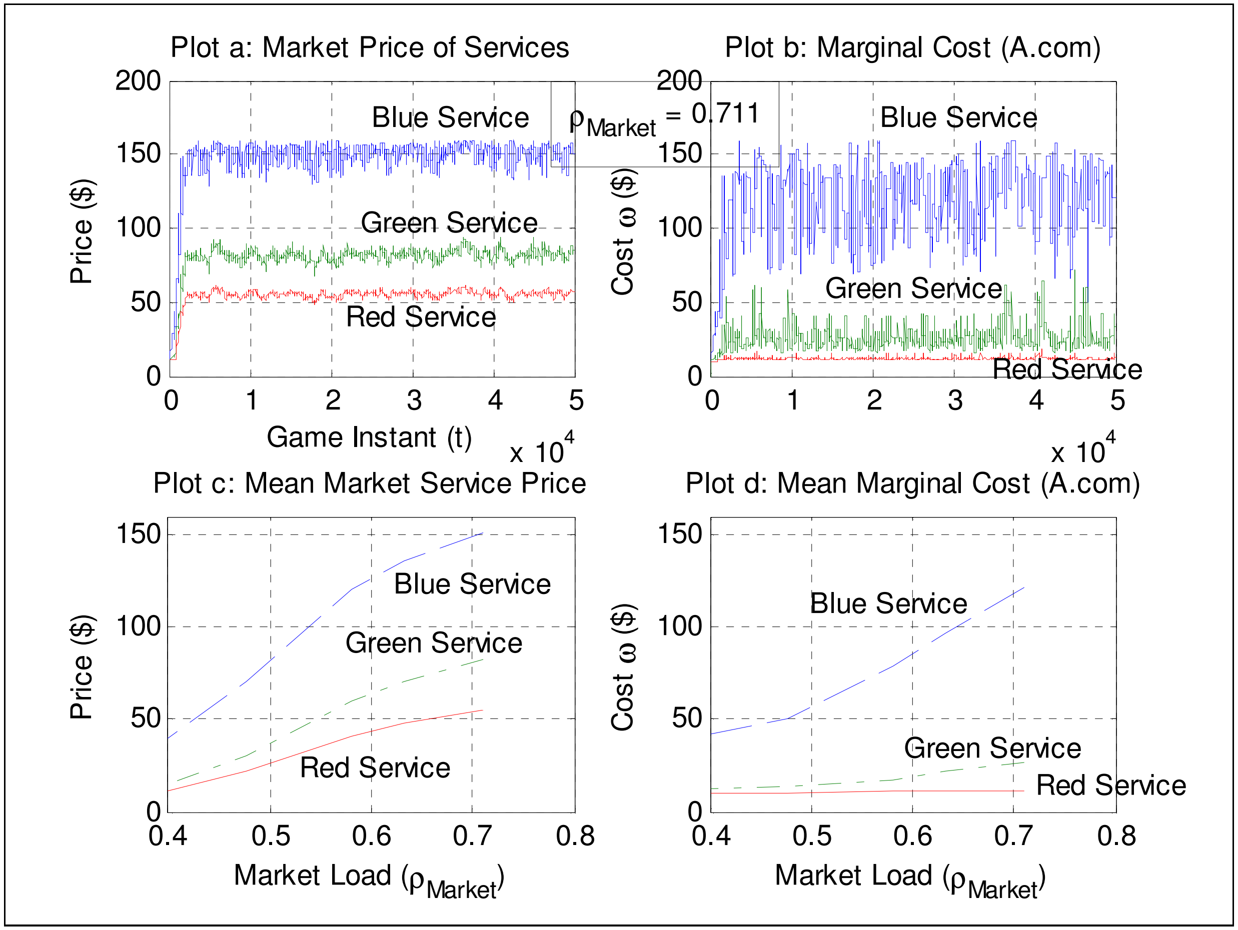
Heterogeneous Service Market (Identical Strategies)

- *Simulation validates analysis*
- $p_b > p_g > p_r$
- *Advantages:*
  - *Market Price less than Reservation Price*
  - *Market price more than Marginal Cost*
  - *Optimizes in positive Profit in Fair market share of*
    - *Market demand and throughput*
    - *Optimum load is around .68 to .70*



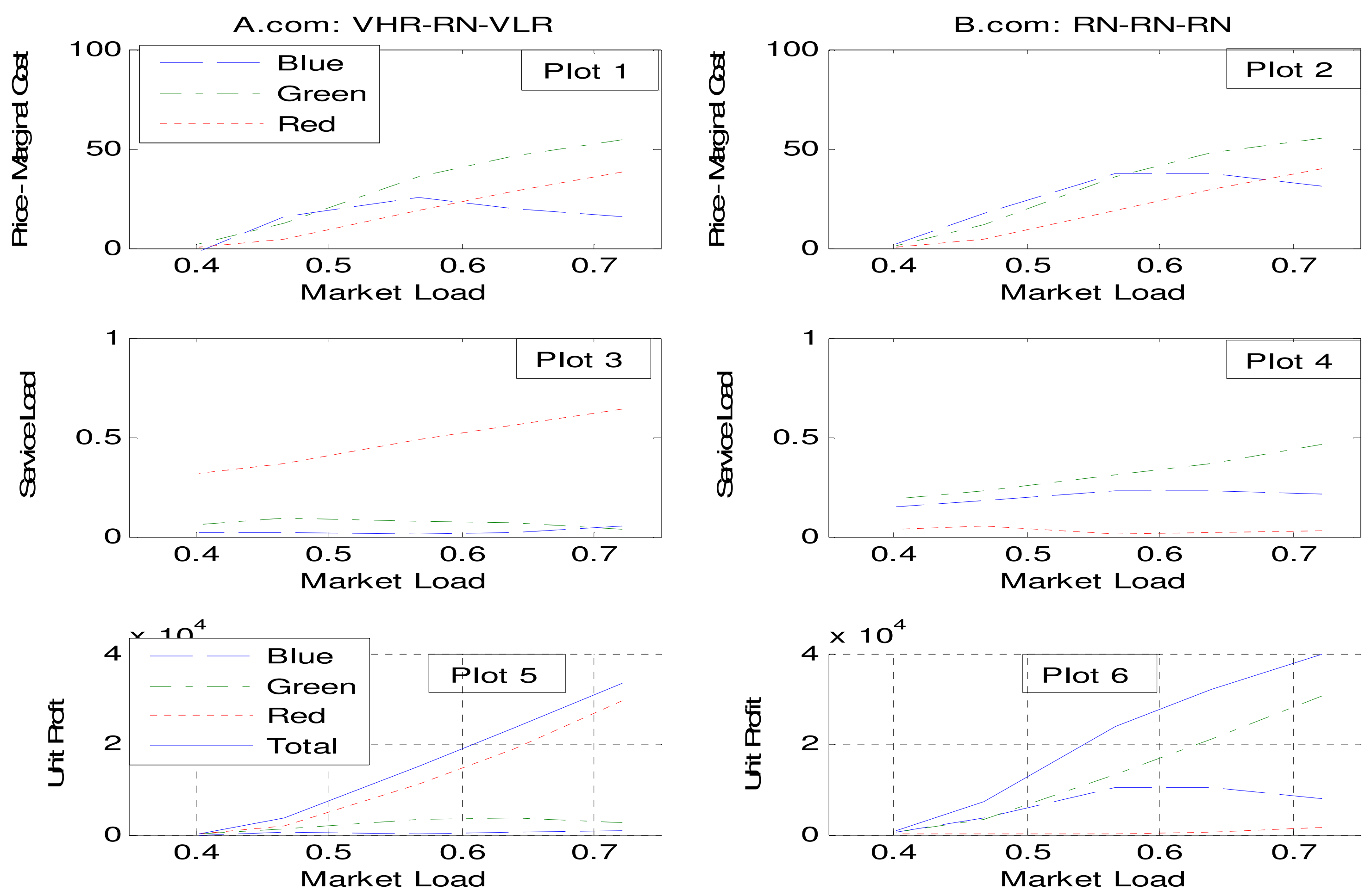
Heterogeneous Service Market (Identical Strategies) {RN,RN,RN}

- $p_b > p_g > p_r$
- *Advantages:*
  - *Market Price less than Reservation Price*
  - *Market price more than Marginal Cost*
  - *Optimizes in positive Profit*



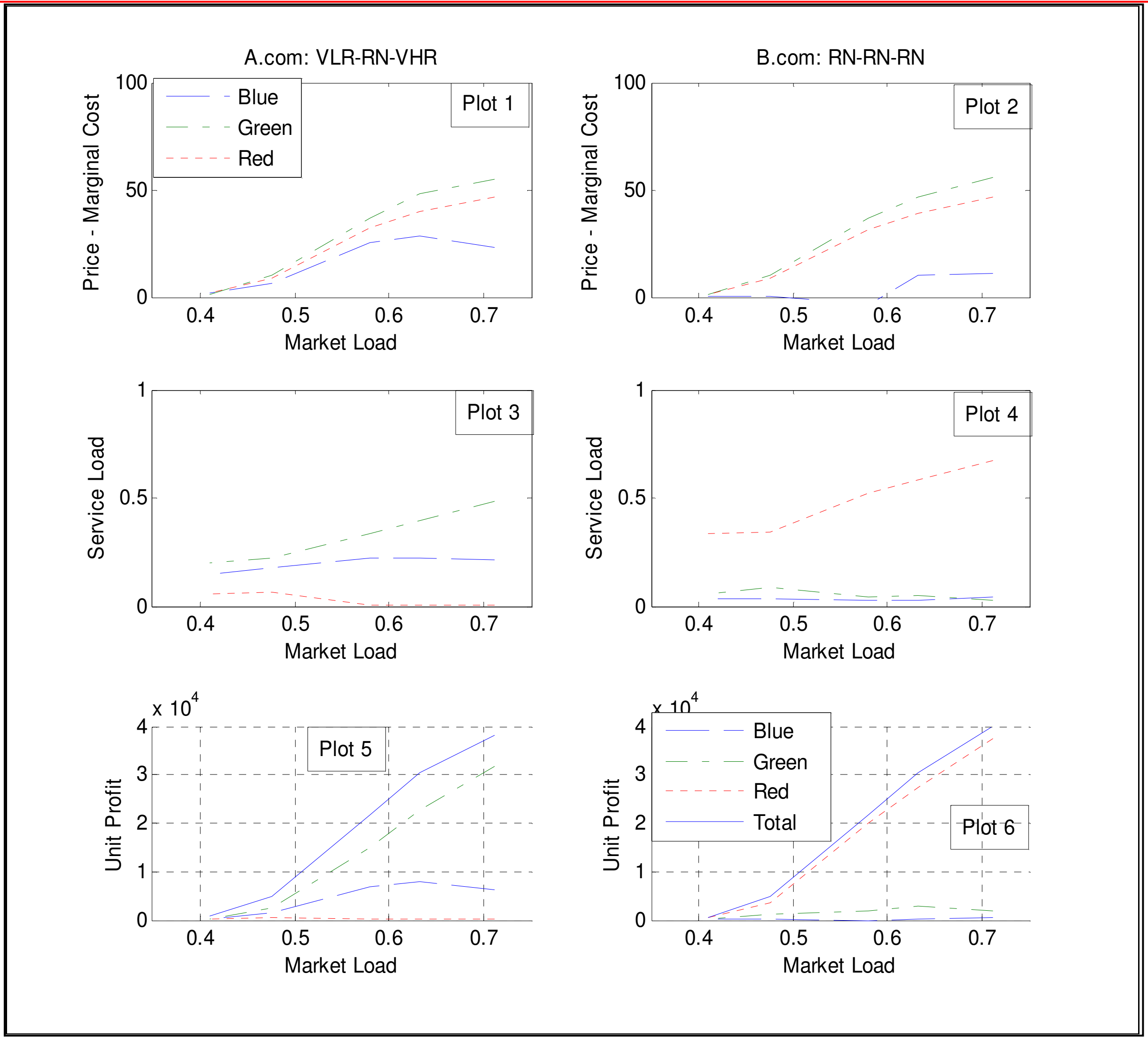
Heterogeneous Service Market (Non-Identical Strategies)

- *Higher Priced Service May Not Bring Higher Profit*
- *Providers' Should Select Lower Rejection Strategy For Higher Profit Yielding Services*
- *Providers' Should Select Higher Rejection Strategy For Lower Profit Yielding services*



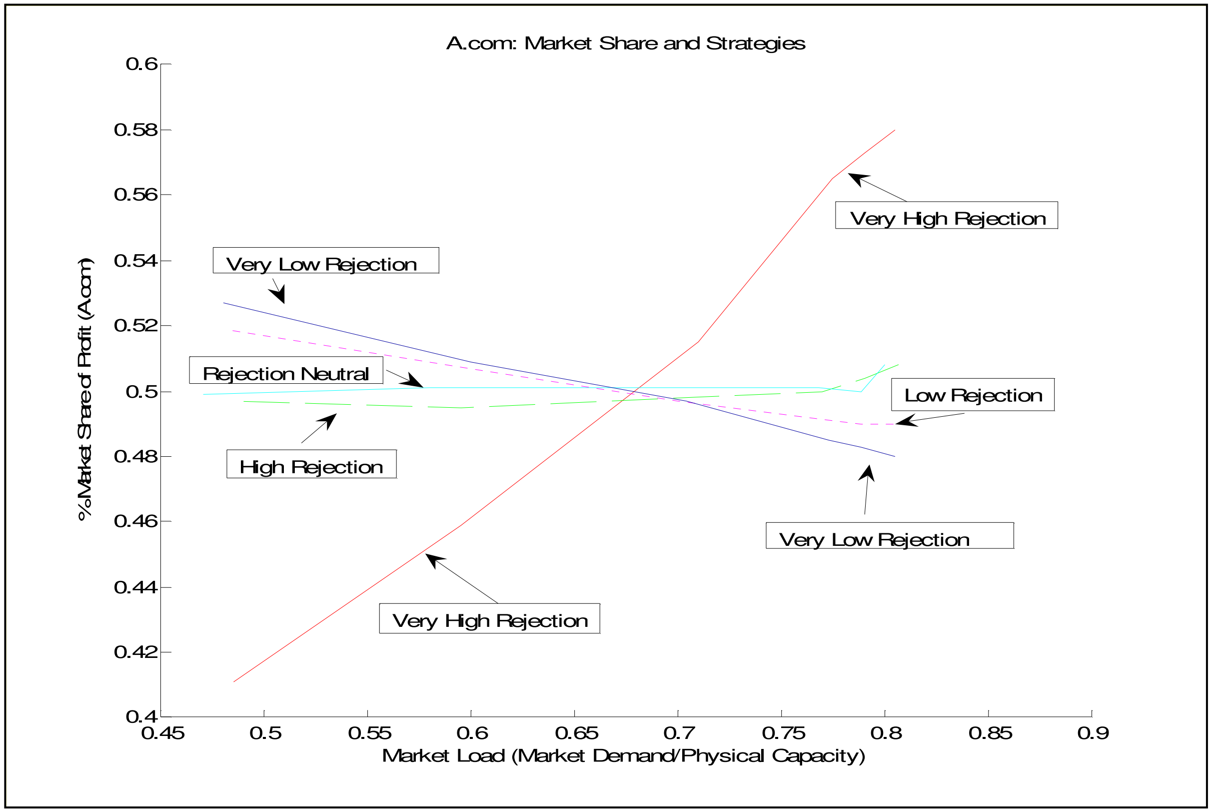
Heterogeneous Service Market (Non-Identical Strategies)

Careful Strategy Selection May Allow a Provider to Optimize the Market Profit Share by Selling Only the Lowest Valued Service



The Market Share of Profit Changes Due to the Change in Market Demand

- Market share in the dynamic internet traffic demand
  - remains invariant for the Rejection Neutral strategy
  - remains close to invariant for the HR and LR strategies
  - changes rapidly for the VHR and VLR strategies
- Assign strategies if traffic demand does not change and known:
  - VHR: for High demand
  - VLR: for Low demand



$$\{h_{Aj}^{\forall j}, h_{Aj}^{RN}\} \text{ vs. } \{h_{Aj}^{RN}, h_{Aj}^{RN}\}$$

The "Best Strategy" Set Should Optimize Profit in all Market Load

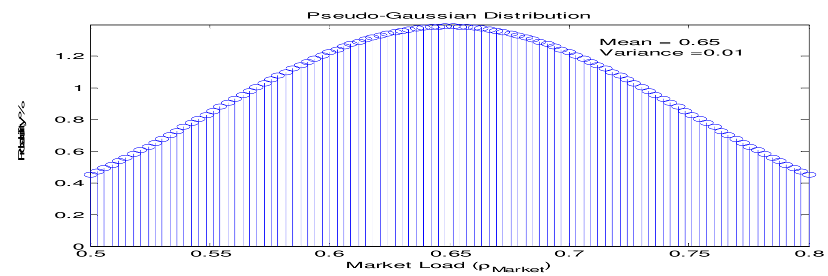
Bayesian-Nash Equilibrium:

Find $\{h_{Aj}^{*}, h_{Bj}^{*}\}$

s.t. $E[u_A(h_{Aj}^{*}, h_{Bj}^{*})] \geq E[u_A(h_{Aj}^{\forall j}, h_{Bj}^{*})]$

- Internet Traffic demand varies and pattern is unknown
- We use a hypothetical market load distribution
  - Gaussian Normal

$$prob(\rho_{Market}) = \frac{1}{\sqrt{2\pi(0.01)}} \exp^{-\frac{(\rho_{Market}-0.65)^2}{2(0.01)}}$$



- Our proposal to compute the expected unit Profit as follows:

$$E[u_A(.)] = \sum_{\forall \rho_{Market}} prob(\rho_{Market}) u_A(.)$$

$$E[u_B(.)] = \sum_{\forall \rho_{Market}} prob(\rho_{Market}) u_B(.)$$

Analytical Algorithm to Find Best Strategy Set

```
FOR γ_Aj = 0.05 to 0.95
        FOR γ_Bj = 0.05 to 0.95
            FOR ρ_Market = Min to Max
                Develop Belief Functions ()
                Find Bid_Prices_A;
                Find Bid_Prices_B;
                Find Market Price;
                Find Network_Load_A;
                Find Network_Load_B;
                Find Marginal Cost_A
                Find Marginal Cost_B;
                Find U_A(.);
                Find U_B(.);
            END;
```

$$E[u_A(.)] = \sum_{\forall \rho_{Market}} prob(\rho_{Market}) u_A(.)$$

$$E[u_B(.)] = \sum_{\forall \rho_{Market}} prob(\rho_{Market}) u_B(.)$$

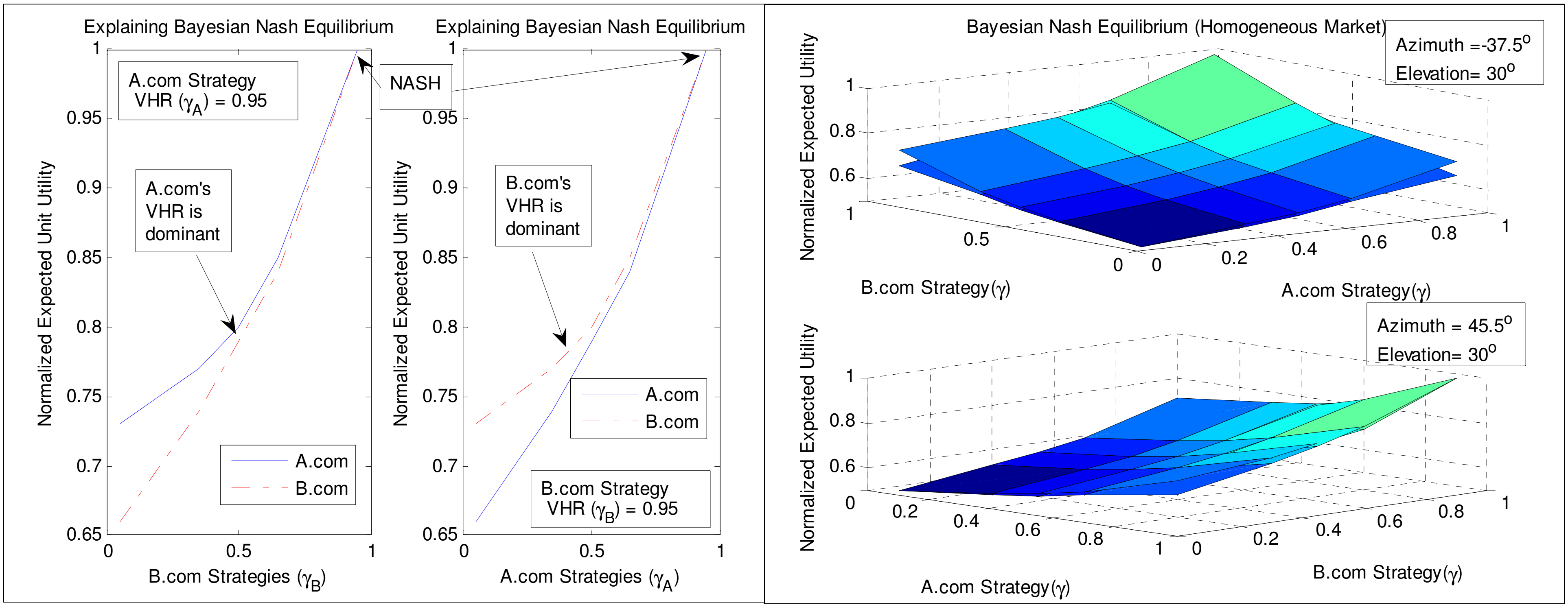
```
        END;
END;
```

$$Find\ \{\gamma_{Aj}, \gamma_{Bj}\}\ s.t.\ E[u_n(\gamma_{Aj}^{*}, \gamma_{Bj}^{*})] \geq E[u_n(\gamma_{Aj}^{\forall j}, \gamma_{Bj}^{*})]$$

Homogeneous Market: Analytical Best Strategy

		B.com				
h_{nj}		*VLR*	*LR*	*RN*	*HR*	*VHR*
A.com	*VLR*	(.50,.50)	(.54,.55)	(.57,.58)	(.60,.61)	(.66,.73)
	LR	(.55,.54)	(.59,.59)	(.62,.62)	(.65,.66)	(.74,.77)
	RN	(.58,.57)	(.62,.62)	(.65,.65)	(.69,.69)	(.79,.80)
	HR	*(.61,.60)*	*(.66,.65)*	*(.69,.69)*	*(.73,.73)*	*(.84,.85)*
	VHR	*(.73,.66)*	*(.77,.74)*	*(.80,.79)*	*(.85,.84)*	*(1.00,1.00)√√*

Unique
Bayesian-Nash Equilibrium
= {VHR, VHR}

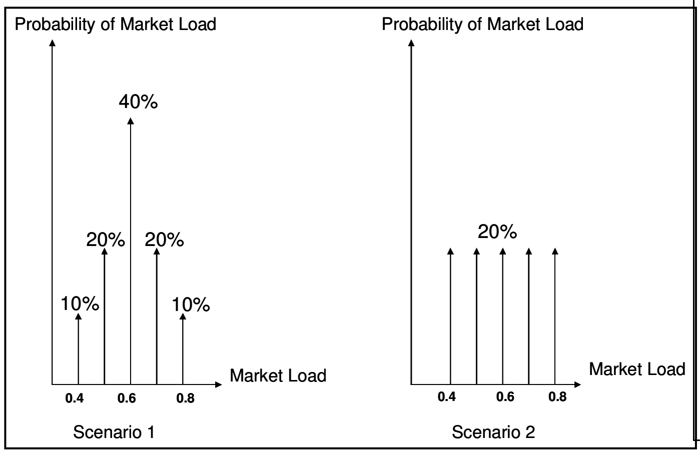


{VHR, VHR} is also Pareto-Efficient Set because there is no other set (α) s.t.
$$u_j(\alpha) > u_j(a = \{Very\_High\_Rejection, Very\_High\_Rejection\}) \quad \forall j$$

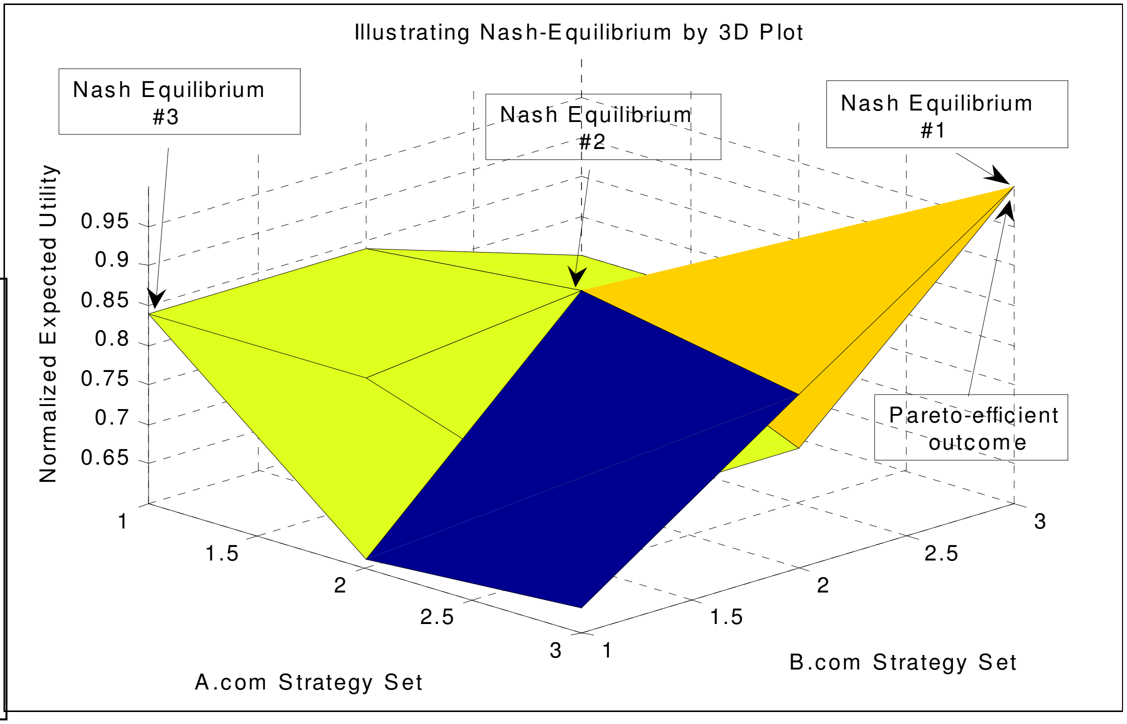
Heterogeneous Market: Best Strategy Set from Simulation

- Three Bayesian-Nash Equilibriums
- Existence of Pareto-Efficient Outcome

1 = {VHR-RN-VLR}
2 = {RN-RN-RN}
3 = {VLR-RN-VHR}

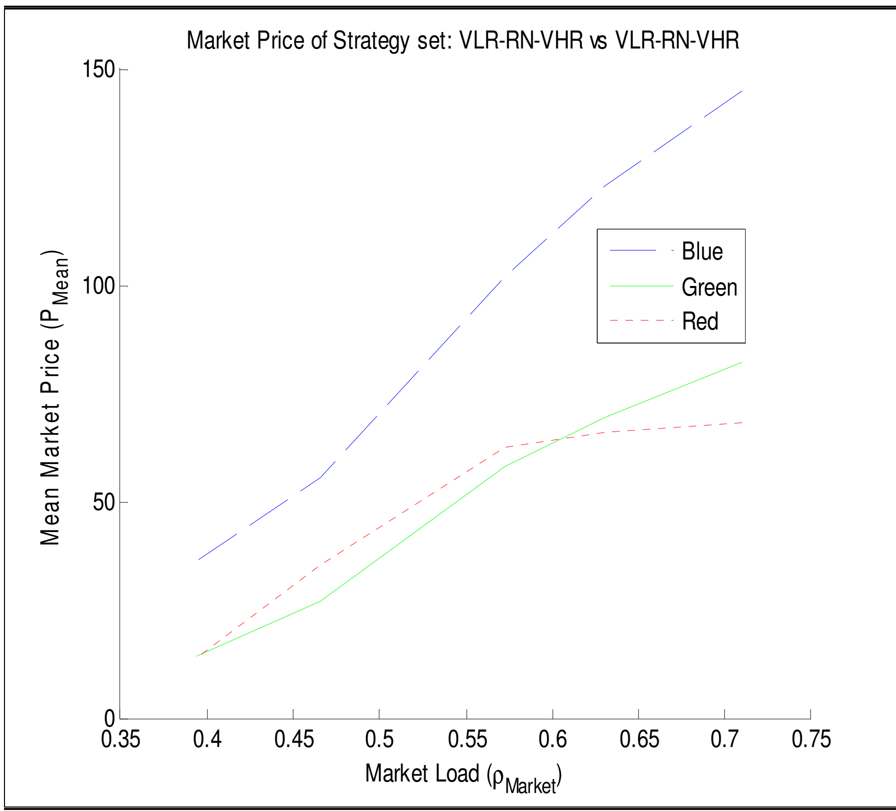


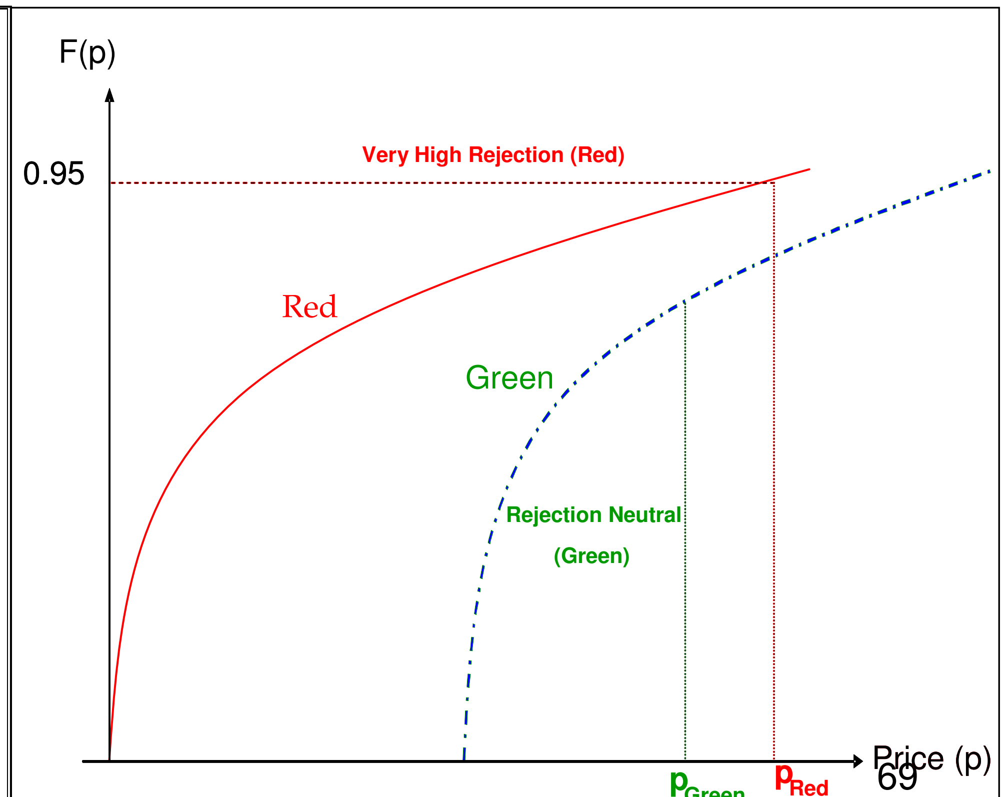
Hypothetical Market Load



Care in Adopting the Best Strategy Set

- Not All Nash-equilibrium is preferred
  - Market price of lower priority service may exceed higher priority service
    - May confuse customers
- The highest Nash equilibrium that meets customers' preference should be selected
- In our study, it is {RN,RN,RN} which is also the same for homogeneous service

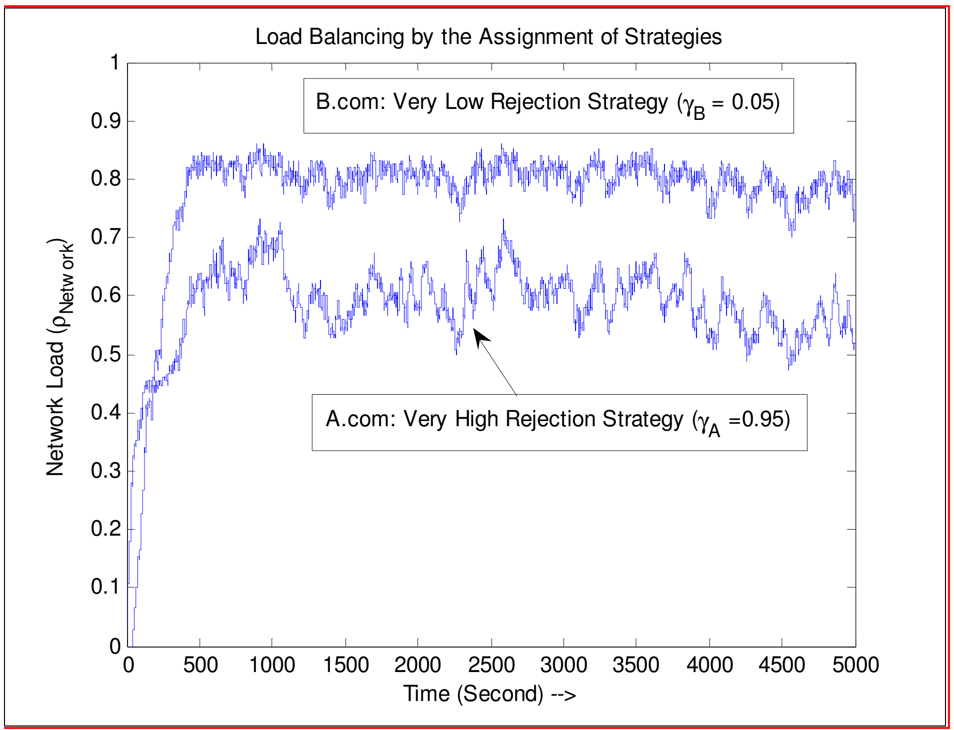


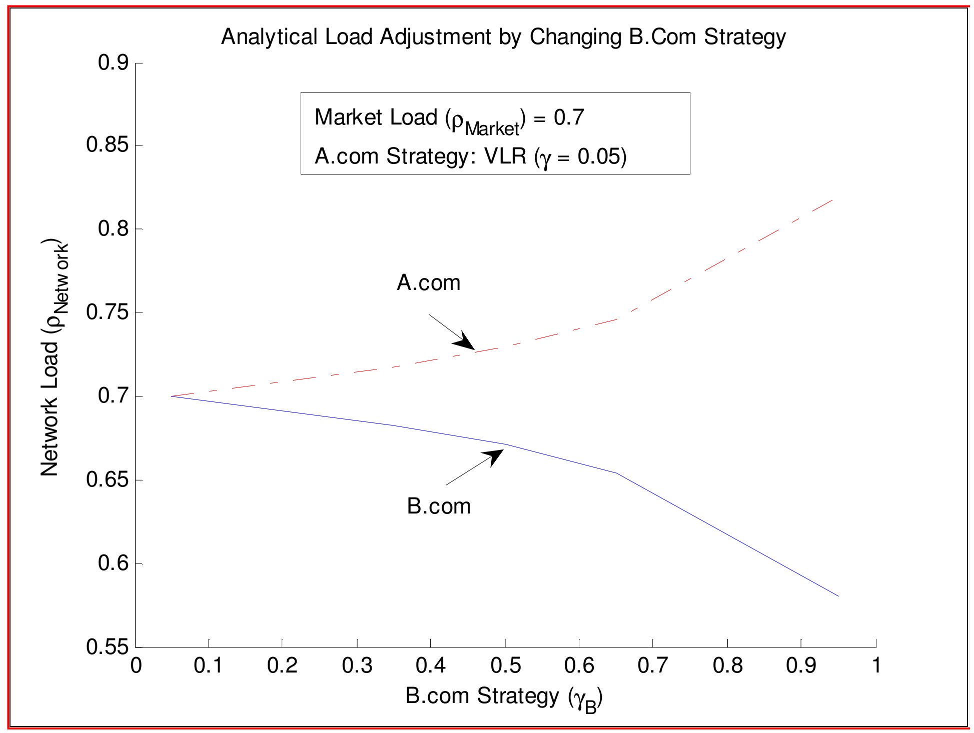


TE Application: Load Distribution

- Load Distribution can be performed
  - By changing strategies

- Assign lower rejection strategy
  - For Higher load in the network
- Assign higher rejection strategy
  - For Lower load in the network
- Assign identical strategy for fair share of load





Conclusion

Contributions

- Developed a New price transaction architecture that benefits customers and providers
  - By automation
  - By providing options to select any provider based on competitive price
  - By allowing customer power to specify budget
  - By introducing new price transaction research in one-to-many architecture
- Developed a mathematical model for providers to
  - To compute competitive price through the best strategy
  - Optimize Profit in dynamic internet traffic demand
- Developed an algorithm and simulation model
  - To verify and study providers' game in flexible environment
- Introduced a New framework to determine Bayesian-Nash equilibrium
  - In dynamic internet traffic demand
- Demonstrated that:
  - Providers improved their Profit
    - Our approach yielded relative advantages over the existing Bertrand Oligopoly Model
  - Providers determined Best strategies (Bayesian-Nash equilibrium and Pareto-efficient outcome) using our approach
  - Providers was able to obtain fair market share of Profit and throughput
  - Providers could implement TE applications such as optimized load balancing in the network
  - Customers could enjoy market price lower than their budgets.
- Introduced new area in Internet pricing research
  - Our research is the first in Internet Oligopoly pricing research for disjoint providers
  - Existing research are for monopoly market
- Introduced pricing research in a complex network model
  - Bi-directional links, multiple paths, Origin-Destination and Destination-Origin Call Legs.

Practical Application

- *Automatic Price-based Services*
- *Profit Optimization and Determining Optimum Throughput*
- *Traffic Load Distribution*
- *Least Price Routing*
- *Forecasting and Capacity Planning*
- *Service Provisioning*

Limitations and Future Work

- Limitations
  - ***Traffic Distribution Pattern***
  - ***The Cost Function***
  - ***Network Queue Model***

- Future Work
  - ***Variable Reservation Price***
  - ***Experiment on 3GPP Network***
  - ***Priority based Queue System***
  - ***Extend model beyond Duopoly***

Appendix

Marginal Cost Function

$$M_{n,t} \leftarrow f(Y_{n,t})$$

- *Service cost coefficient (δs)*
- *Mean Packet Count (M)*
- *Network Throughput (Y)*
- *Provider's Fixed Cost (θ_n)*

Cost (ω):

$$Cost_{n,s,t}(Y_{n,t}) = g(Y_{n,t}) = \delta_s \hat{M}_{n,t} Y_{n,t} + \theta_n Y_{n,t}$$

Marginal Cost (ω):

$$\omega_{ns,t}(\hat{M}^*_{n,t}) = \frac{\partial g(Y_{n,t})}{\partial Y_{n,t}} = \delta_s (Y_{n,t} \frac{\partial \hat{M}^*_{n,t}}{\partial Y_{n,t}} + \hat{M}^*_{n,t}) + \theta_n$$

Simulation:

$$\frac{\partial \hat{M}^*_{n,t+1}}{\partial Y_{n,t+1}} \approx \frac{\hat{M}^*_{n,t+1} - \hat{M}^*_{n,t}}{Y_{n,t+1} - Y_{n,t}}$$

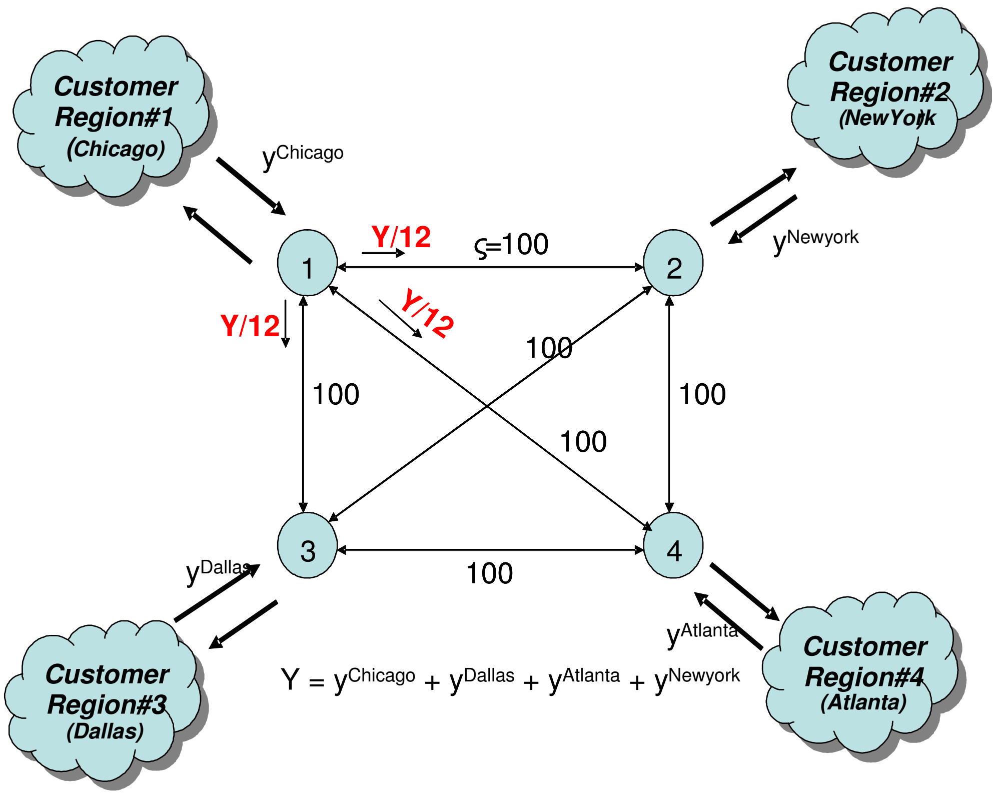
$$\frac{\partial \hat{M}^*_{n,t+1}}{\partial Y_{n,t+1}} \approx \frac{\hat{M}^*_{n,t+1} - \hat{M}^*_{n,t}}{(b_{OD} + b_{DO})}$$

Analysis:

$$\frac{\partial \hat{M}^*_{n,t}}{\partial Y_{n,t}} = \frac{\partial}{\partial Y_{n,t}} \left(\frac{Y_{n,t}}{C - \frac{Y_{n,t}}{12}} \right) = \frac{C}{(C - \frac{1}{12} Y_{n,t})^2}$$

This use of nearsighted one-step history extends the game to a Myopic Markovian-Bayesian Game

Analytical Marginal Cost Function



$$Y_{n,t} = y_{n,t}^{Chicago} + y_{n,t}^{Dallas} + y_{n,t}^{Atlanta} + y_{n,t}^{Newyork}$$

$$y_{l,t} \approx \frac{Y_{n,t}}{12}$$

$$\hat{M}^*_{n,t} = \sum_{l} \frac{\sum_{p:l\in p} x_p}{C_l - \sum_{p:l\in p} x_p} = \sum_{l=1}^{12} \frac{y_{l,t}}{C_l - y_{l,t}}$$

$$= \frac{\frac{Y_{n,t}}{12}}{C - \frac{Y_{n,t}}{12}} + \ldots + \frac{\frac{Y_{n,t}}{12}}{C - \frac{Y_{n,t}}{12}} = \frac{Y_{n,t}}{C - \frac{Y_{n,t}}{12}}$$

$$\frac{\partial \hat{M}^*_{n,t}}{\partial Y_{n,t}} = \frac{\partial}{\partial Y_{n,t}} \left(\frac{Y_{n,t}}{C - \frac{Y_{n,t}}{12}} \right) = \frac{C}{(C - \frac{1}{12} Y_{n,t})^2}$$

$$\omega_{n,s,t}(\hat{M}^*_{n,t}) = \delta_s (Y_{n,t} \frac{2CY_{n,t} - \frac{Y_{n,t}^2}{12}}{(C - \frac{1}{12} Y_{n,t})^2}) + \theta_n$$

Assumption and verified by simulation:
Optimum routing by Gradient Projection, equally load balance network traffic across all links in a market.

Simulation Marginal Cost Function

$$\frac{\partial \hat{M}^*_{n,t+1}}{\partial Y_{n,t+1}} \approx \frac{\hat{M}^*_{n,t+1} - \hat{M}^*_{n,t}}{Y_{n,t+1} - Y_{n,t}}$$

$$\frac{\partial \hat{M}^*_{n,t+1}}{\partial Y_{n,t+1}} \approx \frac{\hat{M}^*_{n,t+1} - \hat{M}^*_{n,t}}{(b_{OD} + b_{DO})}$$

$$\omega_{n,s,t}(\hat{M}^*_{n,t}) = \delta_s (Y_{n,t} \frac{\hat{M}^*_{n,t+1} - \hat{M}^*_{n,t}}{(b_{OD} + b_{DO})} + \hat{M}^*_{n,t}) + \theta_n$$

Profit Functions

Homogeneous Service Market

$$p^{*}_{Market,s,t} = p^{bid}_{A,s,t}(Y_{A,t}) = p^{bid}_{B,s,t}(Y_{B,t})$$

$$Y^{*}_{A} = Y^{*}_{B} = Y^{*}; \quad \Delta^{*} = 2Y^{*}$$

$$u^{*}_{A}(.) = u^{*}_{B}(.)$$

Heterogeneous Service Market

$$Y^{*}_{A} = Y^{*}_{B} = Y^{*}$$

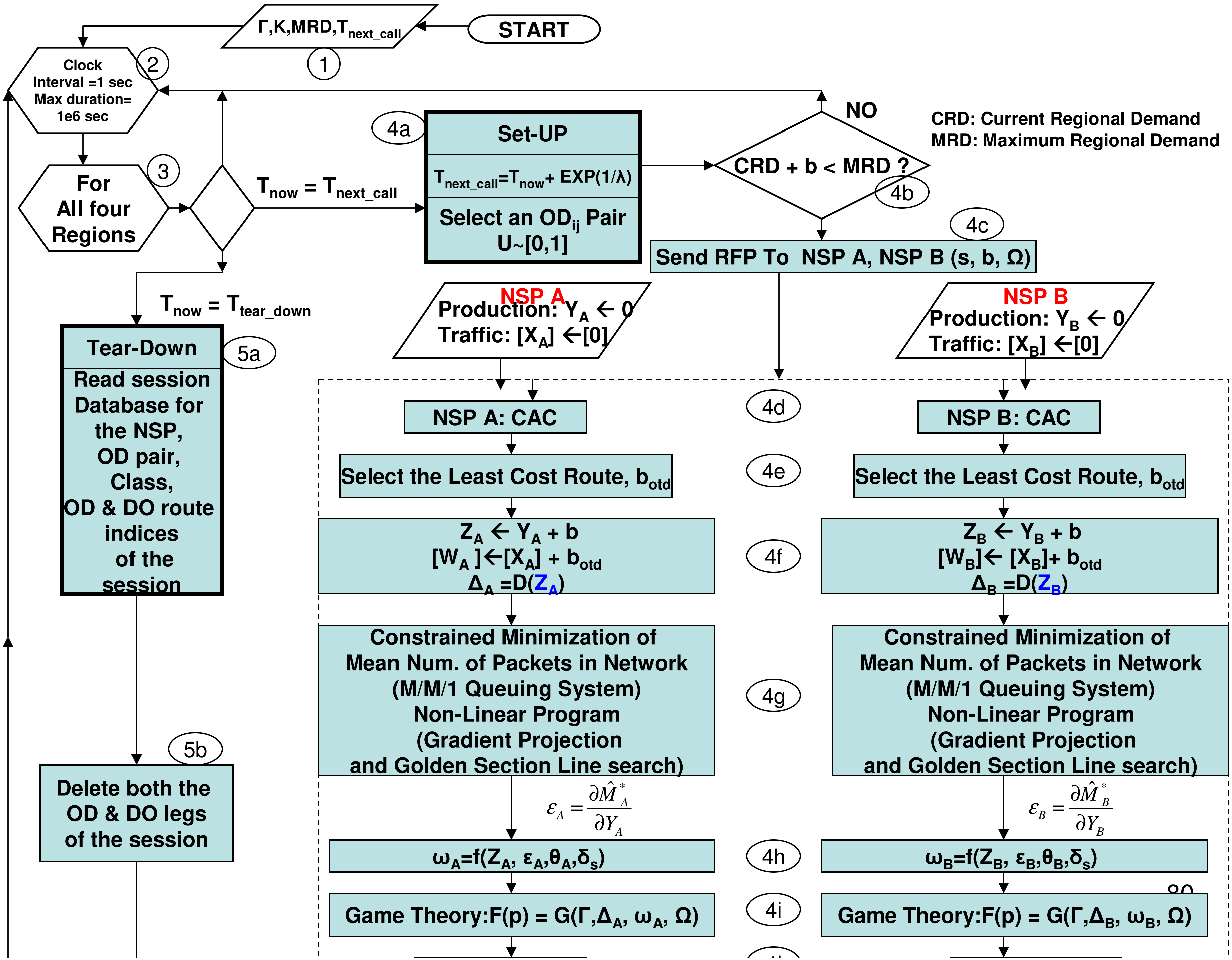
$$\Delta^{*} = 2Y^{*}$$

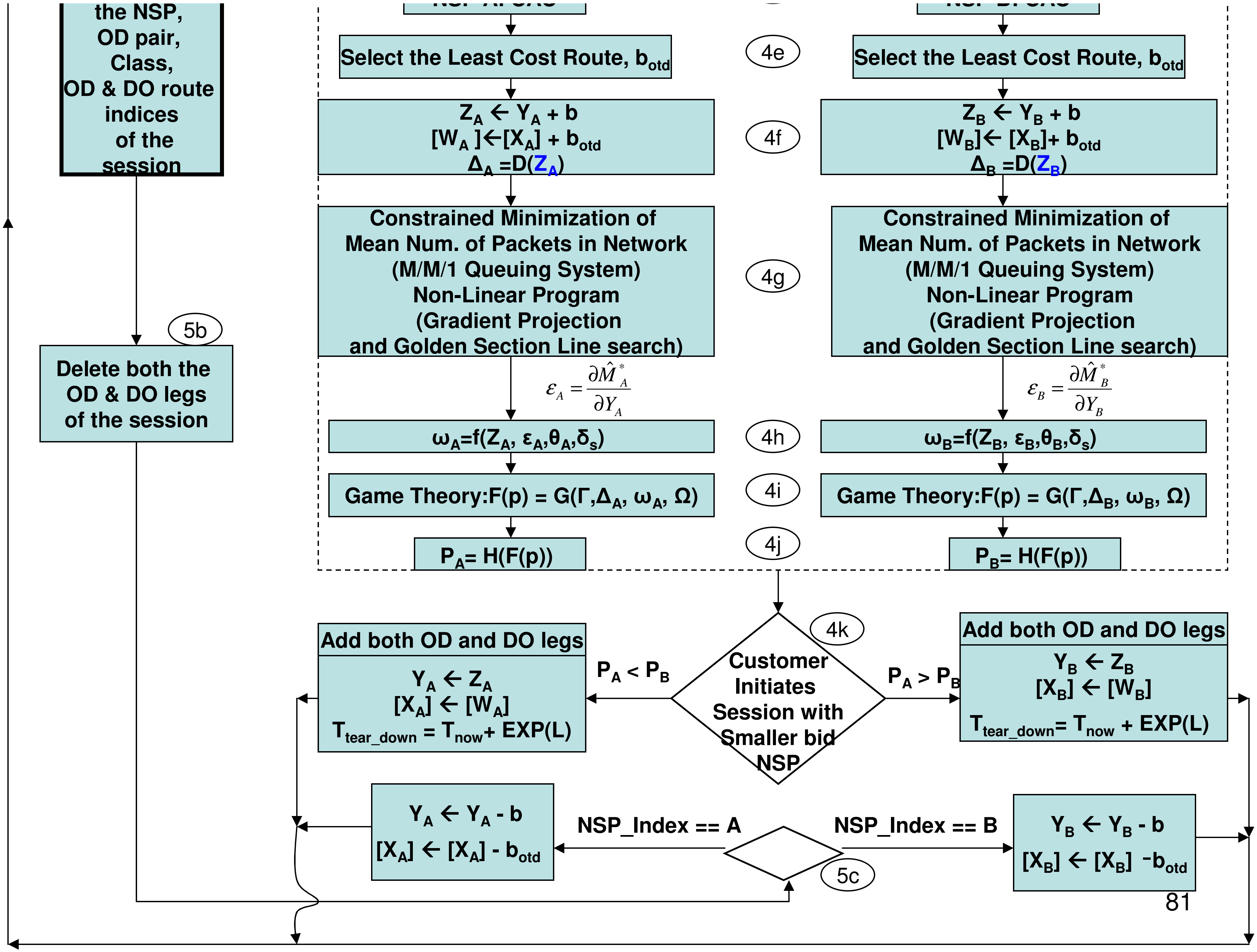
$$u^{*}_{A}(.) = u^{*}_{B}(.)$$

$$Y^{*}_{n,b} = \frac{2}{9}Y^{*}_{n}, Y^{*}_{n,g} = \frac{3}{9}Y^{*}_{n}, Y^{*}_{n,r} = \frac{4}{9}Y^{*}_{n}$$

$$p^{*}_{Market,s,t} = p^{*}_{A,s,t}(Y_{A,t}) = p^{*}_{B,s,t}(Y_{B,t})$$

$$u^{*}_{n}(.) = (p^{*}_{n,b,t} - \omega^{*}_{n,b,t})(\frac{2}{9})Y^{*}_{n,t} + (p^{*}_{n,g,t} - \omega^{*}_{n,g,t})(\frac{3}{9})Y^{*}_{n,t} + (p^{*}_{n,r,t} - \omega^{*}_{n,r,t})(\frac{4}{9})Y^{*}_{n,t}$$


START
Γ,K,MRD,T next_call
1
Clock
Interval =1 sec
Max duration=
1e6 sec
2
For
All four
Regions
3
T now = T next_call
4a
Set-UP
T next_call=T now+ EXP(1/λ)
Select an OD ij Pair
U~[0,1]
NO
CRD + b < MRD ?
4b
CRD: Current Regional Demand
MRD: Maximum Regional Demand
4c
Send RFP To NSP A, NSP B (s, b, Ω)
T now = T tear_down
Tear-Down
5a
Read session
Database for
the NSP,
OD pair,
Class,
OD & DO route
indices
of the
session
NSP A
Production: Y A ← 0
Traffic: [X A] ←[0]
NSP B
Production: Y B ← 0
Traffic: [X B] ←[0]
4d
NSP A: CAC
NSP B: CAC
4e
Select the Least Cost Route, b otd
Select the Least Cost Route, b otd
4f
Z A ← Y A + b
[W A]←[X A] + b otd
Δ A =D(Z A)
Z B ← Y B + b
[W B]← [X B]+ b otd
Δ B =D(Z B)
4g
Constrained Minimization of
Mean Num. of Packets in Network
(M/M/1 Queuing System)
Non-Linear Program
(Gradient Projection
and Golden Section Line search)
Constrained Minimization of
Mean Num. of Packets in Network
(M/M/1 Queuing System)
Non-Linear Program
(Gradient Projection
and Golden Section Line search)
5b
Delete both the
OD & DO legs
of the session
$\varepsilon_A = \frac{\partial \hat{M}^*_A}{\partial Y_A}$
$\varepsilon_B = \frac{\partial \hat{M}^*_B}{\partial Y_B}$
4h
ω A=f(Z A, ε A,θ A,δ s)
ω B=f(Z B, ε B,θ B,δ s)
4i
Game Theory:F(p) = G(Γ,Δ A, ω A, Ω)
Game Theory:F(p) = G(Γ,Δ B, ω B, Ω)


the NSP, OD pair, Class, OD & DO route indices of the session
Select the Least Cost Route, b_otd
4e
Select the Least Cost Route, b_otd
Z_A ← Y_A + b
[W_A] ← [X_A] + b_otd
Δ_A = D(Z_A)
4f
Z_B ← Y_B + b
[W_B] ← [X_B] + b_otd
Δ_B = D(Z_B)
Constrained Minimization of Mean Num. of Packets in Network (M/M/1 Queuing System) Non-Linear Program (Gradient Projection and Golden Section Line search)
4g
Constrained Minimization of Mean Num. of Packets in Network (M/M/1 Queuing System) Non-Linear Program (Gradient Projection and Golden Section Line search)
5b
Delete both the OD & DO legs of the session
ε_A = ∂M̂*_A / ∂Y_A
ε_B = ∂M̂*_B / ∂Y_B
ω_A = f(Z_A, ε_A, θ_A, δ_s)
4h
ω_B = f(Z_B, ε_B, θ_B, δ_s)
Game Theory: F(p) = G(Γ, Δ_A, ω_A, Ω)
4i
Game Theory: F(p) = G(Γ, Δ_B, ω_B, Ω)
P_A = H(F(p))
4j
P_B = H(F(p))
4k
Add both OD and DO legs
Y_A ← Z_A
[X_A] ← [W_A]
T_tear_down = T_now + EXP(L)
P_A < P_B
Customer Initiates Session with Smaller bid NSP
P_A > P_B
Add both OD and DO legs
Y_B ← Z_B
[X_B] ← [W_B]
T_tear_down = T_now + EXP(L)
Y_A ← Y_A - b
[X_A] ← [X_A] - b_otd
NSP_Index == A
NSP_Index == B
5c
Y_B ← Y_B - b
[X_B] ← [X_B] - b_otd