Analytical modeling to evaluate the impact of cross phase modulation in WDM optical networks

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Presentation Outline

- Introduction
  - Fiber optic communication
  - Purpose of Thesis
  - Importance of this work
- Fiber Characteristics
  - Nonlinearities and their effects
  - Eye diagrams
- Analytical Modeling
  - Intensity modulation and time jitter calculation
  - Simulation models
- Results
  - Comparison of Analytical model with VPI
- Conclusion and future work

Introduction

- Fiber optic communication
  - Theoretical bandwidth >10THz
  - Wavelengths at 1550 nm with rates up to 100 Gb/s
  - Dispersion problem solved with DSF
  - Optical amplifiers (EDFA) for large transmission distances
- WDM technology
  - Multiple wavelength channels in a single fiber
  - DWDM to provide more capacity
  - Multiplexer and de-multiplexer to combine and separate channels
- Modulation formats and simulation tools
  - NRZ and RZ formats used for communication
  - VPI and MATLAB to simulate the models are a few used

Introduction

- Purpose of this Thesis
  - Tight spacing of wavelength channels in DWDM
  - Optical signal in one wavelength affects the phase, amplitude and time jitter of the neighboring channel
  - Power in one channel phase modulates the neighboring channel
  - Chromatic dispersion conversion effects
  - XPM induced time jitter and analytical modeling
  - NRZ and RZ format discussion

Introduction

- Importance of this work
  - RWA in optical networks
    - Opaque, Translucent and Transparent networks
    - Greater scalability with transparent networks
    - Linear and nonlinear effects in the optical signal
    - Power dependent and power independent effects
    - Physical links must be characterized
  - Routing considerations
    - An analytical model to characterize these impairments
    - Reject the low quality links, select the better links
    - Model must be fast enough to reduce network delays

Fiber Characteristics

- Fiber basics
  - Core and cladding of different refractive indices
  - Step index and graded index
  - Single mode and multi mode fibers
  - LED and Laser diodes for excitation
- Chromatic Dispersion
  - Different frequency component travel in slightly different speed
  - Wavelength dependent group delays
  - Different amount of time is taken for different spectral components
  - Results in pulse spreading with time
  - Measured in ps/nm-km

Fiber Characteristics

- Fiber nonlinearities
  - Dielectric in response to light will be non linear
  - Increases with increase in data rates, span lengths and channels
  - Gives rise to detrimental effects which reduce the signal quality
- Nonlinear Index of Refraction

$$n = n_0 + n_2 I = n_0 + n_2 \frac{P}{A_{eff}}$$

  - Dependence of refractive index on the intensity of optical signal
  - Produces XPM, SPM, FWM
  - XPM is mainly considered in this thesis

Fiber Characteristics

- Cross Phase Modulation (XPM)
  - Power variations →refractive index changes →phase variations→ time jitter and intensity modulations due to chromatic dispersion
  - Phase shift of the signal depends on power of other channel [1]

$$\theta_j^{NL} = \gamma L_{eff} (P_j + 2\sum_{m \neq j} P_m)$$

  - The nonlinear phase shift depends on the effective length and the power of the probe and pump channels
  - XPM also depends on bit patterns and wavelength separation
  - If separation is wide enough, XPM effects are weak due to walk off
  - DWDM systems have narrow spacing thus increasing XPM
  - Can be analyzed by solving Schrödinger equation

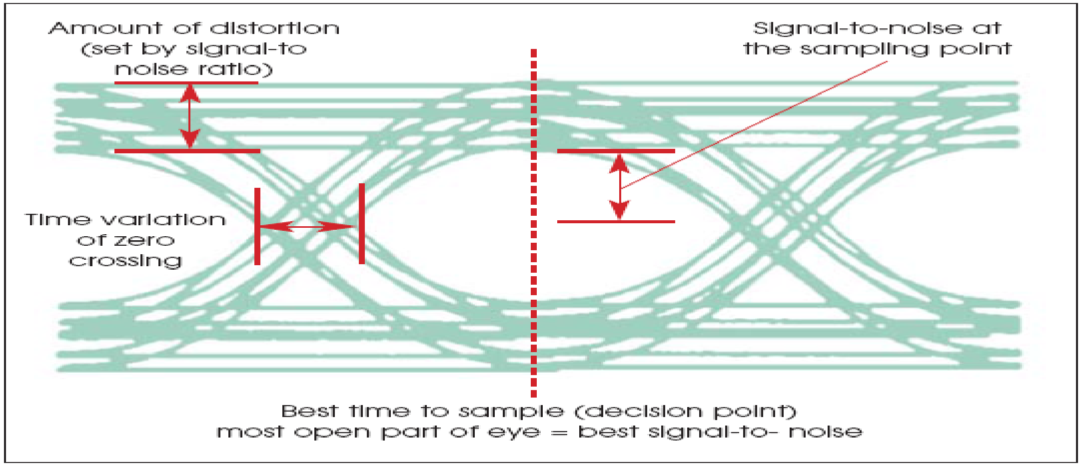
- [1] G.P.Agarwal, *Nonlinear Fiber optics,* 3rd ed, Academic Press, San Diego, CA, 2001.

Fiber Characteristics

- Self Phase Modulation (SPM)
  - Converts power fluctuations into phase fluctuations in the same wave
  - Time varying intensity→ time varying refractive index→ time varying phase change→ frequency shift from initial value
  - Leading edge and falling edge frequency shifts – Blue and red shifts
  - Pulse broadens as it travels down the fiber
- Four Wave Mixing (FWM)
  - Spurious signals fall right on the original signal
  - Dispersion is inversely proportional to mixing efficiency
  - M cross products for N Wavelengths: $M=N^2/2(N-1)$

Fiber Characteristics

- Eye Diagrams
  - Qualitatively measures the signal quality with oscilloscope display
  - Helps in evaluating the system performance
  - Undistorted eye will be clear and wide open
  - Eye with signal impairments such as timing jitter and intensity modulations will have reduced eye opening



Analytical Modeling

- Background
  - Ref [2] and Ref [3]
  - [2] involves only PM-IM and PM-PM mechanisms
  - Considered only intensity and optical phase fluctuation
  - Did not calculate timing jitter
  - [3] involved only intensity distortion
  - Did not consider timing jitter
  - Did not consider RZ format
- Our Analytical model
  - Considers both NRZ and RZ formats
  - Considers both intensity distortions and timing jitter

- [2] G. Goeger, M. Wrage, and W. Fischler, “Cross-Phase Modulation in Multispan WDM systems with Arbitrary Modulation Formats”, IEEE photonics technology letters, Vol 16, No 8, August 2004.
- [3] Rongqing Hui, Kenneth R. Demarest, and Christopher Allen, “Cross phase modulation in multispan WDM optical fiber systems”, J of Lightwave Tech., Vol 17, No. 6, June 1999

Analytical Modeling

- **Basic Considerations and Equations**
  - CW (Continuous Wave) probe waveform
  - Pump with high power in the order of 13 dBm
  - Phase variation of probe channel due to Kerr interaction with pump channel along the fiber is [2]

$$d\theta_i(z,\omega) = -2\gamma_i P_k(z,\omega) dz$$

  - The above equation represents the power of the pump channel and the non linear coefficient
  - The power of the pump channel is expressed as [2]

$$P_k(z,\omega) = P_k(0,\omega).\cos(q_k z).\exp(-\alpha z - i\omega z / v_{g,k})$$

Analytical Modeling

- This phase fluctuation will evolve into intensity modulation (PM-IM) and phase delay (PM-PM) due to chromatic dispersion [4]

$$dP_{XPM,i}(z,\omega) = -2P_i(z)\exp\left[\left(-\alpha - i\omega/v_{g,i}\right)(L-z)\right].\sin\left[q_i(L-z)\right]d\theta_i(z,\omega)$$

$$d\theta_{XPM,i}(z,\omega) = \exp\left[\left(-\alpha - i\omega/v_{g,i}\right)(L-z)\right].\cos\left[q_i(L-z)\right]d\theta_i(z,\omega)$$

  - Since time jitter is the main focus of this thesis, the second equation is taken into consideration to derive the analytical model

- [4] J. Wang and K. Petermann, “Small signal analysis for dispersive optical fiber communication systems,” J.Lightwave Technol., vol 10, pp.99-100, Jan 1992

Analytical Modeling

- Concept of Walk off
  - Non linear interaction is effective only in the first walk off length of the fiber
  - Dispersion takes on from there on
  - Walk off length introduces intensity changes due to the relative alignment of the bit patterns
  - If no change in alignment, more fluctuations due to continuous interaction
  - If the wavelength separation is large, the signals are well separated and results in lesser crosstalk
- Phase noise to intensity noise conversion
  - Conversion matrix by [4] gives the conversion of PM-IM and PM-PM mechanisms

Analytical Modeling

- XPM involving only intensity distortion
  - Assumptions
    - SPM and XPM act separately
    - Usually they act together in a dispersive fiber
    - But for infinitesimal length of fiber (considered here), they act independently
    - Intensity distortions and crosstalk are represented as [3]

$$\Delta s_{jk}^{\ m}(\Omega, L_N) = 4\gamma_i p_j(L_N)\, p_k^{(m)}(\Omega,0)\exp\left[i\Omega \sum_{n=1}^{m-1} d_{jk}^{\ (n)} L^{(n)} \right] . \frac{\sin\left[\Omega^2 \sum_{n=m}^{N} \beta_2^{(n)} L^{(n)} / 2 \right]}{\alpha - i\Omega d_{jk}^{\ (i)}} . \exp(i\Omega L_N / v_j)$$

  - *$m_j(t)$* and *$m_k(t)$* are normalized probe waveform at the output and pump waveform at the input

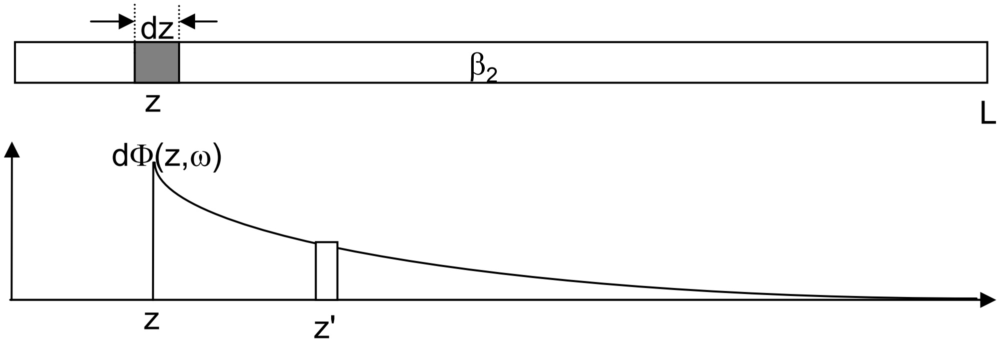
$$C_{jk}(t) = F^{-1}\left\{ F[m_k(t)] \sqrt{\Delta p_{jk}(\Omega, L)} \sqrt{H_j(\Omega)} \right\} m_j(t)$$

Analytical Modeling

- XPM involving both intensity and time jitter
  - Power variations →refractive index changes →phase variations→ time jitter and intensity modulations due to chromatic dispersion
  - Two separate effects combines to distort the signal
  - Intensity distortions – broadening of rails
  - Timing jitter – broadening of edges
  - Frequency shift at the edges depends on channel spacing and intensity

Analytical Modeling

- Model Derivation
  - Pump and probe channels with 0.8 nm spacing



  - Let *dz* be the small section of fiber for the analysis
  - Let *z'* be the portion of the fiber at a distance (z'-z)
  - Phase of probe changes due to power of pump channel

Analytical Modeling

- Model Derivation
  - As a result of cross-phase modulation, the phase variation of the signal channel (*i*) due to the presence of pump channel (*k*) in a short fiber section *dz* at *z* is [9]

$$d\Phi_i(z,\omega) = -2\gamma P_k(z,\omega)dz = -2\gamma P_k(0,\omega)\cos(q_k z)\exp(-\alpha z - j\omega z/v_k)dz$$

  - The equivalent wavelength variation is

$$d\lambda_i(L,z,\omega) = -\frac{\lambda_i^2}{c}dF_i(L,z,\omega)$$

$$= -\frac{\lambda_i^2\omega}{c}\exp\left[-\left(j\omega/v_i\right)(L-z)\right]\cos\left[q_i(L-z)\right]d\Phi_i(z,\omega)$$

Analytical Modeling

- Model derivation
  - After integration, the time jitter at the output of the fiber is given by

$$\Delta t_i(\omega) = \frac{\lambda_i^2 \omega D \cdot \gamma_i P_k(0,\omega)}{c}\left\{A(\omega) + B(\omega)\right\}$$

  - The term A(ω) is given by

$$\frac{\exp(-j\omega/v_i - jq_i)L*D}{2}*$$

$$\left\{\begin{array}{l} L\left[\dfrac{\exp(-\alpha+j\omega d_{ik}+j2q_k)L-1}{-\alpha+j\omega d_{ik}+j2q_k}+\dfrac{\exp(-\alpha+j\omega d_{ik})L-1}{-\alpha+j\omega d_{ik}}\right] \\ -\left[\dfrac{L*\exp(-\alpha+j\omega d_{ik}+j2q_k)L}{-\alpha+j\omega d_{ik}+j2q_k}-\dfrac{1}{(-\alpha+j\omega d_{ik}+j2q_k)^2}\left[\exp(-\alpha+j\omega d_{ik}+j2q_k)L-1\right]\right] \\ -\left[\dfrac{L*\exp(-\alpha+j\omega d_{ik})L}{-\alpha+j\omega d_{ik}}-\dfrac{1}{(-\alpha+j\omega d_{ik})^2}\left[\exp(-\alpha+j\omega d_{ik})L-1\right]\right] \end{array}\right\}$$

Analytical Modeling

- Model derivation
  - Term B(ω) is given by

$$\frac{\exp(-j\omega/v_i+jq_i)L*D}{2}\left\{\begin{array}{l} L\left[\dfrac{\exp(-\alpha+j\omega d_{ik})L-1}{-\alpha+j\omega d_{ik}}+\dfrac{\exp(-\alpha+j\omega d_{ik}-j2q_k)L-1}{-\alpha+j\omega d_{ik}-j2q_k}\right] \\ -\left[\dfrac{L*\exp(-\alpha+j\omega d_{ik}-j2q_k)L}{-\alpha+j\omega d_{ik}-j2q_k}-\dfrac{1}{(-\alpha+j\omega d_{ik}-j2q_k)^2}\left[\exp(-\alpha+j\omega d_{ik}-j2q_k)L-1\right]\right] \\ -\left[\dfrac{L*\exp(-\alpha+j\omega d_{ik})L}{-\alpha+j\omega d_{ik}}-\dfrac{1}{(-\alpha+j\omega d_{ik})^2}\left[\exp(-\alpha+j\omega d_{ik})L-1\right]\right] \end{array}\right\}$$

  - For Multi-span system, the n^th^ span will see the Dispersion in the (n+1) spans.

Analytical Modeling

- Model derivation
  - With Dispersion compensation $A_n(\omega)$ is

$$\frac{\exp(-j\omega/v_i - jq_i)L_n * D_n}{2}\left\{\begin{array}{l} L_n\left[\dfrac{\exp(-\alpha + j\omega d_{ik} + j2q_k)L_n - 1}{-\alpha + j\omega d_{ik} + j2q_k} + \dfrac{\exp(-\alpha + j\omega d_{ik})L_n - 1}{-\alpha + j\omega d_{ik}}\right] \\ -\left[\dfrac{L_n * \exp(-\alpha + j\omega d_{ik} + j2q_k)L_n}{-\alpha + j\omega d_{ik} + j2q_k} - \dfrac{1}{(-\alpha + j\omega d_{ik} + j2q_k)^2}\left[\exp(-\alpha + j\omega d_{ik} + j2q_k)L_n - 1\right]\right] \\ -\left[\dfrac{L_n * \exp(-\alpha + j\omega d_{ik})L_n}{-\alpha + j\omega d_{ik}} - \dfrac{1}{(-\alpha + j\omega d_{ik})^2}\left[\exp(-\alpha + j\omega d_{ik})L_n - 1\right]\right] \end{array}\right\}$$

$$* \sum_{x=n+1}^{y} D_i L_i + \left[\frac{\exp(-\alpha + j\omega d_{ik} + j2q_k)L_n - 1}{-\alpha + j\omega d_{ik} + j2q_k} + \frac{\exp(-\alpha + j\omega d_{ik})L_n - 1}{-\alpha + j\omega d_{ik}}\right]$$

Analytical Modeling

- Model derivation
  - With Dispersion compensation $B_n(\omega)$ is

$$\frac{\exp(-j\omega/v_i + jq_i)L_n * D_n}{2}\left\{\begin{array}{l} L_n\left[\dfrac{\exp(-\alpha+j\omega d_{ik})L_n-1}{-\alpha+j\omega d_{ik}}+\dfrac{\exp(-\alpha+j\omega d_{ik}-j2q_k)L_n-1}{-\alpha+j\omega d_{ik}-j2q_k}\right]\\ -\left[\dfrac{L_n*\exp(-\alpha+j\omega d_{ik}-j2q_k)L_n}{-\alpha+j\omega d_{ik}-j2q_k}-\dfrac{1}{(-\alpha+j\omega d_{ik}-j2q_k)^2}\left[\exp(-\alpha+j\omega d_{ik}-j2q_k)L_n-1\right]\right]\\ -\left[\dfrac{L_n*\exp(-\alpha+j\omega d_{ik})L_n}{-\alpha+j\omega d_{ik}}-\dfrac{1}{(-\alpha+j\omega d_{ik})^2}\left[\exp(-\alpha+j\omega d_{ik})L_n-1\right]\right]\end{array}\right\}$$

$$\sum_{x=n+1}^{y} D_i L_i * \left[\frac{\exp(-\alpha+j\omega d_{ik})L_n-1}{-\alpha+j\omega d_{ik}}+\frac{\exp(-\alpha+j\omega d_{ik}-j2q_k)L_n-1}{-\alpha+j\omega d_{ik}-j2q_k}\right]$$

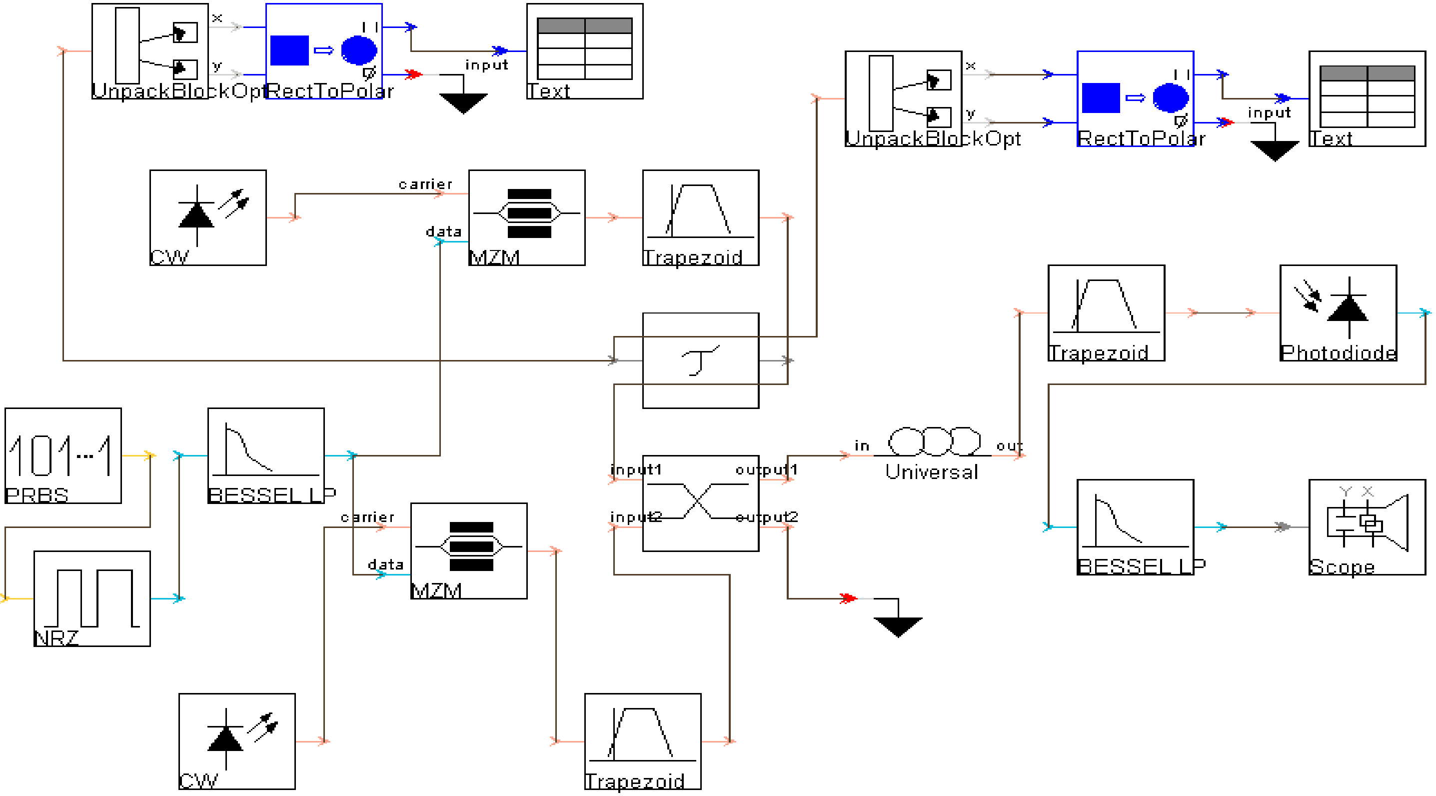
  - Time jitter = $\displaystyle\sum_{n=1}^{N}\frac{\lambda_i^2\omega^n\gamma_i P_k{}^n(0,\omega)}{c}\left[\mathrm{A}_{\mathrm{n}}+B_n\right]$

Simulation Models

- Simulation models available
  - VPI (Virtual Photonics Inc)
  - Almost physical realization of a system
  - Test the existing systems and build networks
  - Provides laser diodes, modulators, filters etc
  - Simulates SPM, XPM, FWM, SBS, SRS, PMD etc
  - Disadvantage: Very slow in simulating the impairments

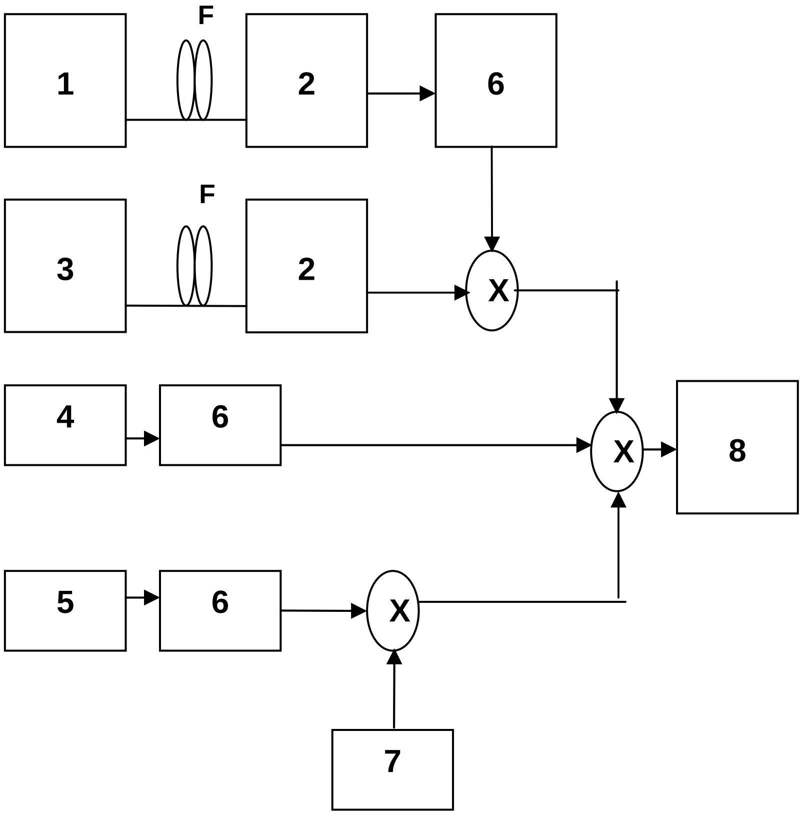
Simulation Models

- VPI Block diagram



Simulation Models

- MATLAB
  - Our analytical model is implemented in MATLAB
  - Text output from VPI is used for fair comparison
  - 1. Intensity fluctuation transfer function
  - 2. Bessel low pass filter
  - 3. Time jitter transfer function
  - 4. Pump 1549.6 nm
  - 5. Probe 1550.4 nm
  - 6. Normalization block
  - 7. Dispersion and SPM effects
  - 8. Eye diagram



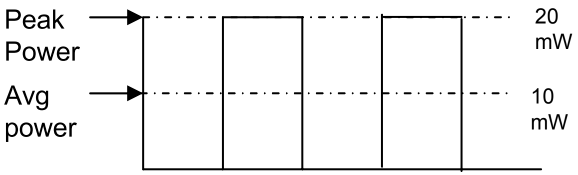
Simulation Models

- Model Differences
  - Not possible to measure the time jitter in case of CW
  - Both channels are modulated in VPI
  - Output probe waveform is affected by SPM, dispersion, XPM
  - SPM and dispersion implemented separately in MATLAB
  - Split step Fourier transform is used for SPM
  - Time jitter and Intensity modulation are added onto the probe waveform to see the effects

Simulation Models

Comparison criteria for NRZ

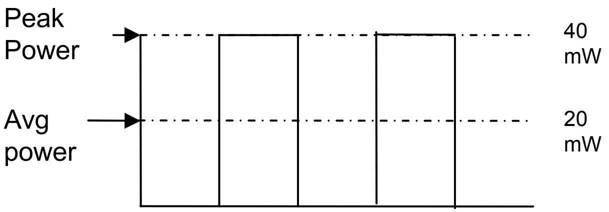
MATLAB



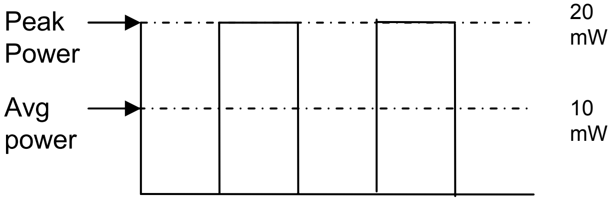
Average power = Peak power / 2

VPI

At laser diode output



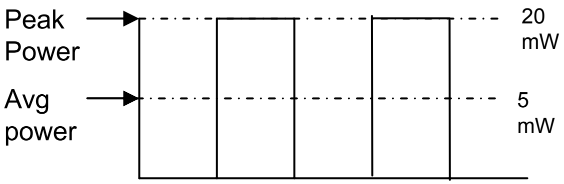
At the coupler output



Simulation Models

Comparison criteria for RZ

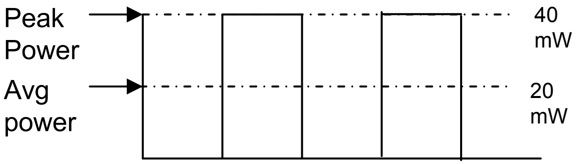
MATLAB



Average power = Peak power / 4

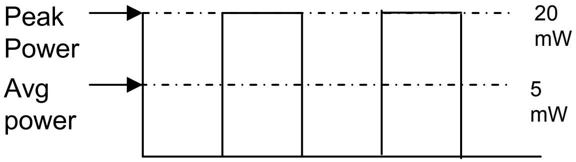
VPI

At laser diode output



Average power = Peak power / 2

At the coupler output



Average power = Peak power / 8

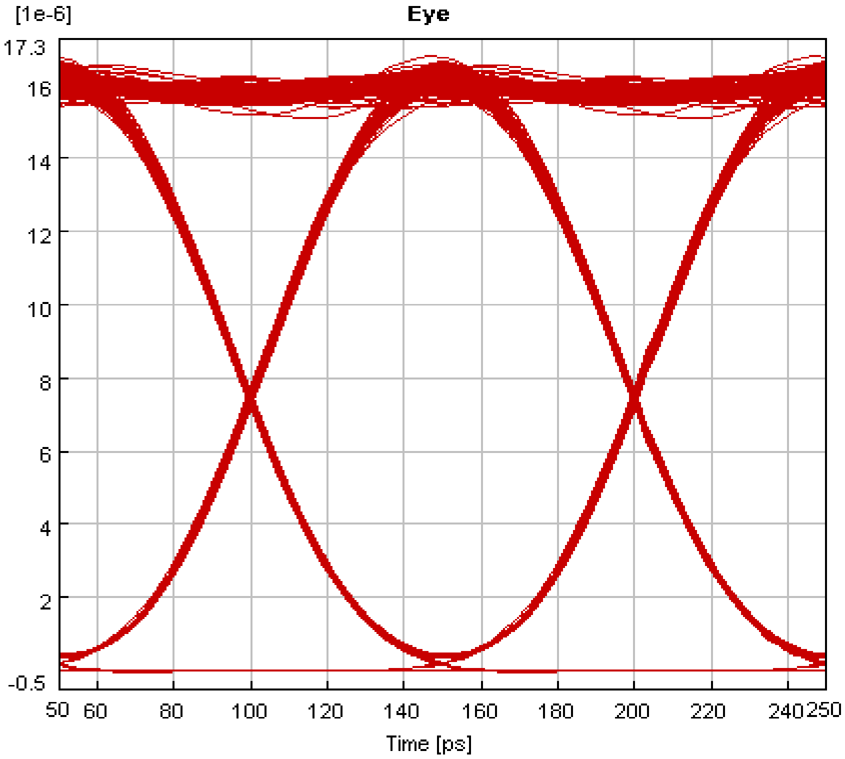
Results

- Transmission of single probe channel
- Comparison with [5]
- NRZ analysis
  - Single span
  - Three span
- RZ analysis
  - Single span
  - Three span

- [5] - Micheal Eiselt, Mark Shtaif, and Lara D. Garrett, “Contribution of timing jitter and amplitude distortion to XPM system penalty in WDM systems”, IEEE photonics tech letters, vol 11, no 6, June 1999.

Results

- Transmission of single probe channel
  - Exhibits no distortions

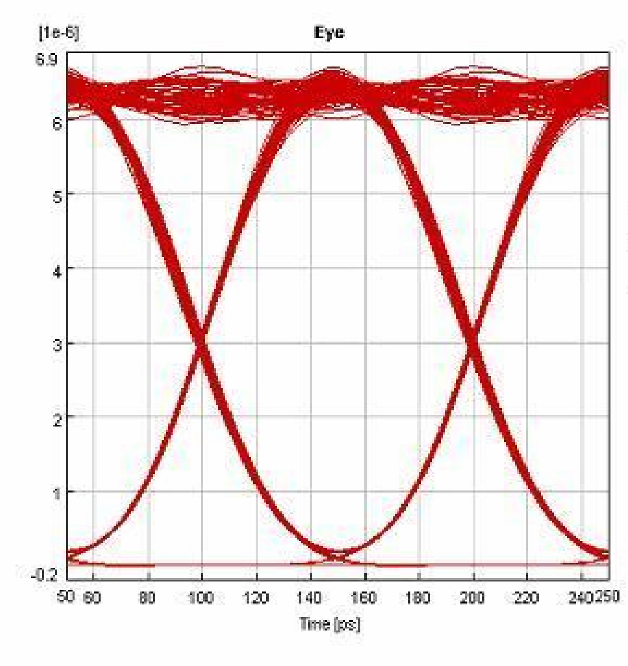


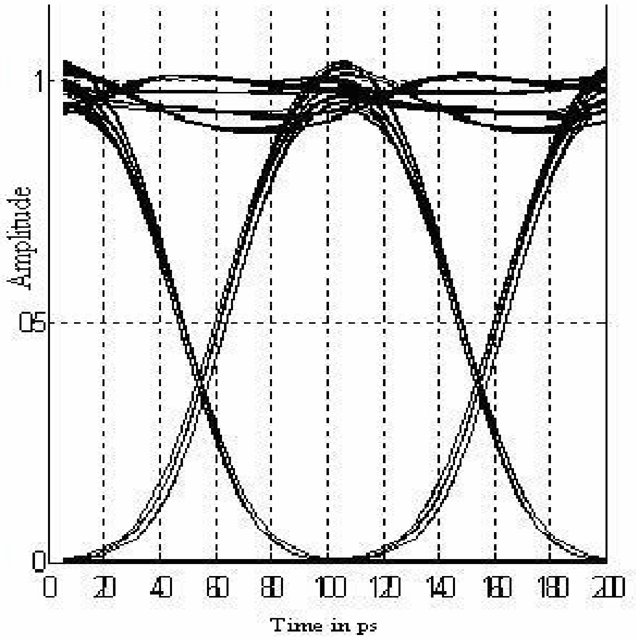
Results

- Comparison with [5]
  - Dispersion of 2.7 ps/nm-km, length of 100 Km, NRZ format
  - Calculates intensity and timing jitter for NRZ systems
  - Focuses mainly on intensity distortions
  - Horizontal eye closure of ± 10 ps
  - Eye closure of 10 ps with the model and 13.6 ps with VPI for 180 ps delay between pump and probe
  - Gradual walk off effect account for difference in values
  - Power levels from 5 to 20 mW were considered
  - Comparison is made between the analytical model and VPI model

Results

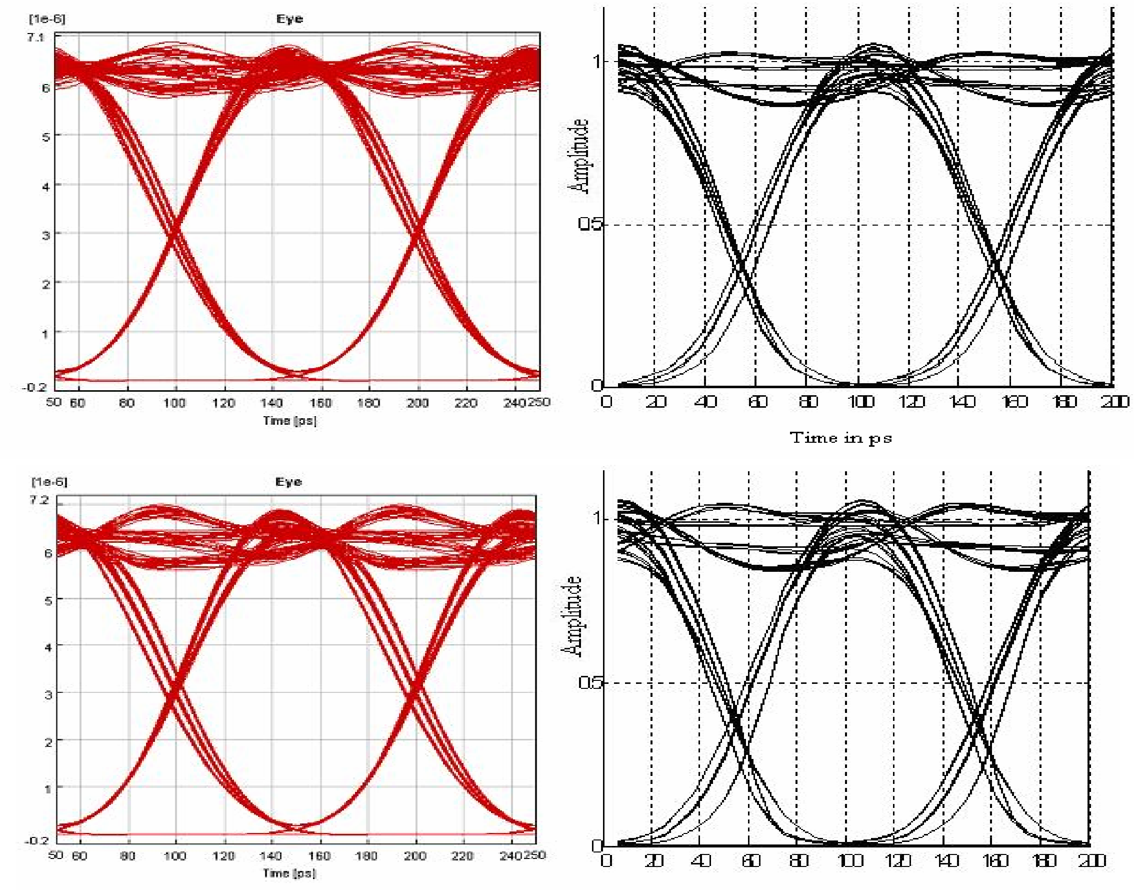
- Comparison for 5 mW of pump power level





Results

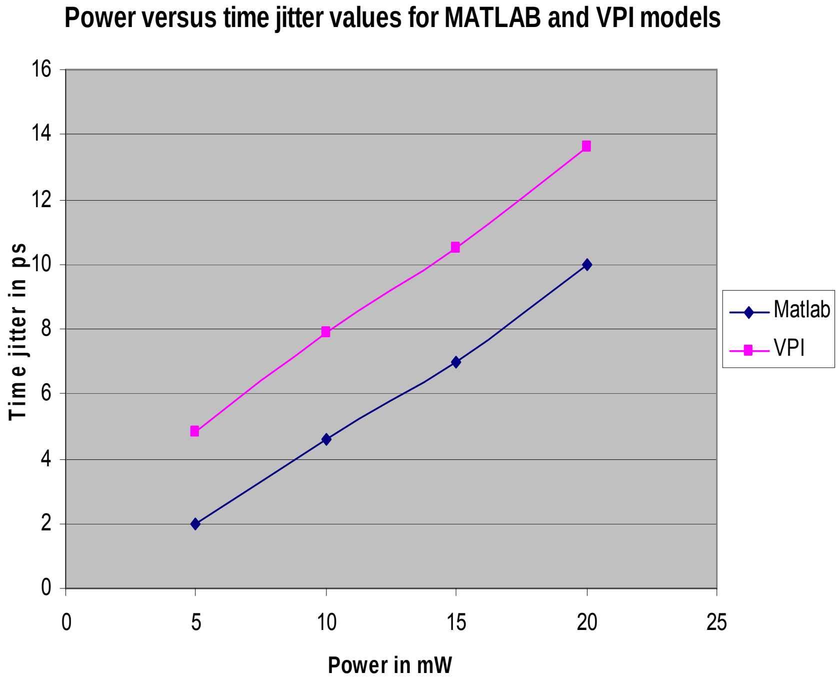
- Comparison for 10 and 20 mW pump power



Results

- Comparison of time jitter values for 180 ps delay

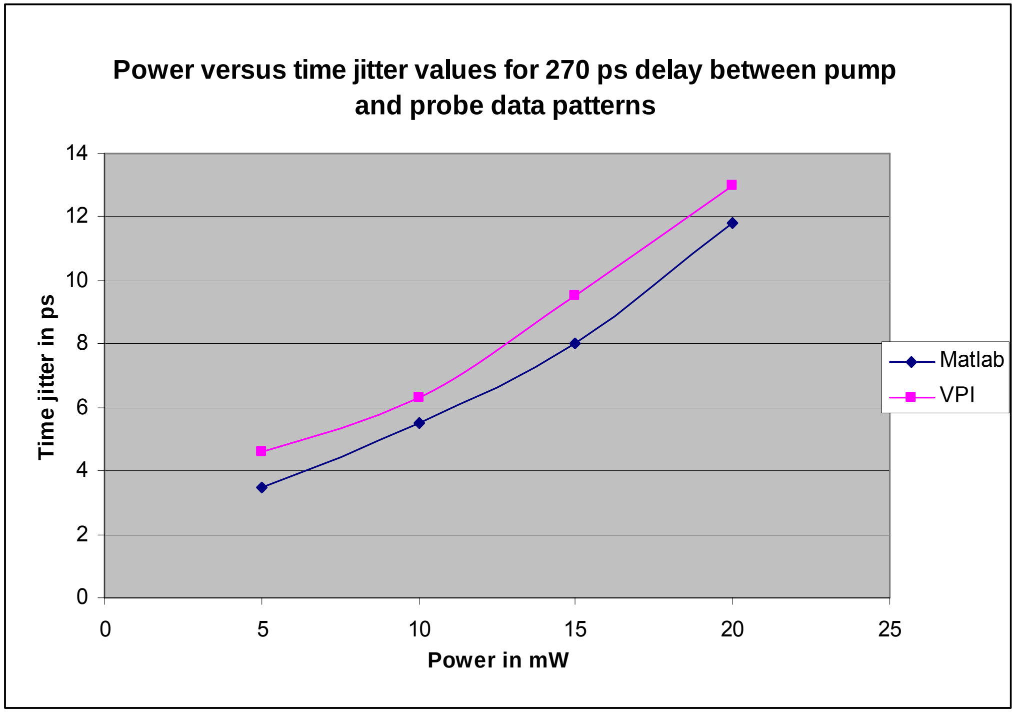
Power versus time jitter values for MATLAB and VPI models



Power in mW	MATLAB time jitter in ps	VPI time jitter in ps
5	2	4.8
10	4.6	7.9
15	7	10.5
20	10	13.6

Results

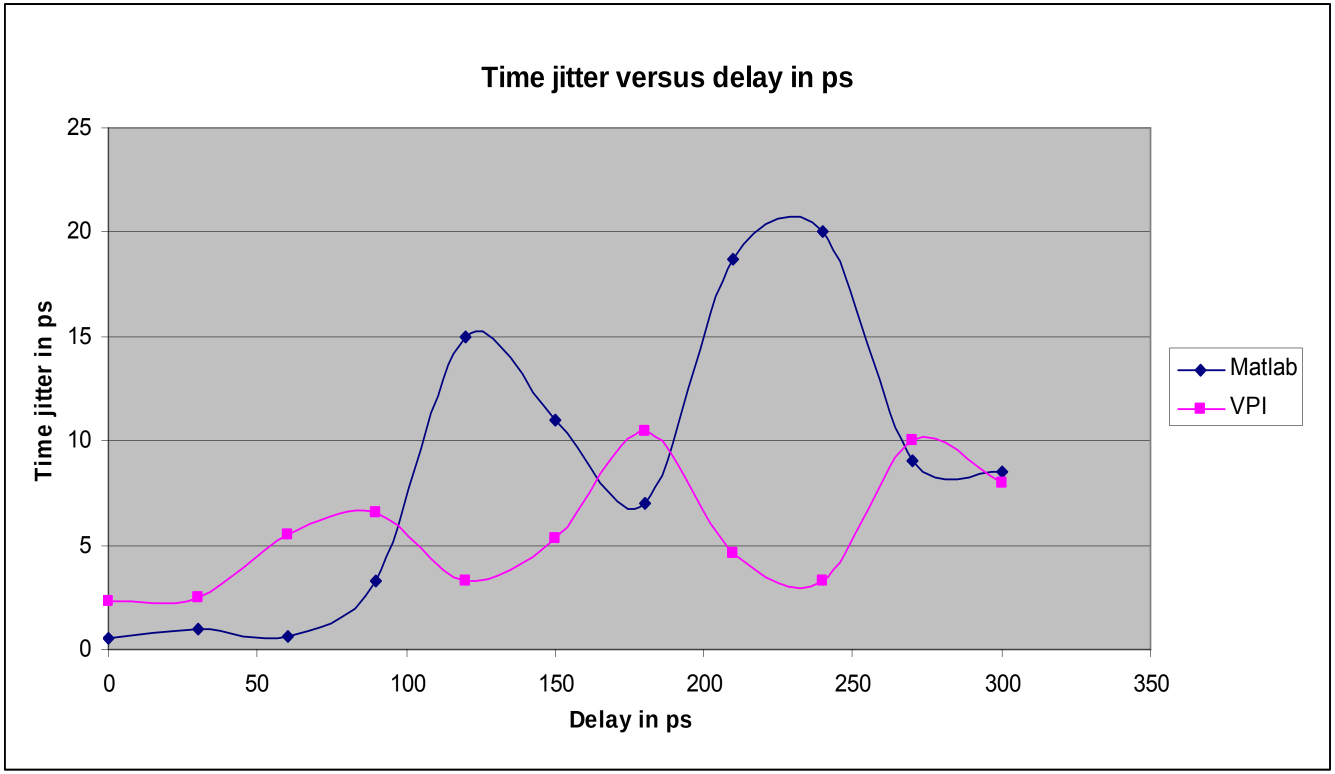
- Comparison of time jitter values for 270 ps delay



Power in mW	MATLAB time jitter in ps	VPI time jitter in ps
5	3.5	4.6
10	5.5	6.3
15	8	9.5
20	11.8	13

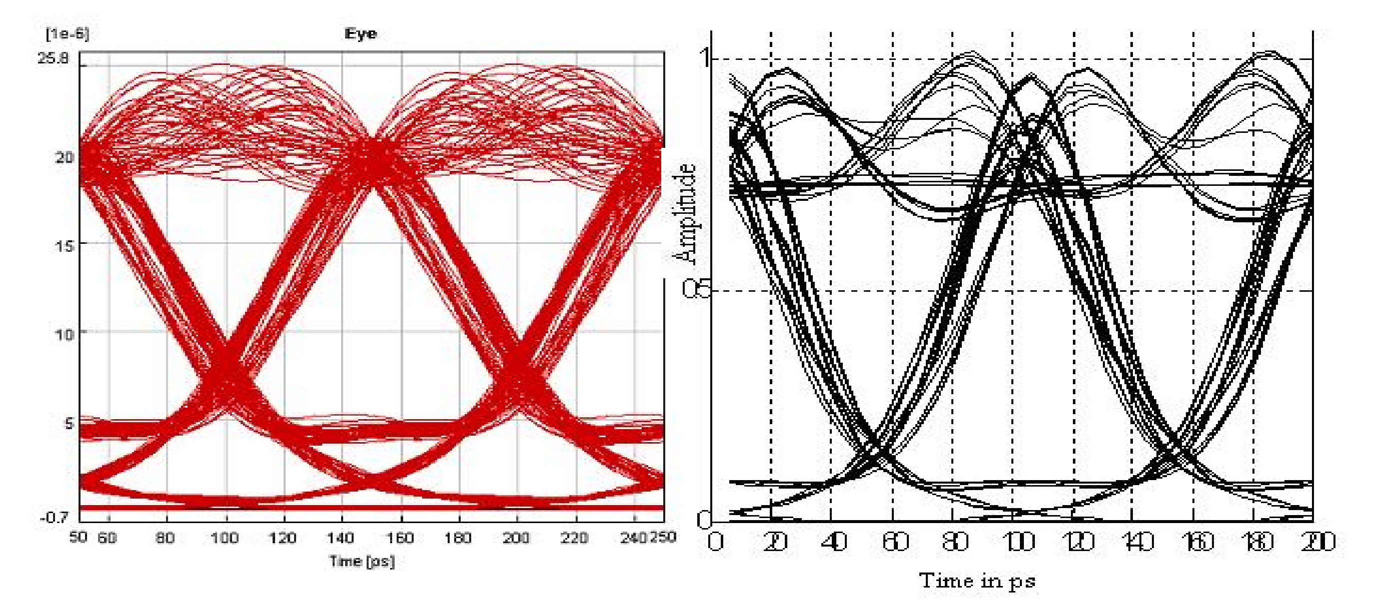
Results

- Delay versus time jitter values
  - Gradual walk off effect accounts for differences



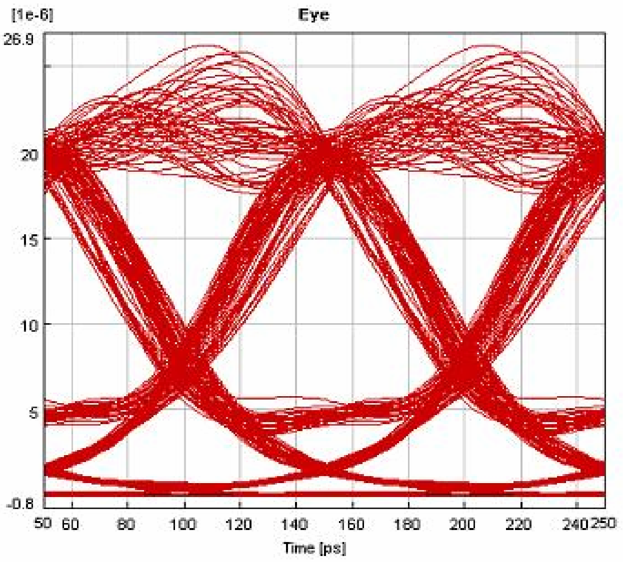
Results

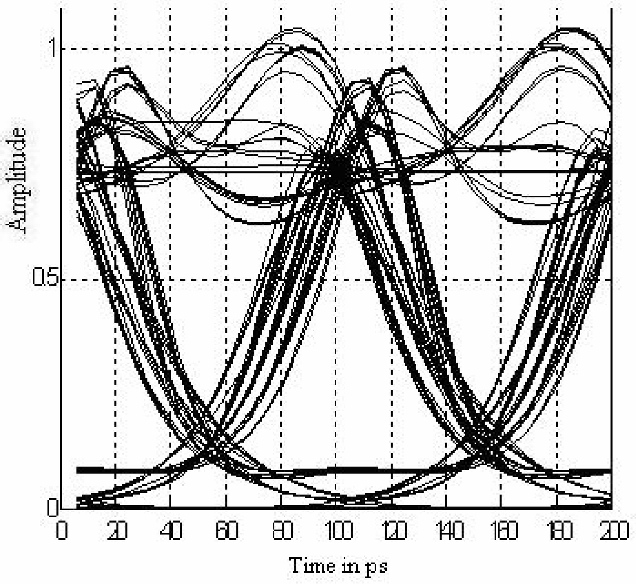
- NRZ 1 span comparison for 5 mW pump power



Results

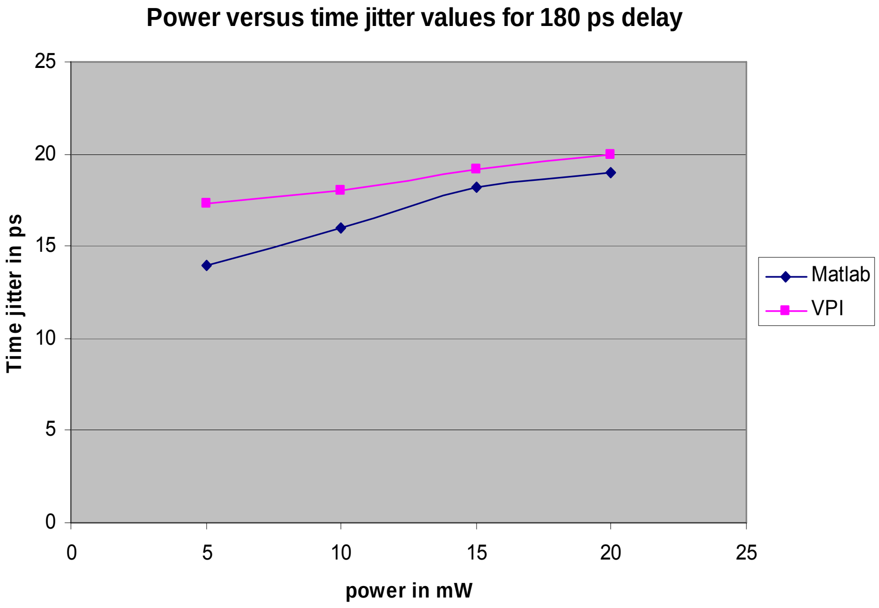
- 20 mW comparison





Results

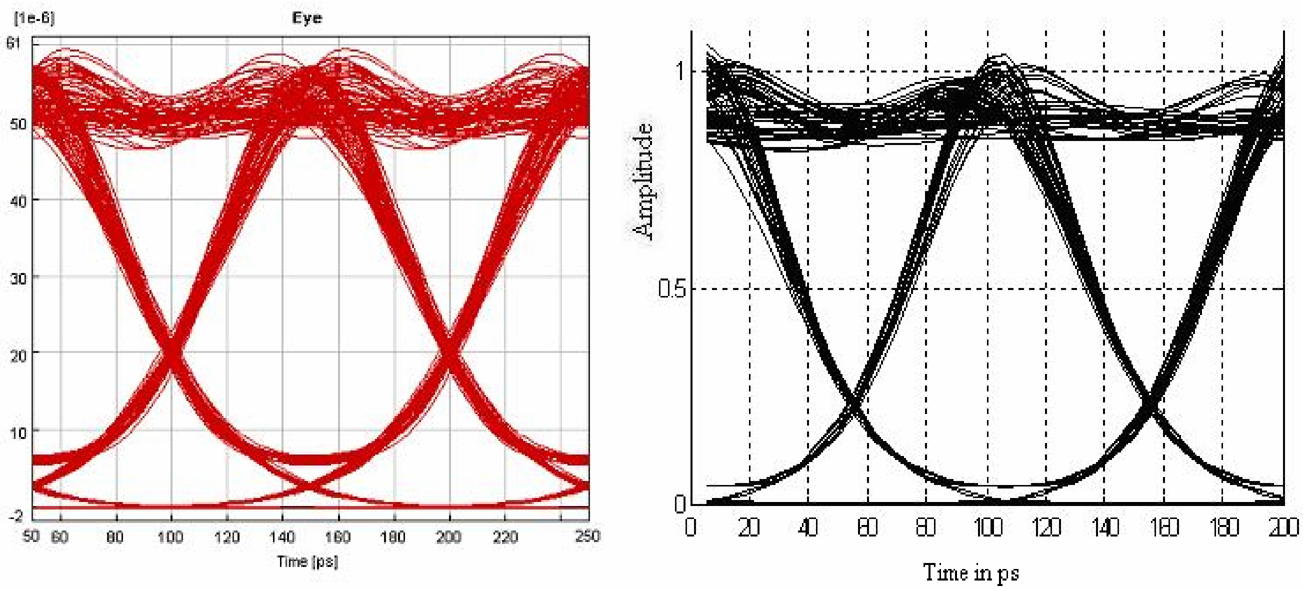
- NRZ 1 span time jitter comparison for 180 ps delay



Power in mW	MATLAB time jitter in ps	VPI time jitter in ps
5	14	17.3
10	16	18
15	18.2	19.2
20	19	20

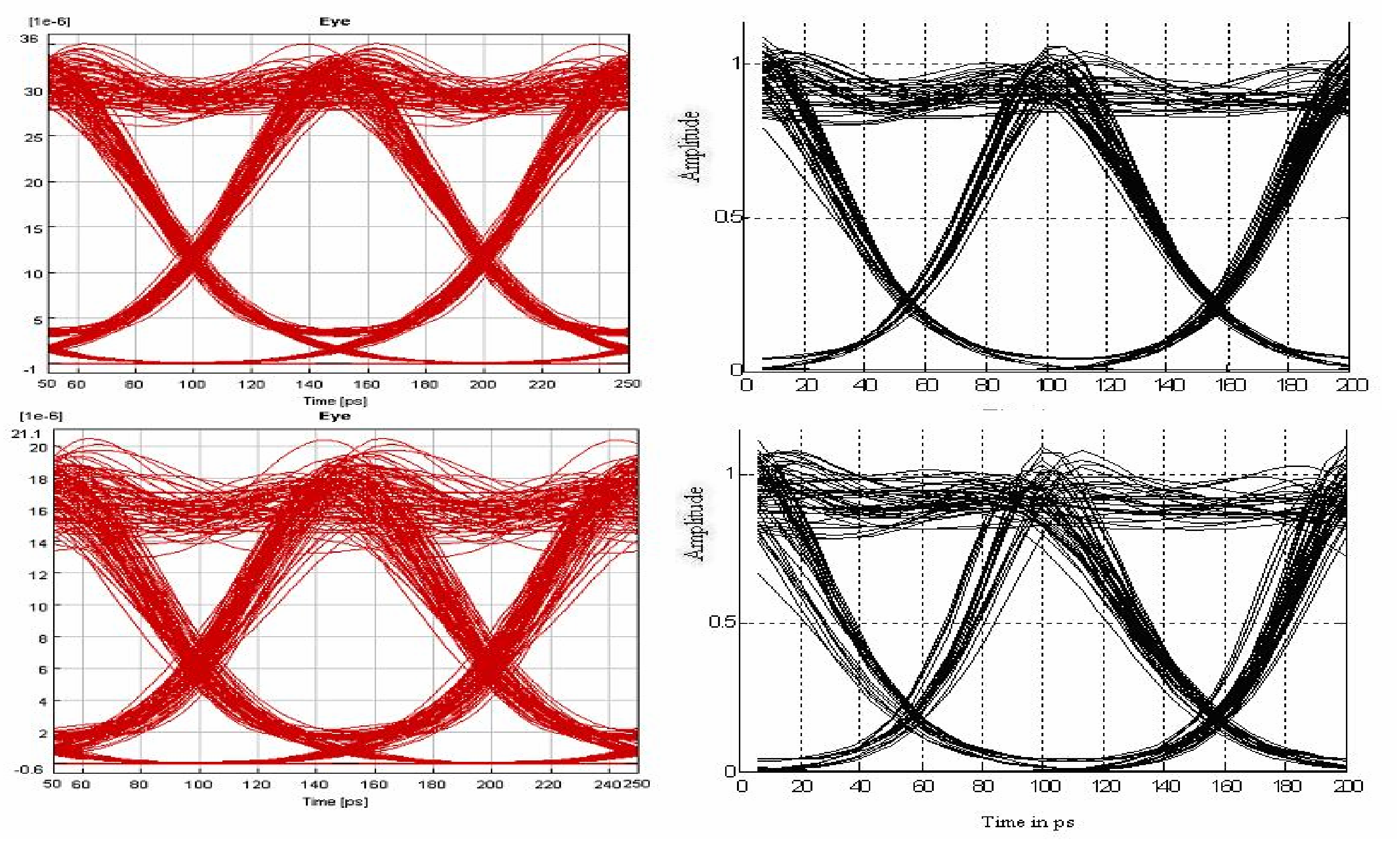
Results

- NRZ 3 span system 5 mW comparison



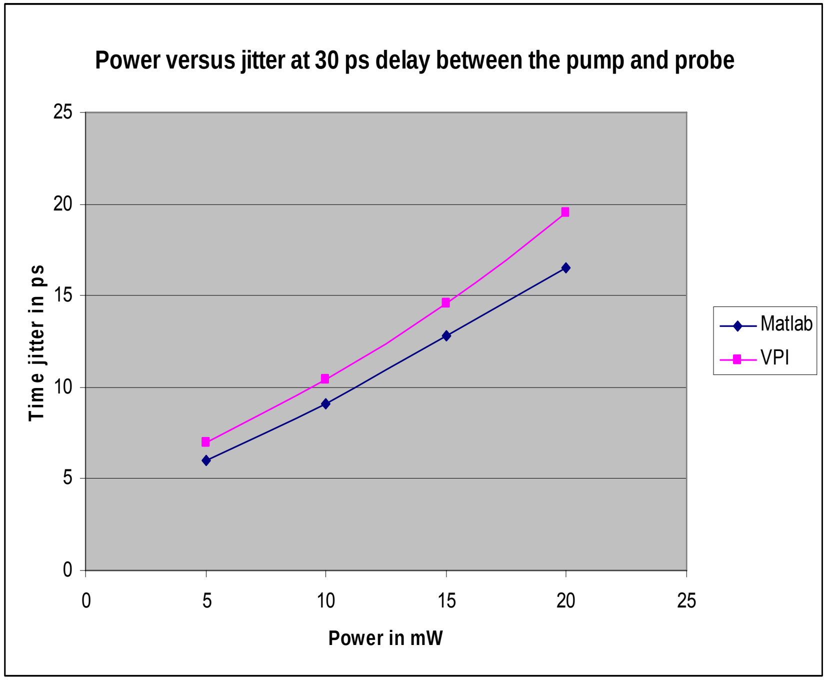
Results

- 10 mW and 20 mW comparison

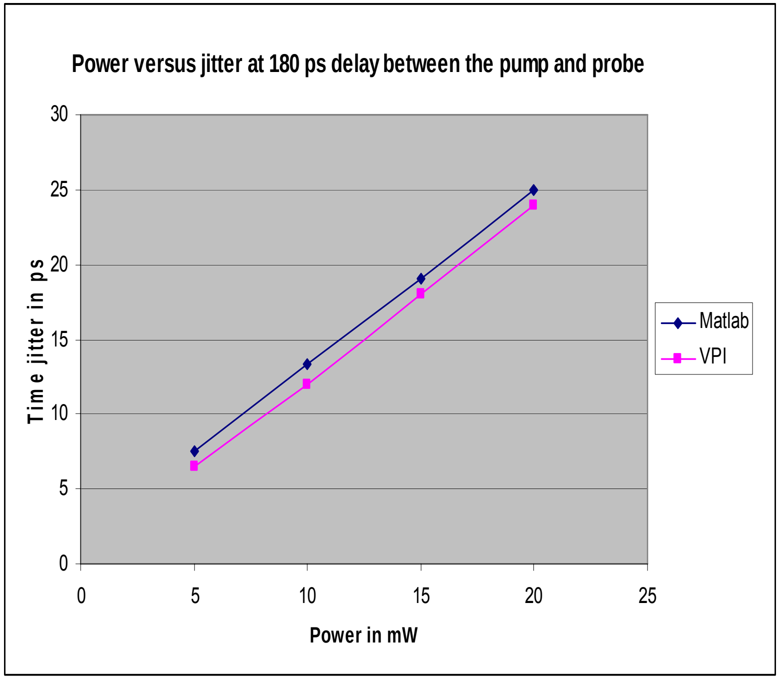


Results

- Time jitter comparison for 30 ps delay

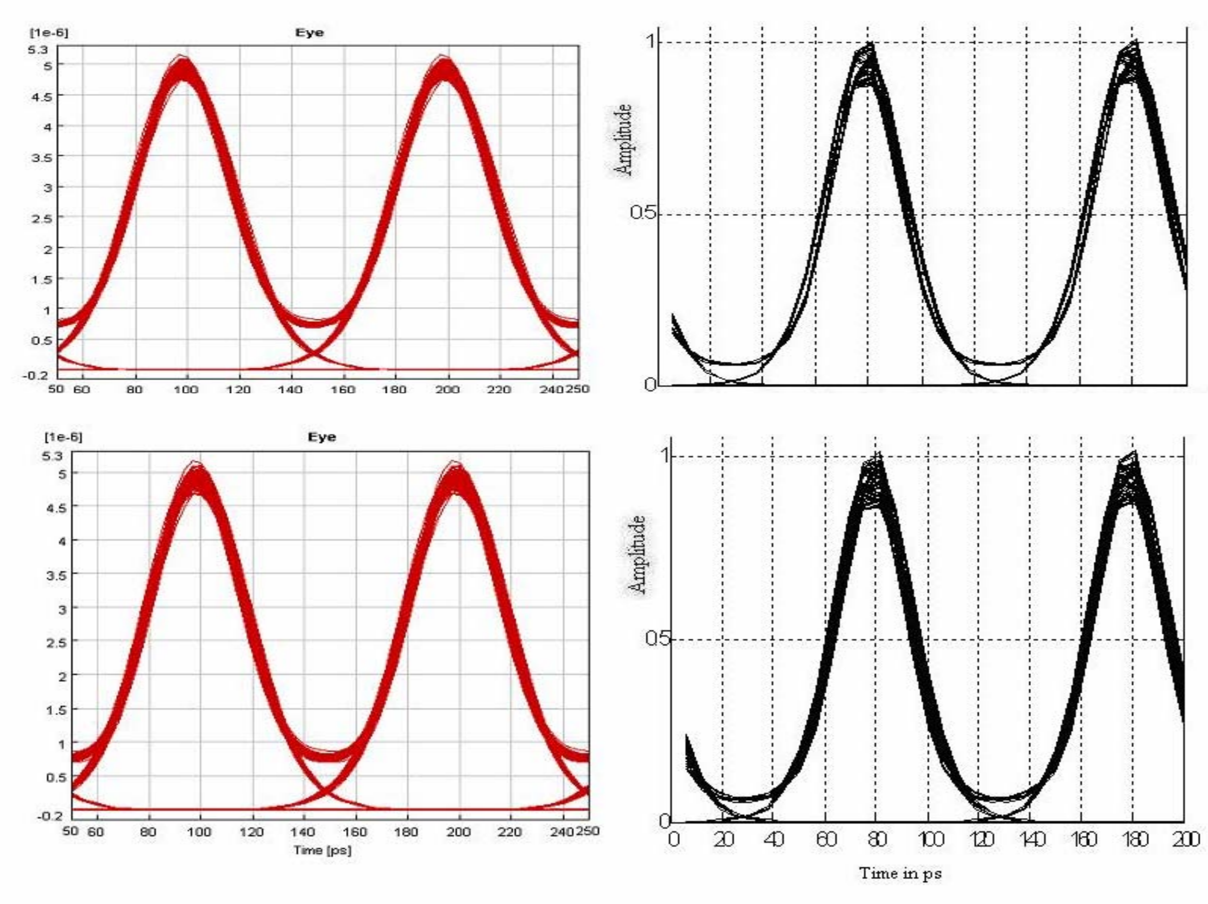


- Time jitter comparison for 180 ps delay



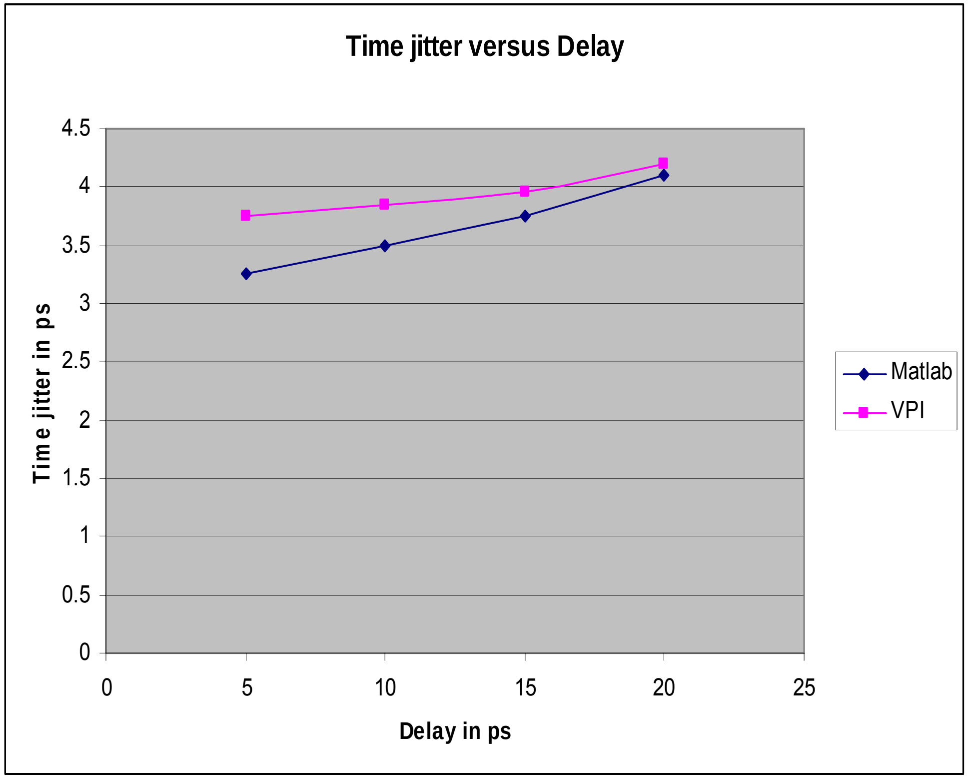
Results

- RZ 1 span 5 mW and 20 mW comparison



Results

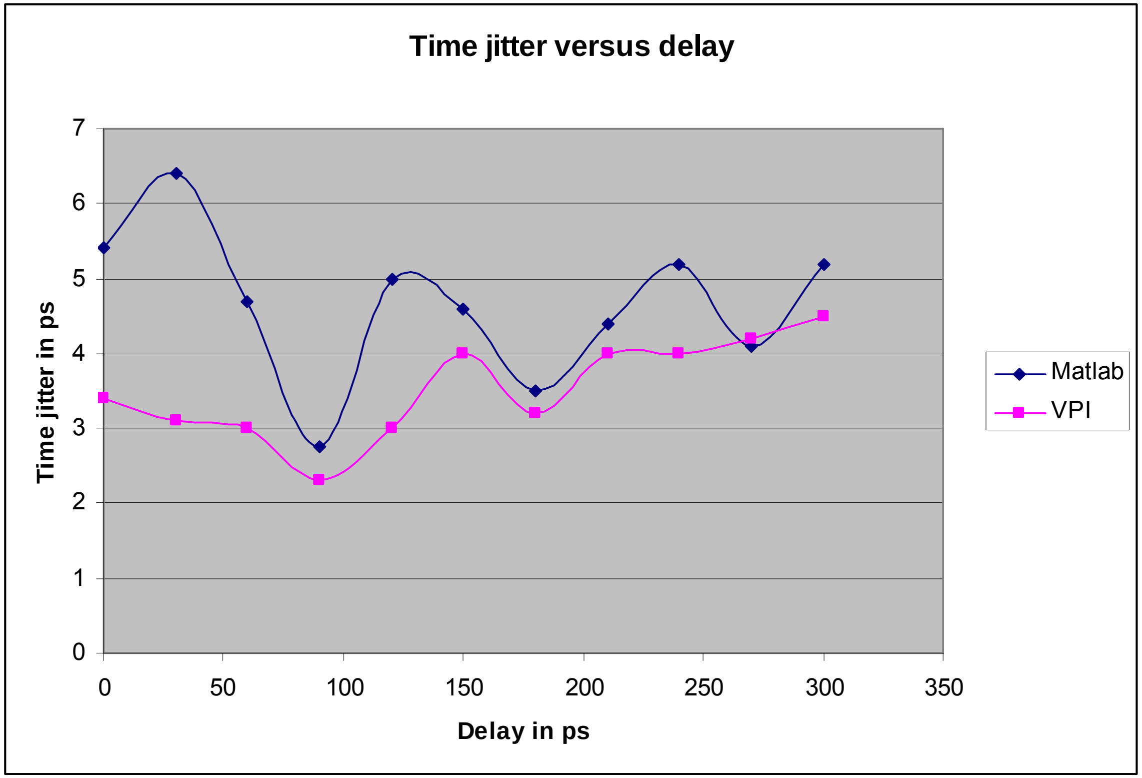
- Comparison for time jitter values for 270 ps delay



Power in mW	MATLAB time jitter in ps	VPI time jitter in ps
5	3.25	3.75
10	3.5	3.85
15	3.75	3.95
20	4.1	4.2

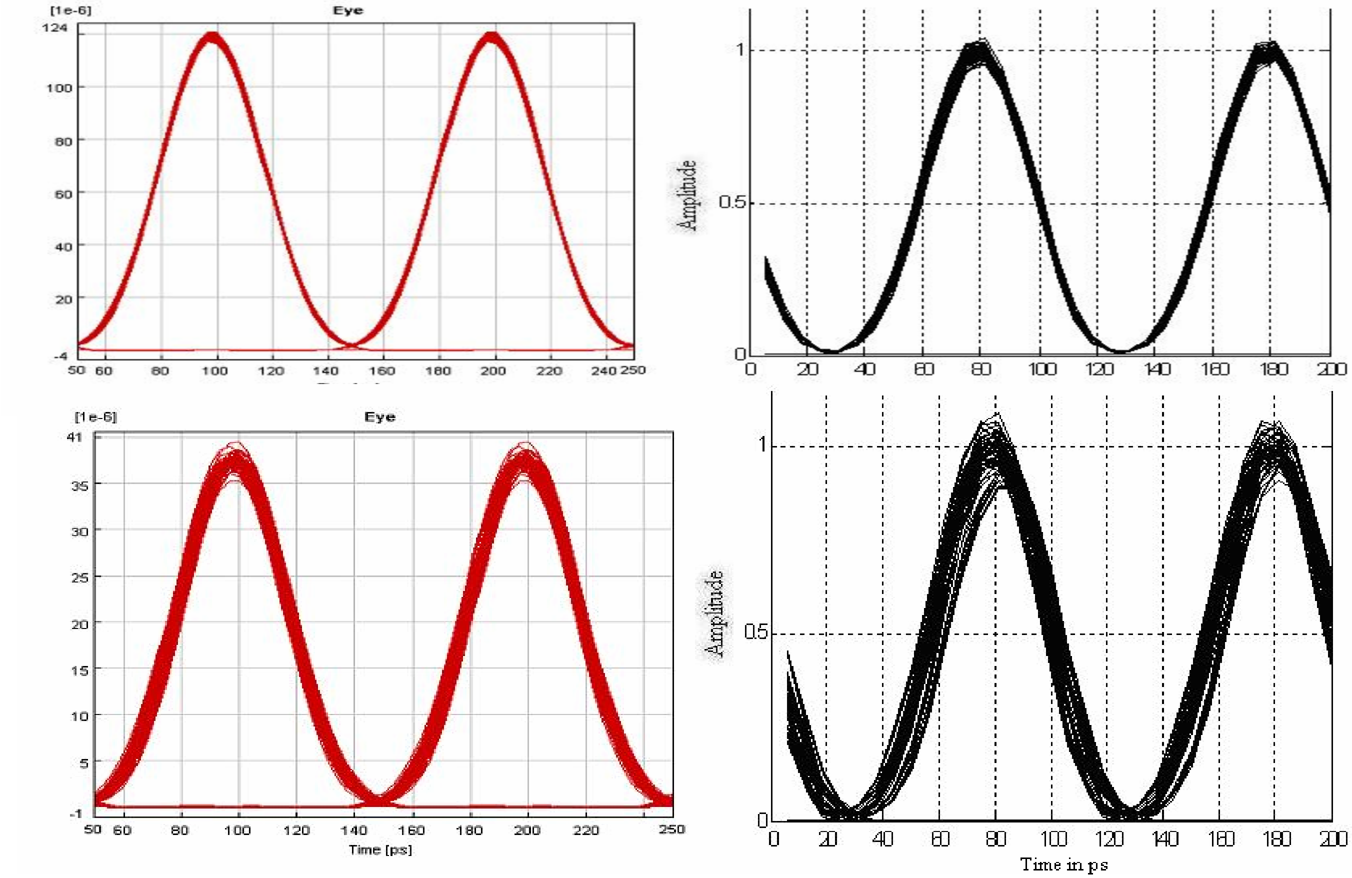
Results

- Delay versus time jitter comparison for 1 span
  - Gradual walk off effect accounting for differences



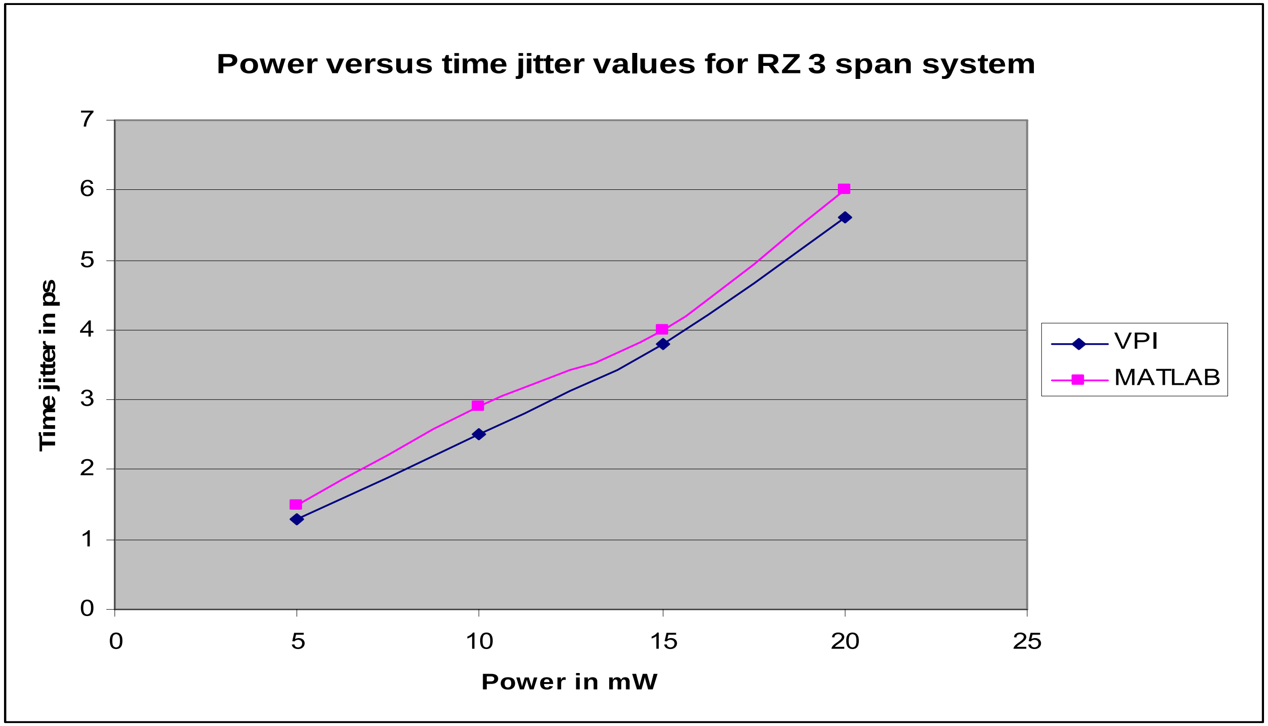
Results

- RZ 3 span analysis; comparison for 5 and 20 mW power



Results

- Comparison of time jitter values for 270 ps delay
  - 3 dB difference



Results

- Execution time

Number of spans	VPI execution time in s	MATLAB execution time in s
1	~ 12	1.4
3	~ 38	2.5

- We can see that our analytical model performs a lot faster than the VPI model

Conclusion and Future work

- An analytical model is developed which efficiently models the XPM impairment
- Improvement in execution times therefore can be efficiently used in RWA algorithms
- RZ format performance was analyzed which was not dealt with previous research papers
- Future work includes implementing the gradual walk off effect in the analytical model to resolve the differences

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- Others references in the thesis document

Thank you

Questions??