

PREDICTING FIRST-ORDER PMD OUTAGE RATES ON LONG-HAUL OPTICAL FIBER LINKS USING MEASURED AND MODELED TEMPORAL AND SPECTRAL DGD DATA

Ph.D. Final Oral Exam

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Outline

- Introduction
- Literature review and previous work
- Proposed PhD work
- Simplified first-order PMD outage rate expression
- Enhanced PMD numerical model
- Enhanced-model validation
- Simulation study of first-order PMD outage rate variation with link length
- Conclusions and future work

Introduction

- Need for high-speed transmission systems
 - In spite of the recent telecom bubble, the net traffic growth (combined Internet, data and voice traffic) remains steady
 - Major carriers are looking at increasing the transmission speeds on their networks
 - Sprint recently had 40 Gbps trials in their network
- Major challenges at high bit rates
 - Polarization-mode dispersion (PMD) and chromatic dispersion (CD)
 - PMD, unlike CD, is stochastic in nature and is difficult to compensate for
 - PMD is not as severe at 10 Gbps as it is at 40 Gbps and beyond

Introduction (contd...)

- Polarization-mode dispersion (PMD)
 - Caused by birefringence; complicated by mode coupling
 - Signal energy at a given λ is resolved into two orthogonal polarization modes with different refractive indices
 - Difference in propagation times between both modes is differential group delay (DGD)
 - Principal states of polarization (PSPs) - a light pulse launched in any PSP results in an output pulse that is undistorted to first order
 - PMD vector: magnitude of DGD and direction of fast PSP
 - Changes stochastically with λ and time due to randomness of mode coupling and external stresses
 - DGD has Maxwellian PDF

Literature review

Reported PMD measurements

Research Group	Fiber installation type	Fiber length / type	Measurement Repetition	Measurement Period	Mean DGD (ps)	Correlation Times	Measurement method
Karlsson et al 2000	Buried	127-km DSF 2 fibers	2 hrs	36 days	2.75 2.89	3 days 5.7 days	Jones matrix
Nagel et al. 2000	Buried	114-km SMF	5-10 min	70 days	41	19 hrs	Custom algorithm
Cameron et al. 1998	Buried	48.8-km SMF	58 sec	15 hrs	2.002	1-2 hrs	Interferometric
De Angelis et al. 1992	Buried	17-km		27 hrs	~0.5	20 min	
Bulow et al. 1999	Buried	52-km			7.3	6 to 13 ms	
Takahashi et al. 1993	Submarine	119-km	15 min	7 hrs	2.2	~ 1hr	Jones matrix
Kawazawa et al. 1994	Submarine	62-km DSF	10 min	9 months	~1.4	~ 2 months	Wavelength-scanning
Cameron et al. 1998	Aerial	96-km SMF	1.37 min	23 hrs	8.849	5 to 90 min	Interferometric
Bahsoun et al. 1990	Spool	10-km DSF	5 min	7 days	23	~ 3hrs	Wavelength-scanning
Poole et al. 1991	Spool	31.6-km DSF			34	~30 min	Wavelength-scanning

Literature review (contd...)

- Reported temporal and spectral characteristics of PMD
 - Temporal
 - On buried fibers DGD varies slowly, but randomly with time
 - Strong correlation between the changes in DGD and PSPs
 - Rate of temporal change of the PMD increases with the cable length and the mean DGD
 - Correlation between the temperature fluctuations and DGD variations is much stronger if links include connectors exposed to temperature variations
 - Spectral
 - DGD varies significantly with wavelength
 - High DGD events are spectrally localized
 - DGD bandwidth: $4/\langle\Delta\tau\rangle$ where $\langle\Delta\tau\rangle$ is the mean DGD
 - PMD in EDFAs is deterministic and less significant

Literature review (contd...)

- First-order PMD outage analysis

- Definition of an outage

- Caponi et al. (2002) definition: an event where DGD exceeds the given threshold value
 - Other definitions are also in use: power penalty, OSNR penalty, eye-opening penalty, Q penalty, BER penalty, etc.

- PMD outage probability

$$P_{\text{out}} = P(\Delta\tau \geq \Delta\tau_{\text{th}}) = 1 - \int_0^{\Delta\tau_{\text{th}}} f_{\Delta\tau}(\Delta\tau) d\Delta\tau \quad \text{units: minutes/year}$$

- Caponi et al. studied first-order PMD outage analysis

- Expression for mean outage rate

$$R_{\text{out}} = \frac{1}{2} f_{\Delta\tau}(\Delta\tau_{\text{th}}) \int_{-\infty}^{\infty} f_{\Delta\tau'}(\Delta\tau') |\Delta\tau'| d\Delta\tau' \quad \text{units: 1/year}$$

$\Delta\tau'$ is DGD time derivative

- Expression for mean outage duration

$$T_{\text{out}} = P_{\text{out}} / R_{\text{out}} \quad \text{units: minutes}$$

Literature review (contd...)

■ Numerical PMD model

- Dal Forno et al. (2000) developed a model for numerical simulation of PMD using coarse-step method
 - SMF is modeled as a concatenation of several fiber segments with a given mean birefringence and random coupling angles
 - Jones matrix at the end of the fiber is determined using

$$T(\omega) = \prod_{n=1}^N \begin{bmatrix} e^{j\left(\sqrt{\frac{3\pi}{8}}b\omega\sqrt{h_n}/2+\phi_n\right)} & 0 \\ 0 & e^{-j\left(\sqrt{\frac{3\pi}{8}}b\omega\sqrt{h_n}/2+\phi_n\right)} \end{bmatrix} \begin{bmatrix} \cos\alpha_n & \sin\alpha_n \\ -\sin\alpha_n & \cos\alpha_n \end{bmatrix}$$

■ where

- N: number of segments; h_n : length of n^{th} segment;
- b: PMD coefficient; ω : optical frequency;
- ϕ_n : temperature fluctuations, uniform distribution between 0 and 2π ;
- α_n : coupling angle between the segment axes, uniform distribution between 0 and 2π

Literature review (contd...)

- Dal Forno et al.'s PMD model
 - DGD is determined using the expression

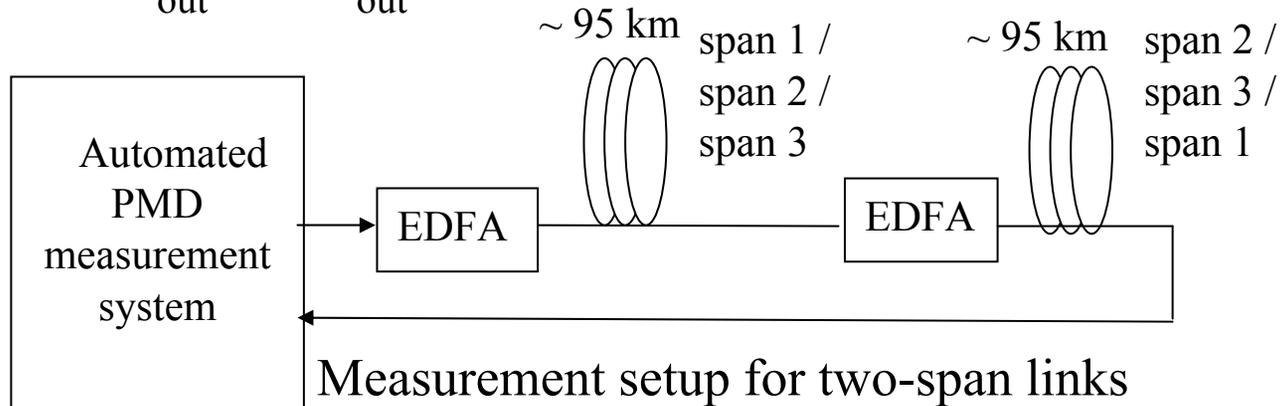
$$\Delta\tau = \left| \frac{\tan^{-1}\left(\frac{\rho_1}{\rho_2}\right)}{\Delta\omega} \right|$$

ρ_1 and ρ_2 are the Eigenvalues of the matrix $T_\omega(\omega)*T^{-1}(\omega)$, where $T_\omega(\omega)$ is the frequency derivative of $T(\omega)$

- The model gives the Maxwellian PDF of DGD and the DGD spectral dependence
- But the model does not have a temporal component
- To simulate realistic temporal DGD characteristics the free variables (namely b , ϕ_n , and α_n) should be varied in accordance with the environmental variations

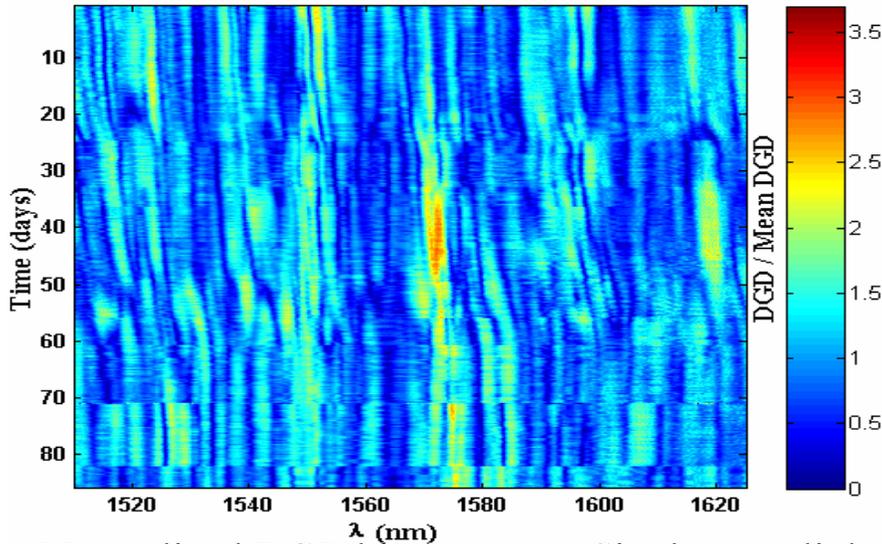
Previous work (Master's-level research)

- Long-term PMD measurements on buried fibers
 - Temporal and spectral measurements using 3 different 95-km fibers (1, 2, and 3) within a slotted-core, direct buried, standard SMF optic cable
 - 7 different fiber configurations: three single-span links 1, 2, 3, three two-span links 1-2, 2-3, 1-3 and one three-span link 1-2-3
 - EDFAs were used on multi-span links
- First-order PMD outage analysis using measured data
 - Predicted R_{out} and T_{out} values for the 7 different links

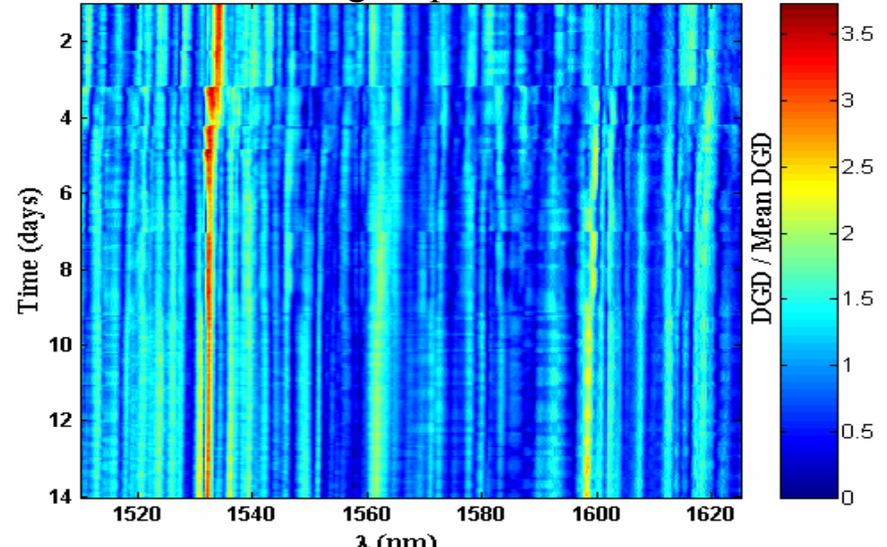


Measured DGD colormaps

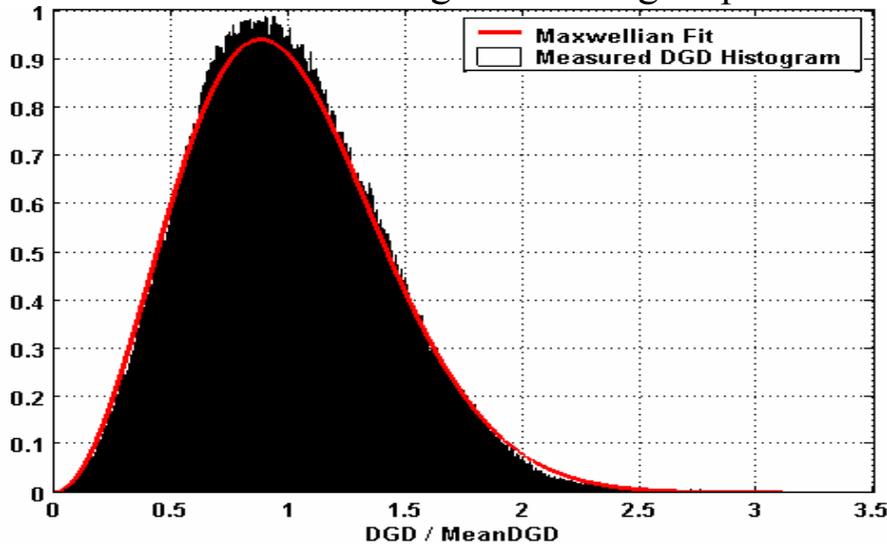
Single-span link 1



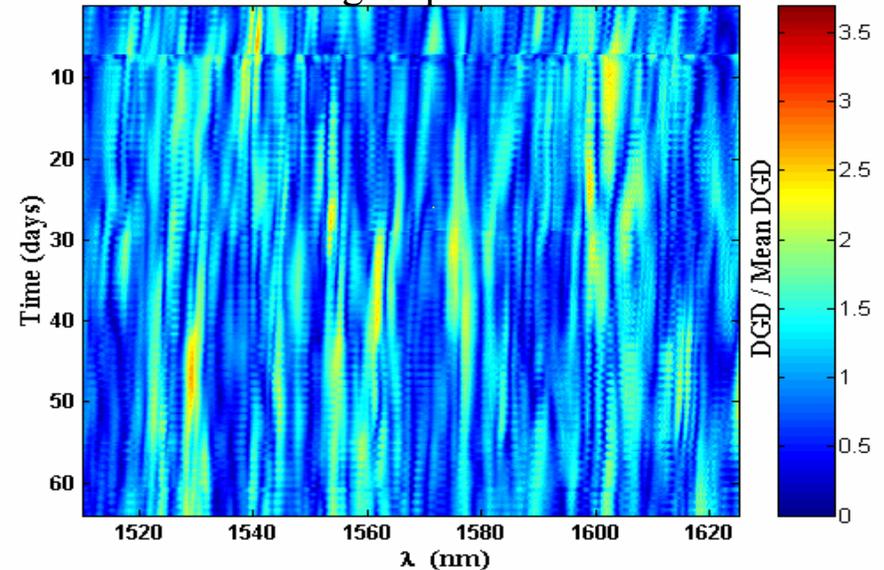
Single-span link 2



Normalized DGD histogram on Single-span link 1

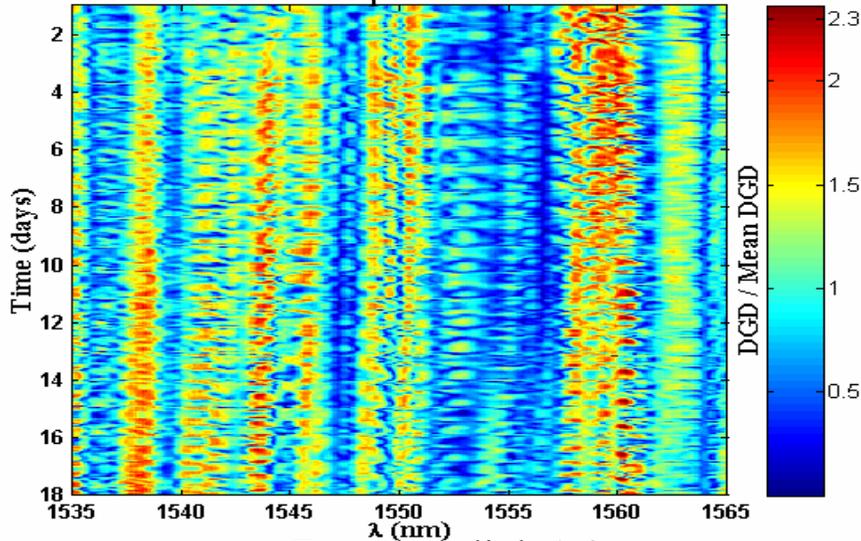


Single-span link 3

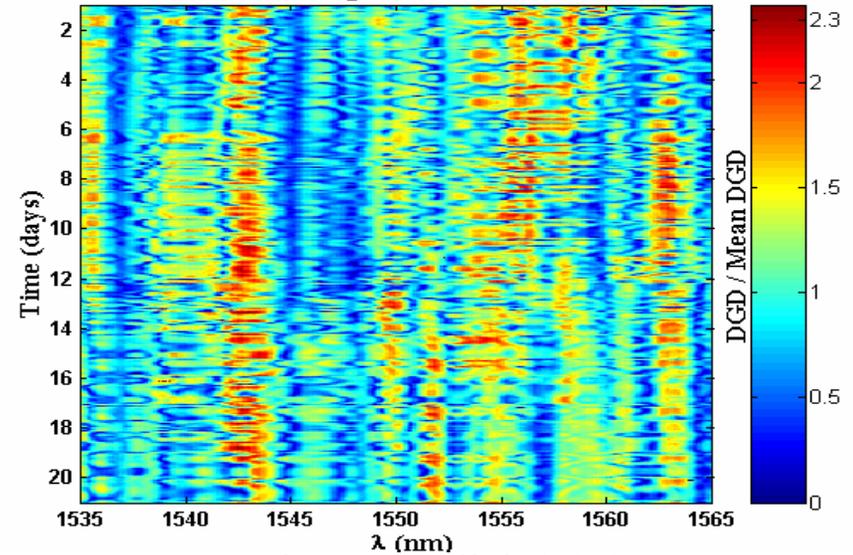


Measured DGD colormaps (contd.)

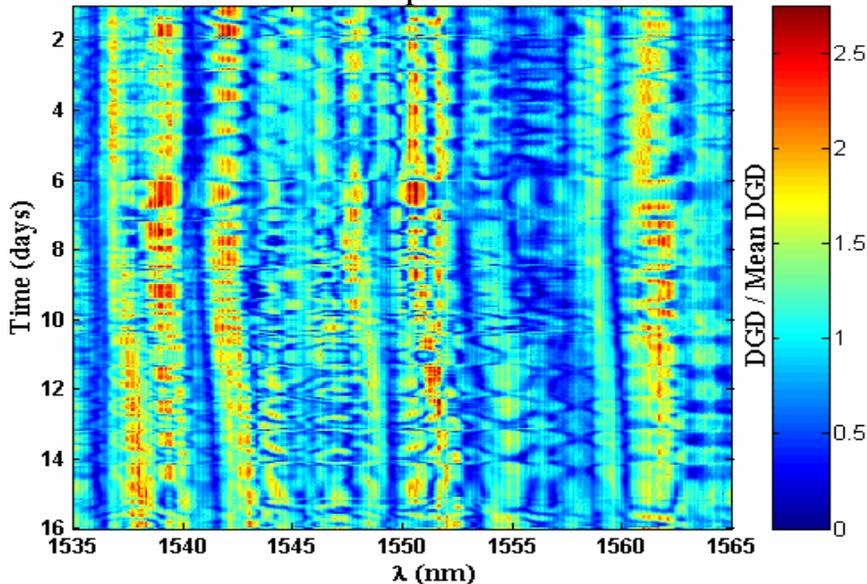
Two-span link 1-2



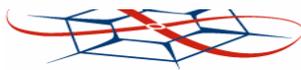
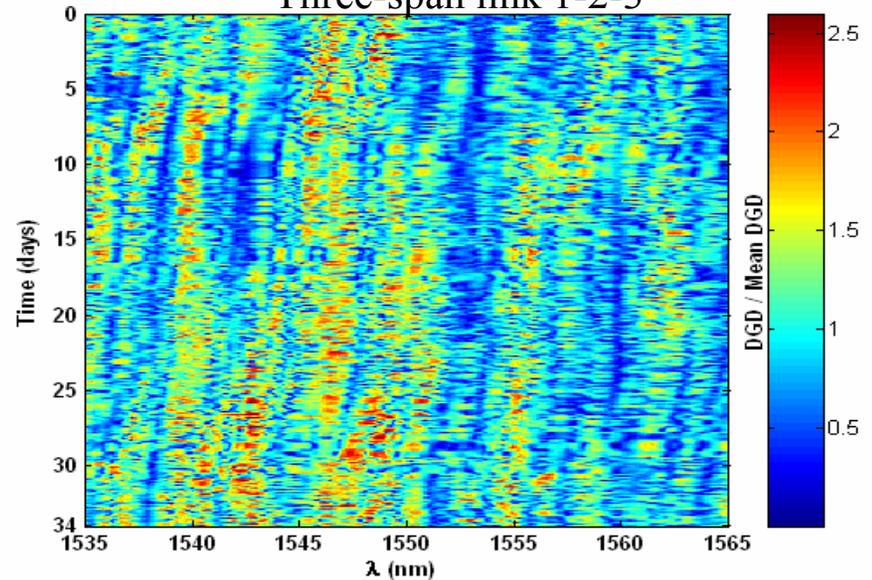
Two-span link 2-3



Two-span link 1-3



Three-span link 1-2-3



Proposed PhD work

- In my comprehensive exam, I have proposed to expand our understanding of the temporal behavior of PMD and predict first-order PMD outage rates on long-haul fiber-optic links
- Three-fold process to achieve this:
 - Simplify the first-order PMD outage rate expression given by Caponi et al. into a simple closed-form expression
 - Enhance the existing numerical model for PMD to simulate the real temporal variations observed from measurements
 - Use the simplified expression and the enhanced model to predict first-order PMD outage rates on long-haul fiber-optic links and study the variation of outage rates with link length

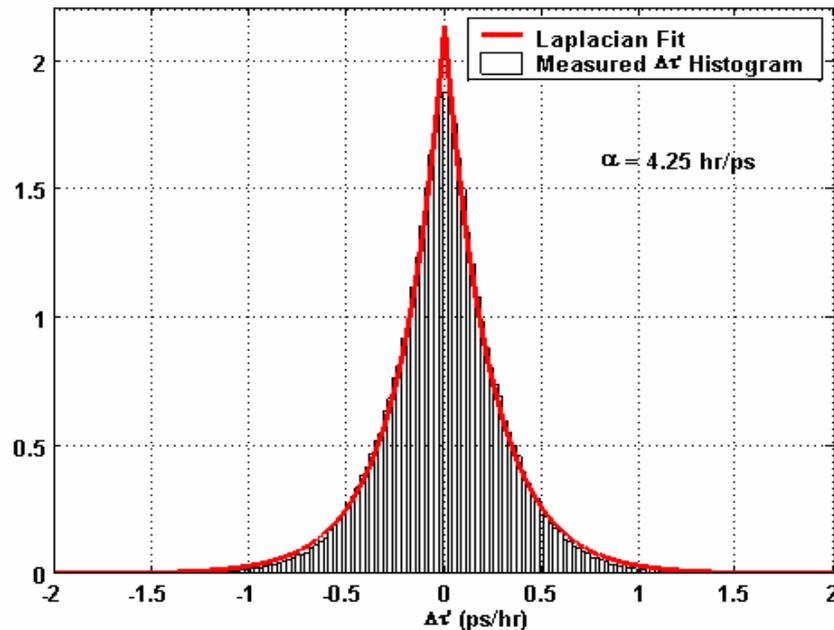
Significance of the proposed work

- Major carriers, like Sprint, are pushing for high-speed, all-optical, ultra long-haul fiber links
- To ensure signal quality on their fiber at higher rates, network engineers must anticipate the impact of PMD
- Solid understanding of PMD-induced system outages is lacking in PMD community
- Proposed work enables us to simulate the temporal and spectral PMD characteristics on any arbitrary length fiber-optic link and fully understand the impact of first-order PMD outages
- Higher-order PMD information could be extracted from the proposed enhanced PMD model for higher-order outage analysis

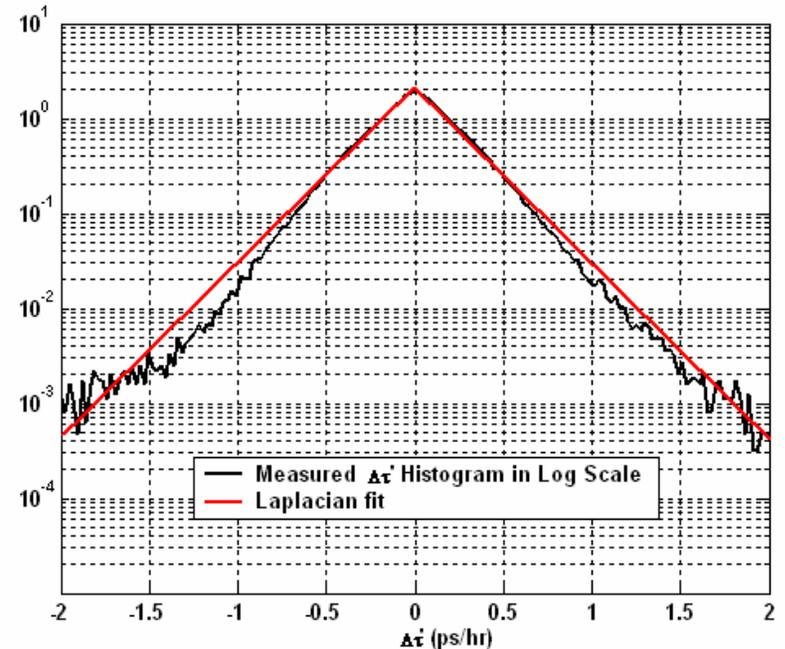
Simplified first-order PMD outage rate expression

The PDF of DGD time derivative ($\Delta\tau'$)

$$R_{\text{out}} = \frac{1}{2} f_{\Delta\tau}(\Delta\tau_{\text{th}}) \int_{-\infty}^{\infty} f_{\Delta\tau'}(\Delta\tau') |\Delta\tau'| d\Delta\tau'$$



Histogram of measured $\Delta\tau'$ data from Link 2 and its Laplacian fit in linear scale

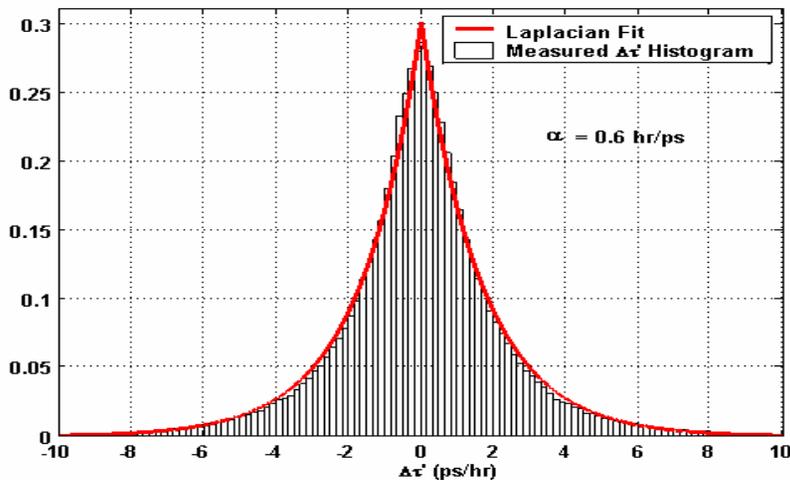


Histogram of measured $\Delta\tau'$ data from Link 2 and its Laplacian fit in log scale

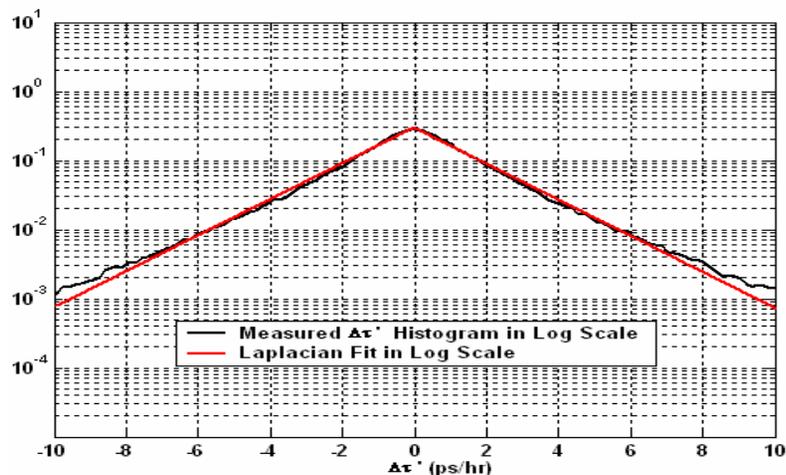
- Laplacian PDF: (a two-sided, first-order exponential)

$$f_{\tau'}(\Delta\tau') = \frac{\alpha}{2} e^{-\alpha|\Delta\tau'|} \quad \text{where } \alpha = \frac{\sqrt{2}}{\sigma} \quad \text{and is the Laplacian parameter with units of hr/ps and } \sigma \text{ is the standard deviation of } \Delta\tau'$$

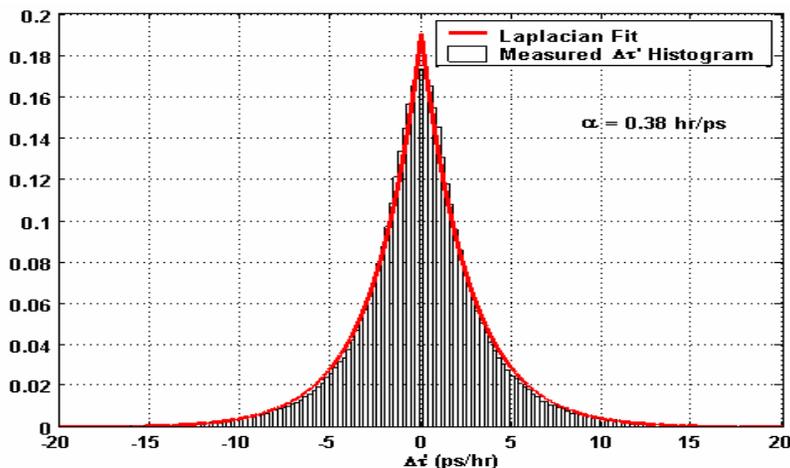
Simplified first-order PMD outage rate expression



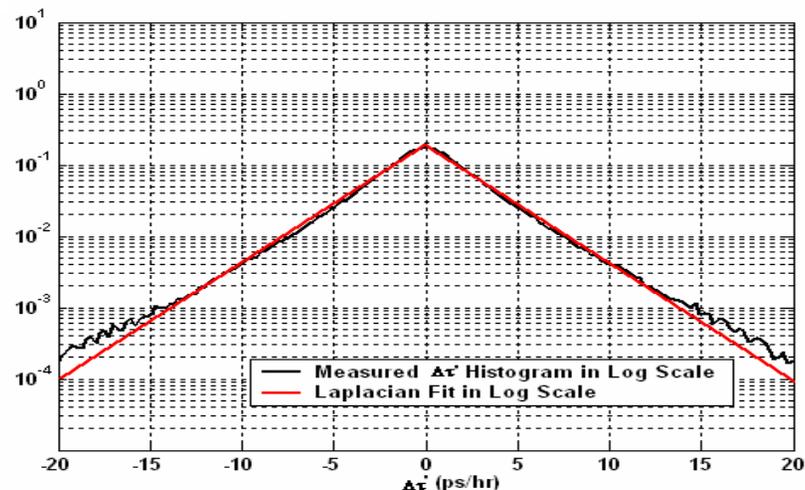
Histogram of measured $\Delta\tau'$ data from Link 1-2 and its Laplacian fit in linear scale



Histogram of measured $\Delta\tau'$ data from Link 1-2 and its Laplacian fit in log scale



Histogram of measured $\Delta\tau'$ data from Link 1-2-3 and its Laplacian fit in linear scale

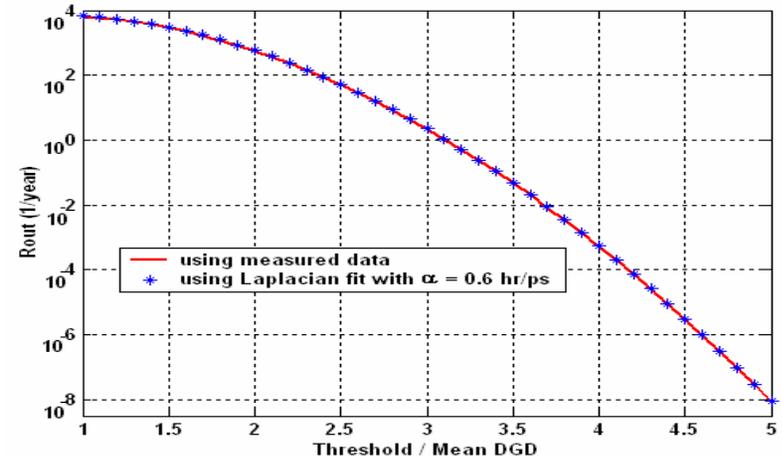


Histogram of measured $\Delta\tau'$ data from Link 1-2-3 and its Laplacian fit in linear scale

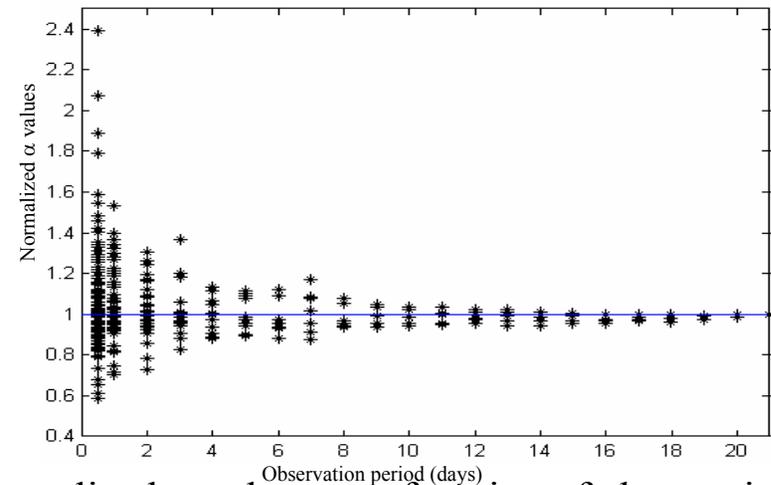
Simplified first-order PMD outage rate expression

Closed-form expression for R_{out}

- Using Laplacian PDF for $\Delta\tau'$, the original expression for R_{out} simplifies to
$$R_{out} = \frac{1}{2\alpha} f_{\tau}(\Delta\tau_{th})$$
- Simplified expression depends only on two parameters: mean DGD and Laplacian parameter
- Required measurement period for a good estimate of α
 - At least 10 to 14 days to estimate the value of α to within 10% of its actual value



Comparison of R_{out} values on Link 1-2 calculated using original and simplified expressions



Normalized α values as a function of observation period using measured data from two-span links

Enhanced PMD numerical model

Characterizing the existing base PMD model

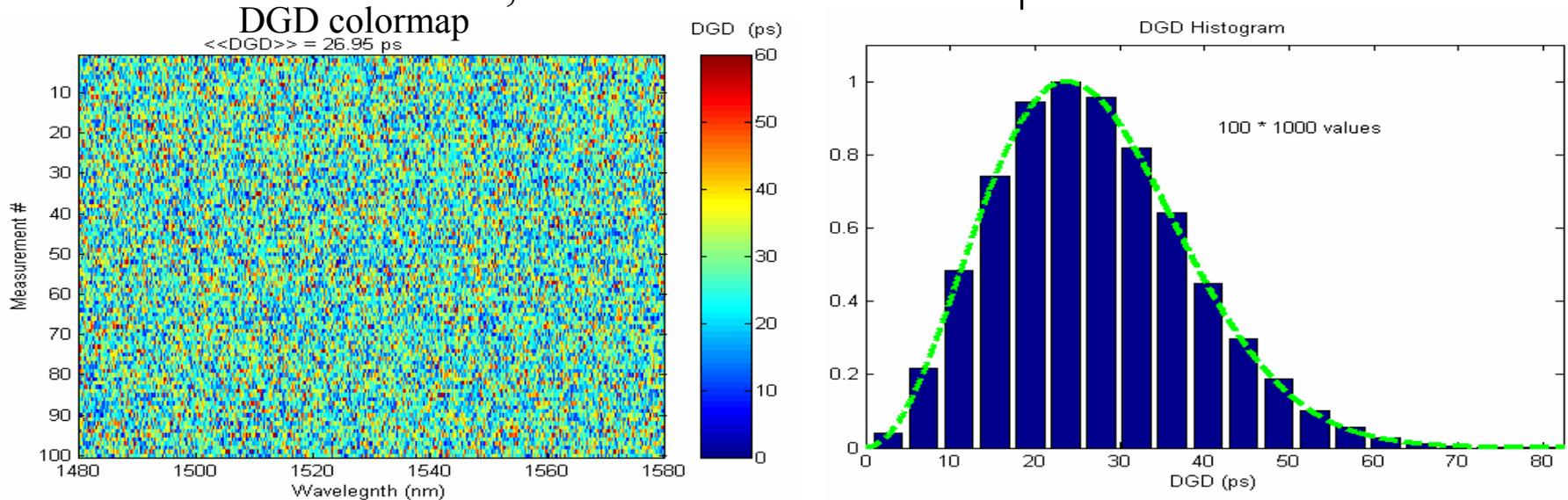
$$T(\omega) = \prod_{n=1}^N \begin{bmatrix} e^{j\left(\sqrt{\frac{3\pi}{8}}b\omega\sqrt{h_n}/2+\phi_n\right)} & 0 \\ 0 & e^{-j\left(\sqrt{\frac{3\pi}{8}}b\omega\sqrt{h_n}/2+\phi_n\right)} \end{bmatrix} \begin{bmatrix} \cos\alpha_n & \sin\alpha_n \\ -\sin\alpha_n & \cos\alpha_n \end{bmatrix}$$

- Simulation parameters: (Matlab)

PMD coefficient (b): 2.7 ps/ $\sqrt{\text{km}}$; fiber length: 100 km; N = 100;

λ band: 1480-1580 nm (100 nm); λ step: 0.1 nm; $h_n = 1$ km (fixed)

100 simulation runs; Different sets of α and ϕ values for each run



Enhanced PMD numerical model (contd..)

- Need to add the temporal component to the base model
- Studies have shown that PMD temporal variation strongly correlates with the ambient temperature variations with no time lag
- Such behavior is believed to be driven by a few segments of the fiber, like man holes, EDFA huts, bridge attachments, etc., being exposed to the outside air temperature variations
- Other stress-inducing factors like atmospheric pressure, rain events, surface vibrations, etc., also affect the temporal behavior of PMD, but in a small way
- The three parameters, α_n , ϕ_n , and b , of the base model should be made functions of the ambient temperature and the other stress-inducing factors

Enhanced PMD numerical model (contd..)

- The coupling angle α_n , is determined by the manufacturing and installation procedures
- An installed fiber does not see appreciable change in coupling angles over time and so α_n for an installed fiber can be modeled as a static set of uniform random values between 0 and 2π
- Making PMD coefficient 'b' a function of temperature results in a drift in spectral domain (illustrated on the next slide)
- Angle ϕ_n is the crucial parameter to model PMD temporal behavior
 - ϕ_n of few segments of each span should be made time-variant
 - Central limit theorem could be used to model all the stress-inducing factors other than temperature, which is modeled as a linear term

$$\phi_n = U[0 \ 2\pi] + k * \text{Air Temperature } (^{\circ}\text{F}) + N[0, G_var]$$

Uniform random variable
between 0 and 2π

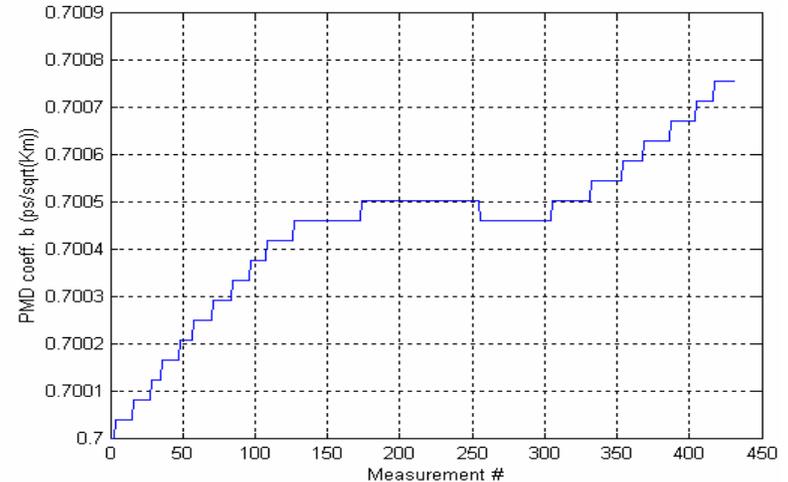
Proportionality constant
Units: radians/($^{\circ}\text{F}$)

Gaussian random variable
Mean: 0; Variance: G_var
Units: Radian²

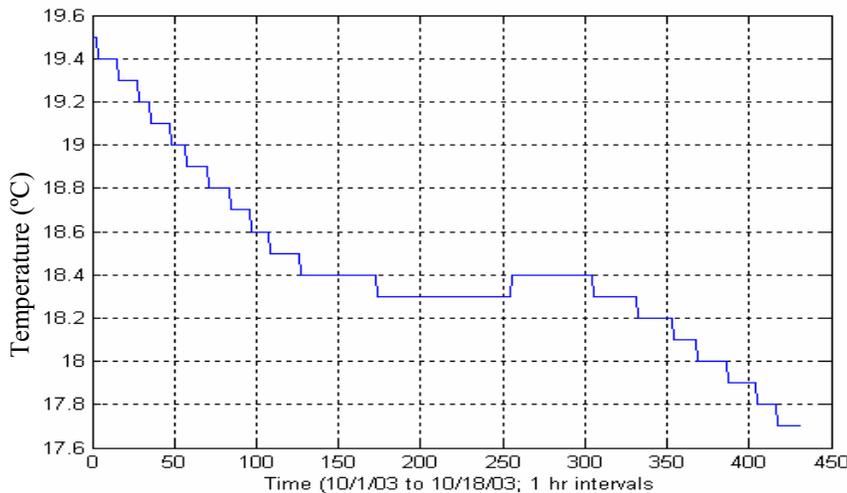
Enhanced PMD numerical model (contd.)

Effect of temperature-dependent 'b'

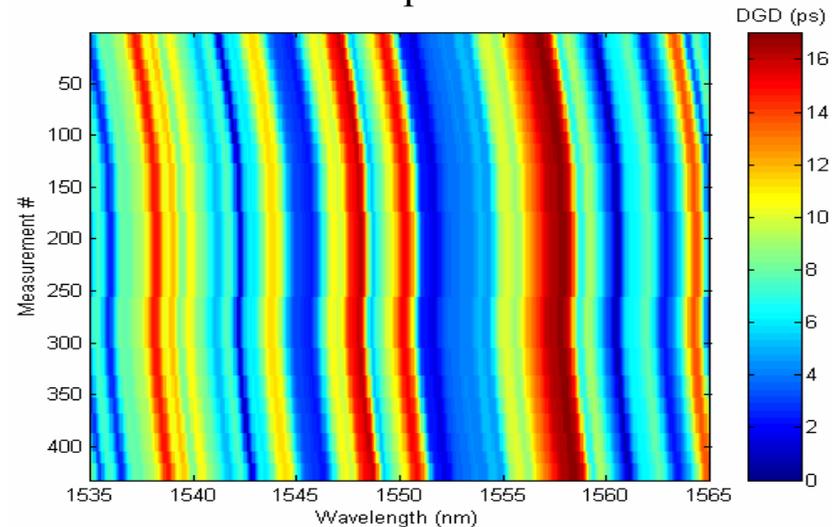
- Simulations were run with an initial PMD coefficient of 0.7 ps/ $\sqrt{\text{km}}$
- Value of Relative temperature sensitivity of 'b' used: $6 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$
- Fixed sets of uniform random values for α_n and ϕ_n



PMD coefficient variation modeled based on the temperature variation



Hourly temperature at one location.
Source: NRCS website



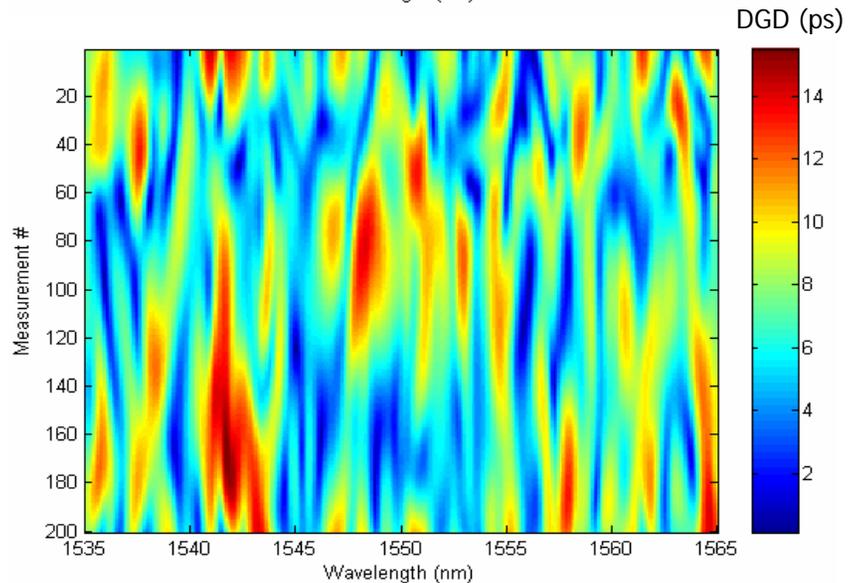
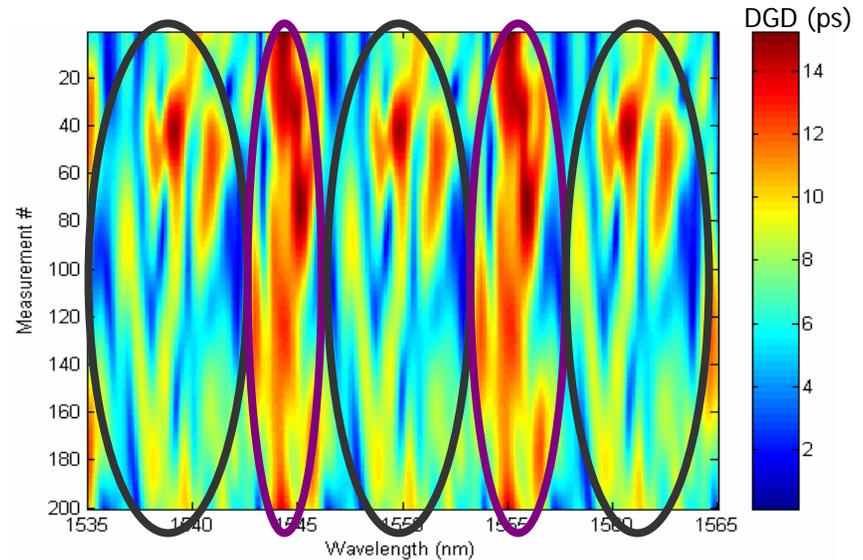
DGD vs. wavelength and time using the modeled PMD coefficient

Enhanced PMD numerical model (contd..)

- Segment length h_n
 - Using fixed value for h_n results in artificial periodicity in spectral domain
 - Could be avoided by making h_n a Gaussian variable

■ Illustration

- Simulation parameters
 - $N=100$; $b=0.7$ ps/ $\sqrt{\text{km}}$
 - 4 segments with time-dependent ϕ_n
 - h_n : 1 km fixed value (top fig)
 - Gaussian values (bottom fig)
 - mean: 1 km
 - variance: 20 % of mean



Enhanced-model validation

- Validate the model by comparing the simulation results with the measured results on the 7 links used in measurements
- Model accuracy metrics
 - Mean DGD (time and wavelength averaged DGD) value
 - Goodness of Maxwellian PDF fit to the simulated DGD data
 - Goodness of Laplacian PDF fit to the simulated DGD time derivative data
 - Laplacian parameter value
 - Decorrelation time and bandwidth
 - Overall appearance of the DGD colormap

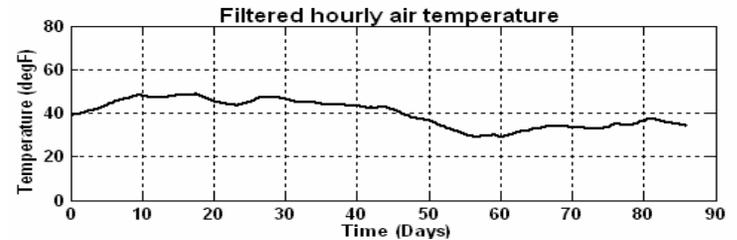
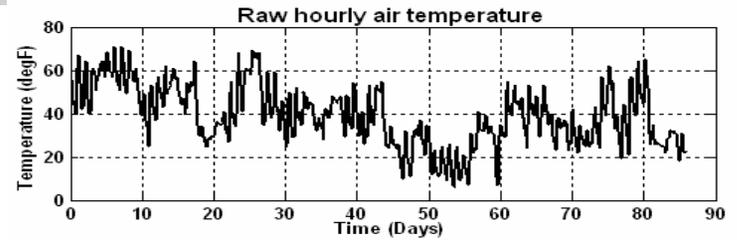
Enhanced-model validation: Single-span link 1

Temperature profile →

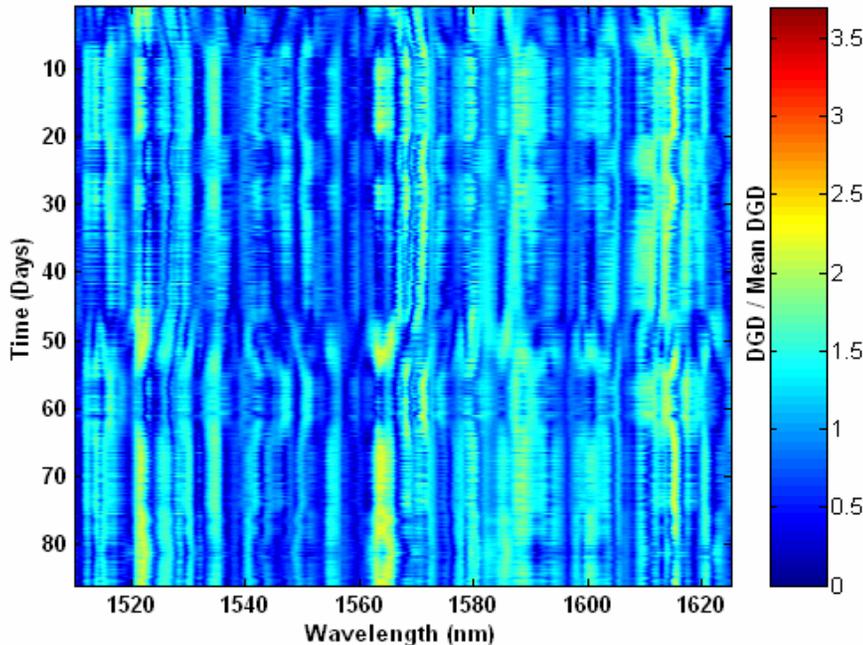
N=500 per span; Time-invariant 'b'
 Sampling interval:
 3 hrs (simulations); 2 hrs 55 min (measurements)

Model free parameters

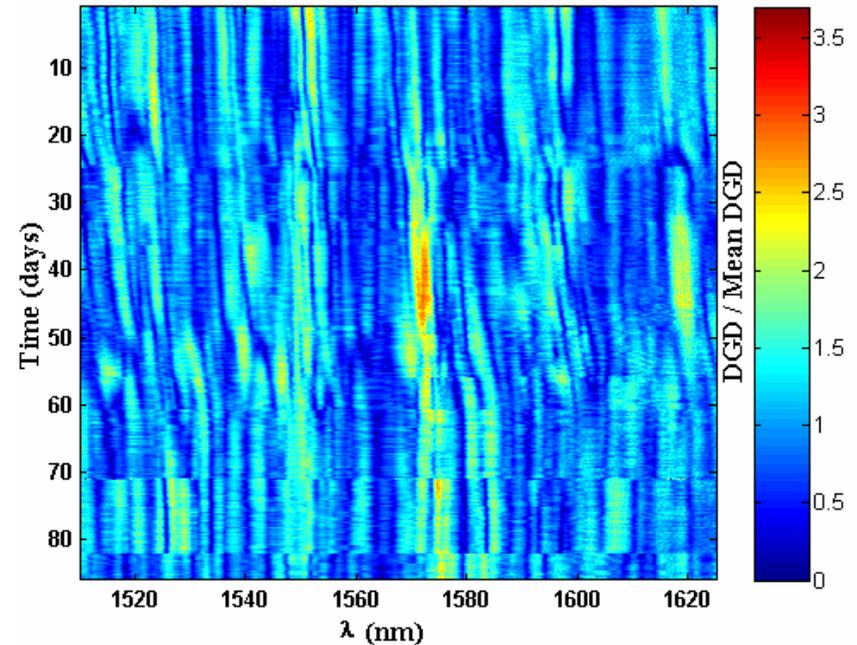
Proportionality constant k (radians/°F)	Number of time-varying sections	Relative filter bandwidth parameter	Gaussian std. deviation (radians)
0.2	4	0.001	$\pi/22$



From Simulations

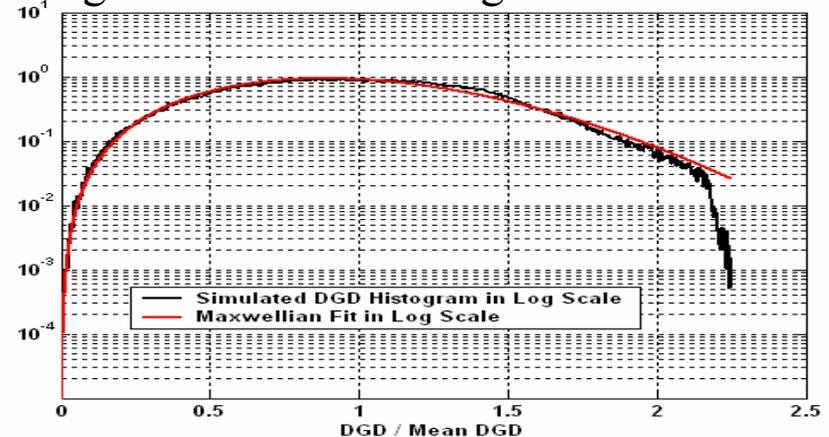
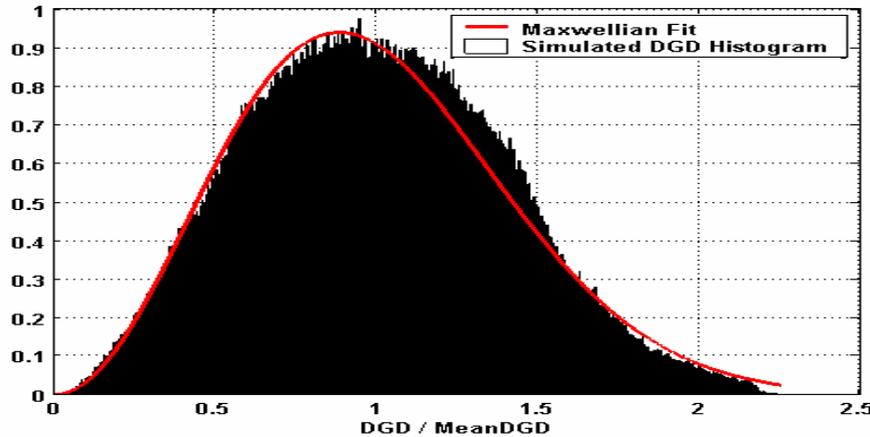


From Measurements

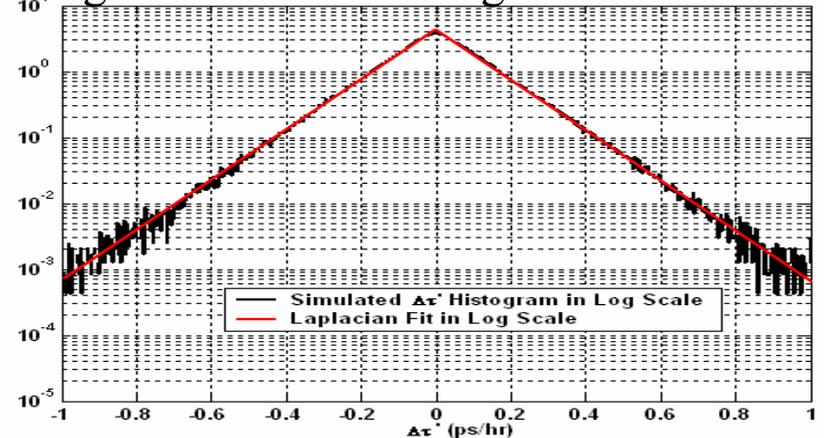
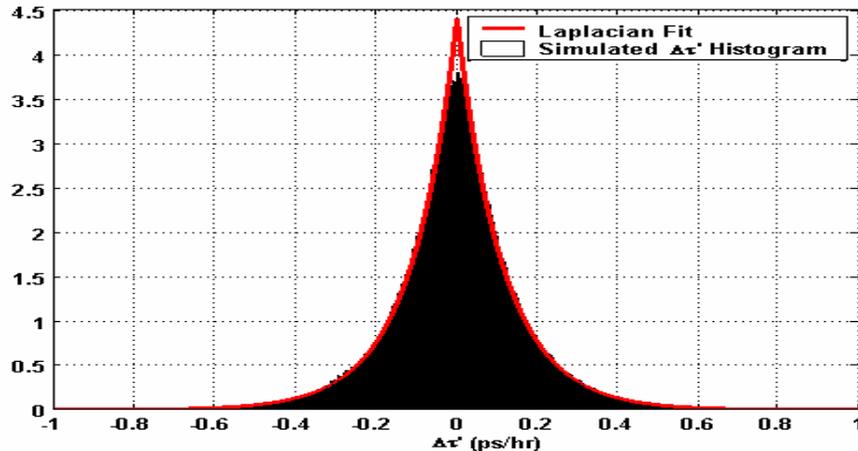


Enhanced-model validation: Single-span link 1

Simulated normalized DGD histogram in linear and log scales



Simulated DGD time derivative histogram in linear and log scales



- Mean DGD from simulations within 1 % of the value from measurements
- Laplacian parameter: from simulations 8.7 hr/ps; from measurements 7.5 hr/ps
- Decorrelation time: from simulations 4 days; from measurements 4.6 days;
- Decorrelation BW from simulations within 30 % of the value from measurements

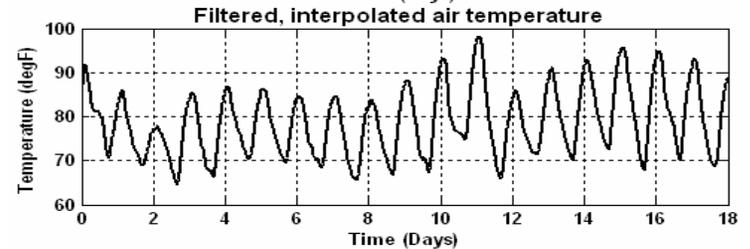
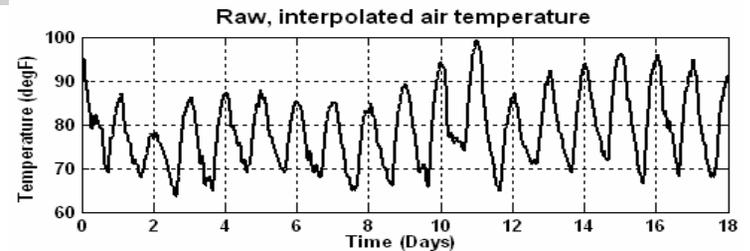
Enhanced-model validation: Two-span link 1-2

Temperature profile →

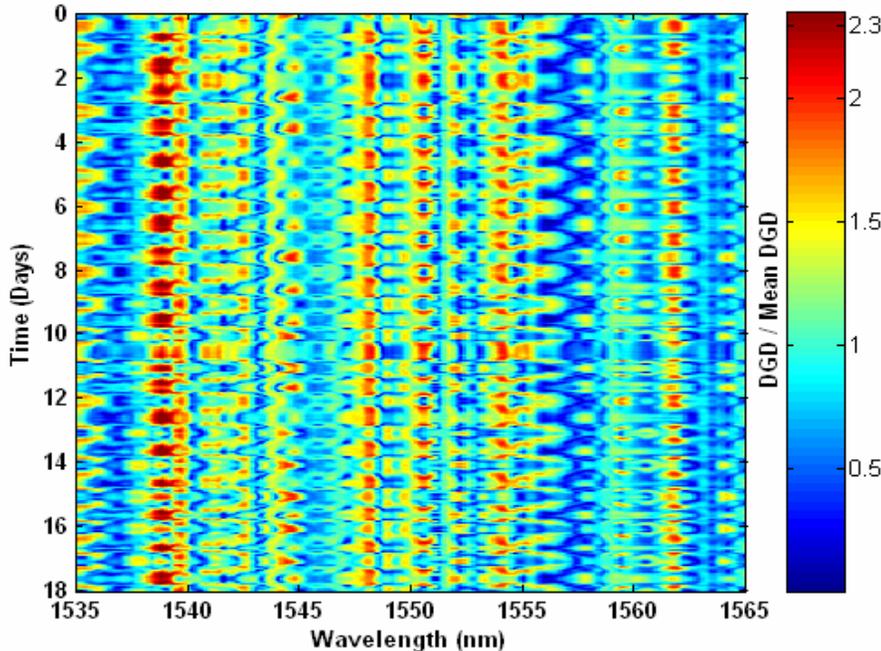
N=500 per span; Time-invariant 'b' for each span
 Sampling interval:
 20 min (simulations); 23 min (measurements)

Model free parameters

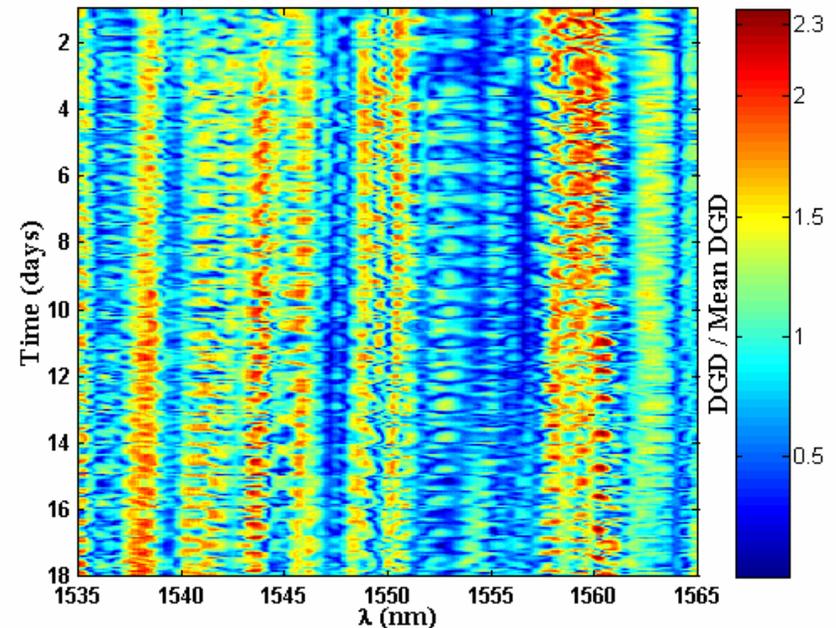
Proportionality constant k (radians/°F)	Number of time-varying sections	Relative filter bandwidth parameter	Gaussian std. deviation (radians)
0.11	8	0.08	$\pi/135$



From Simulations

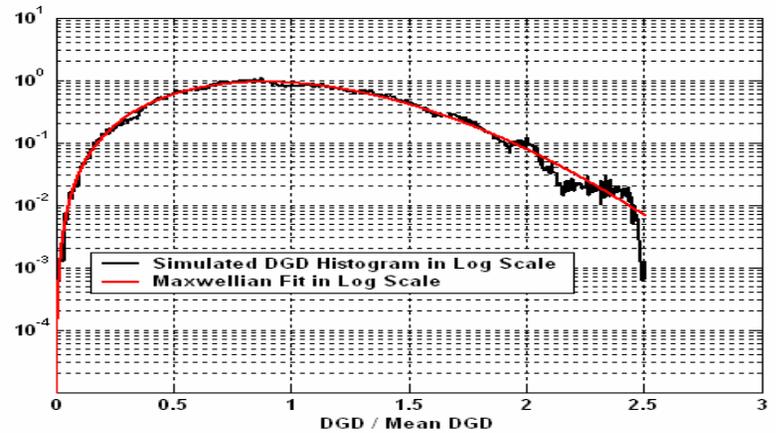
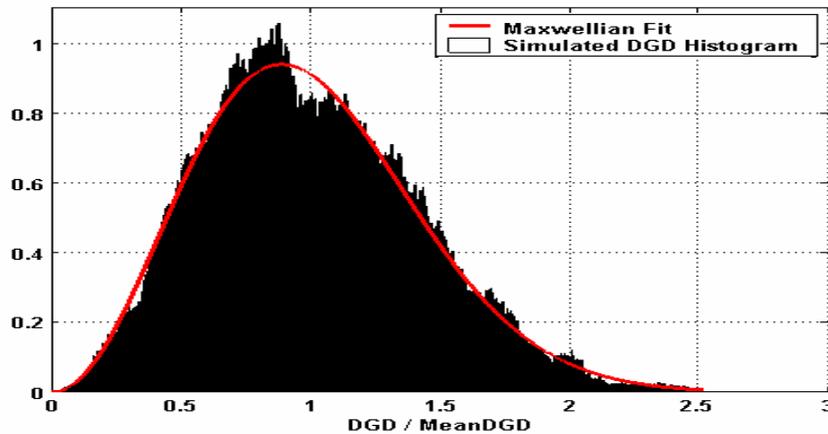


From Measurements

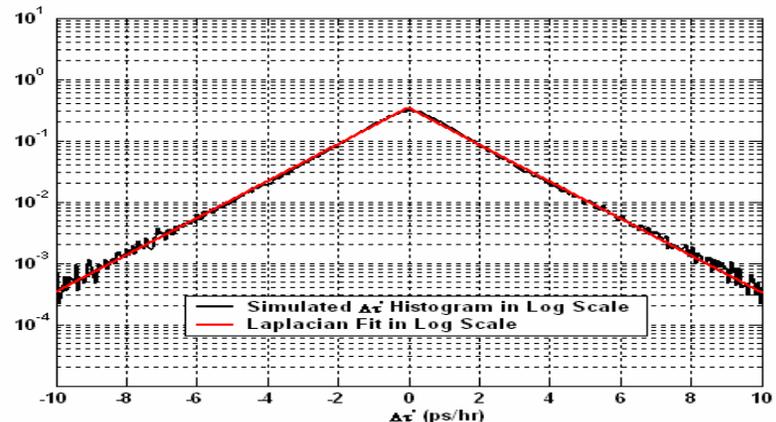
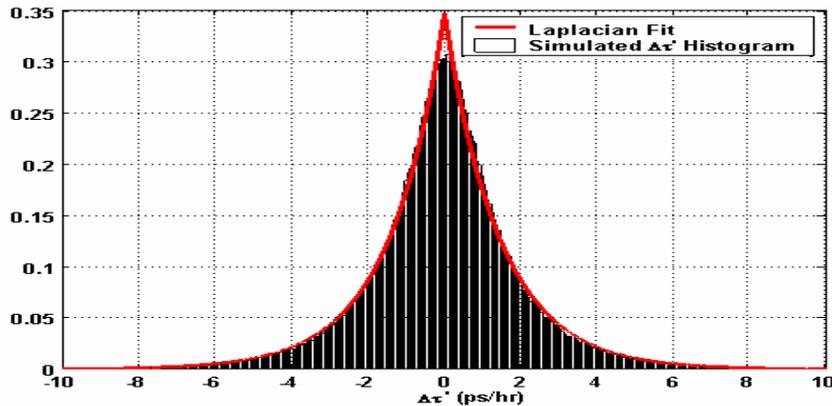


Enhanced-model validation: Two-span link 1-2

Simulated normalized DGD histogram in linear and log scales



Simulated DGD time derivative histogram in linear and log scales



- Mean DGD from simulations within 2 % of the value from measurements
- Laplacian parameter: from simulations 0.69 hr/ps; from measurements 0.6 hr/ps
- Decorrelation time: from simulations 1.66 hours; from measurements 1.53 hours;
- Decorrelation BW from simulations within 10 % of the value from measurements

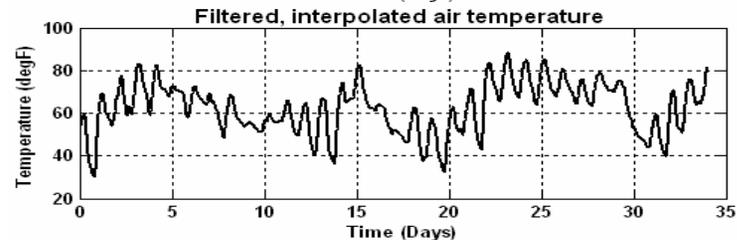
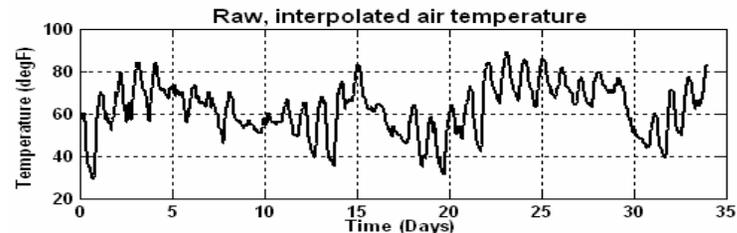
Enhanced-model validation: Three-span link 1-2-3

Temperature profile →

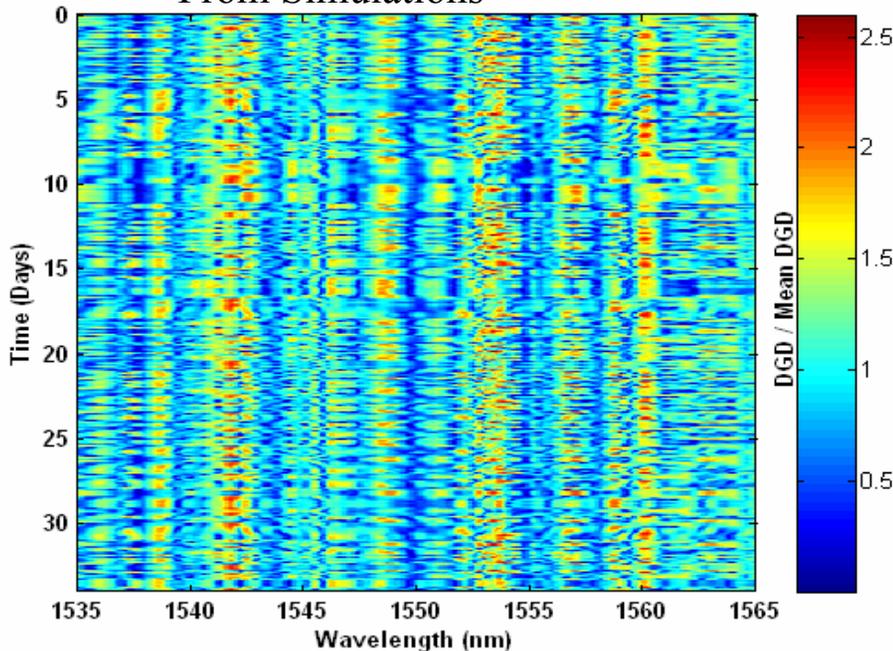
N=500 per span; Time-invariant 'b' for each span
 Sampling interval:
 20 min (simulations); 23 min (measurements)

Model free parameters

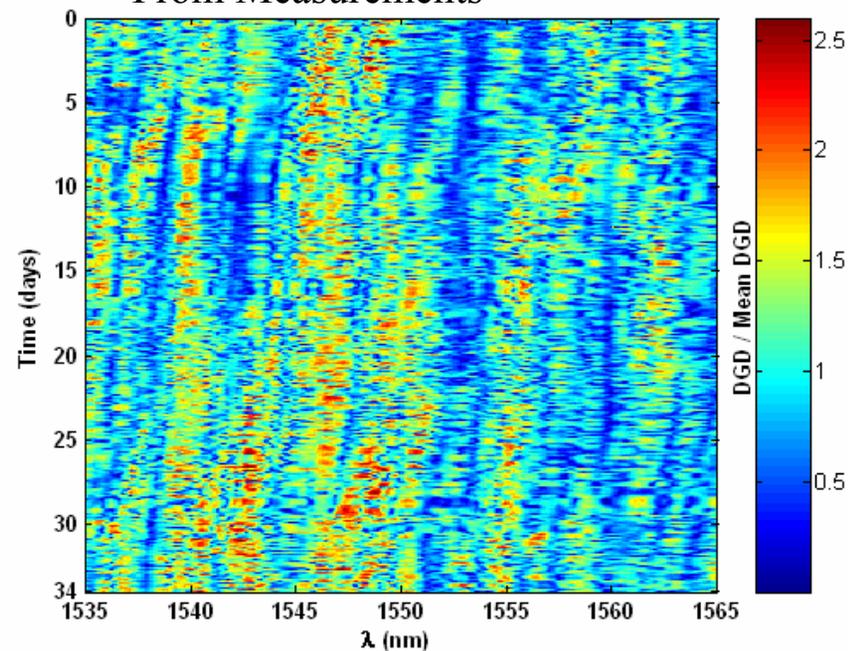
Proportionality constant k (radians/°F)	Number of time-varying sections	Relative filter bandwidth parameter	Gaussian std. deviation (radians)
0.15	12	0.08	$\pi/90$



From Simulations

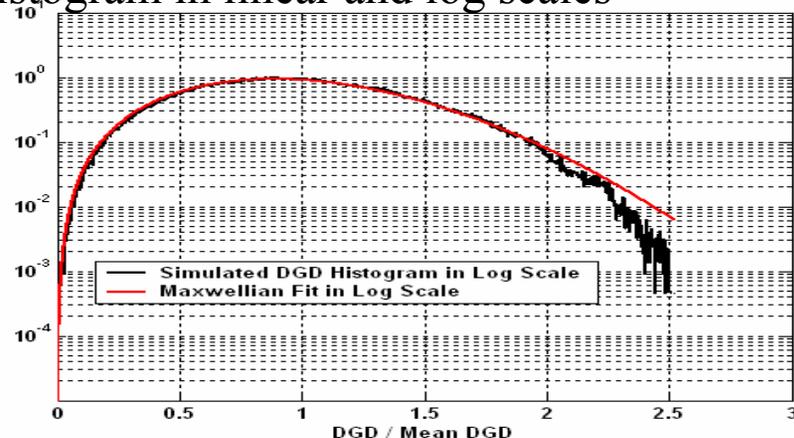
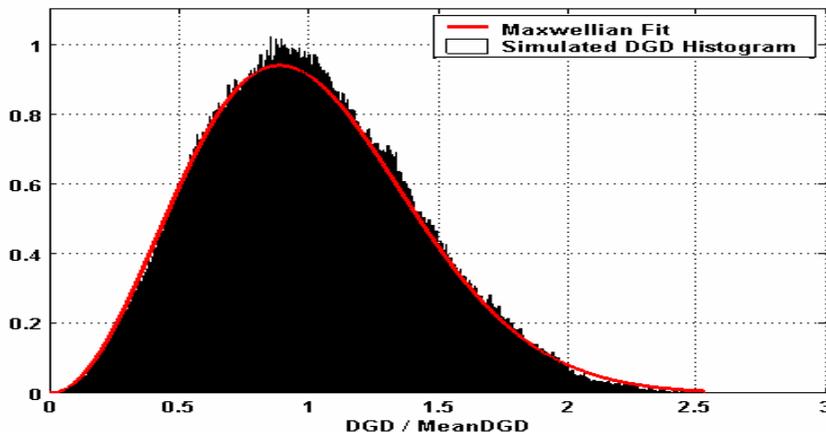


From Measurements

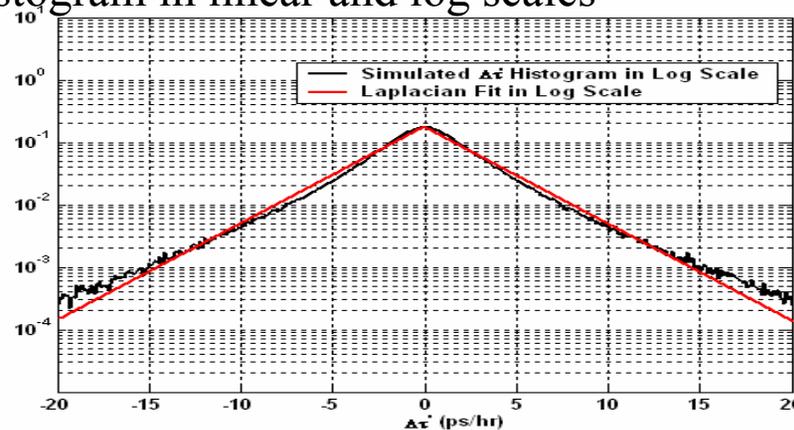
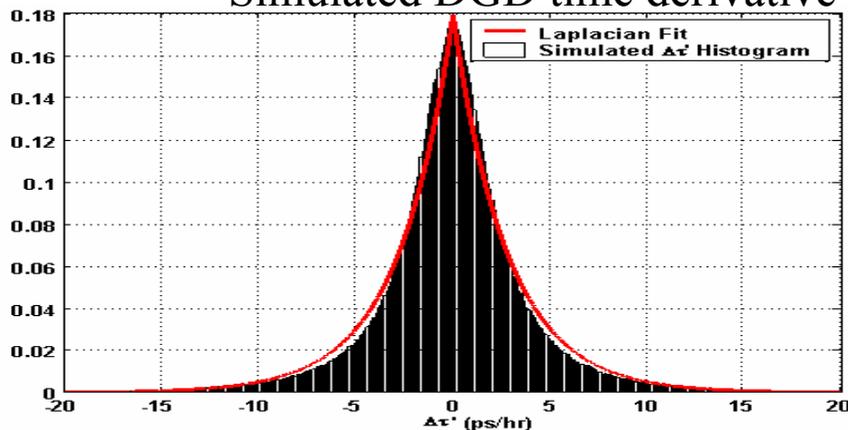


Enhanced-model validation: Three-span link 1-2-3

Simulated normalized DGD histogram in linear and log scales



Simulated DGD time derivative histogram in linear and log scales



- Mean DGD from simulations within 3 % of the value from measurements
- Laplacian parameter: from simulations 0.35 hr/ps; from measurements 0.38 hr/ps
- Decorrelation time: from simulations 1.33 hours; from measurements 1.83 hours;
- Decorrelation BW from simulations is same as the value from measurements

Enhanced-model validation: Summary

Link configuration	Free parameters of the model				Percent difference between measurements and simulations			
	Constant k (radians/ °F)	# of time-varying sections	Relative filter BW parameter	Gaussian standard Deviation (radians)	Mean DGD	Laplacian parameter α	Decorrelation time	Decorrelation bandwidth
Link 1	0.2	4	0.001	$\pi/22$	1 %	14 %	13 %	30 %
Link 2	0.2	4	0.001	$\pi/48$	5 %	7 %	4.5 %	10 %
Link 3	0.2	4	0.002	$\pi/120$	5 %	10.5 %	11 %	15 %
Link 1-2	0.11	8	0.08	$\pi/135$	2 %	13 %	8 %	10 %
Link 2-3	0.1	8	0.08	$\pi/120$	3 %	3 %	23 %	0 %
Link 1-3	0.08	8	0.08	$\pi/120$	1 %	12.5 %	13 %	0 %
Link 1-2-3	0.15	12	0.08	$\pi/90$	3 %	8 %	27 %	0 %

■ Conclusion

- Enhanced-model reproduced very well the temporal and spectral characteristics of DGD observed from the measurements
- Need for a narrow LPF for single-span links should be further investigated
- Enhanced-model can be used to predict first-order PMD outage rates on long-haul optical fiber links

Simulation study of first-order PMD outage rate variation with link length

- Objective was to study the variation of Laplacian parameter, and thereby the first-order PMD outage rate with link length
- Simulations on two-span (190 km), four-span (380 km), five-span (475 km), seven-span (665 km), nine-span (855 km) and eleven-span (1045 km)
- Single temperature profile, from the 34-day measurement period of link 1-2-3
- PMD coefficients of the 3 single-span links (b1, b2, b3) cycled through for multi-span links. Ex: five-span link b1-b2-b3-b1-b2
- Values of h_n , α_n , and ϕ_n for all the links were derived from a single set of values used for the eleven-span link

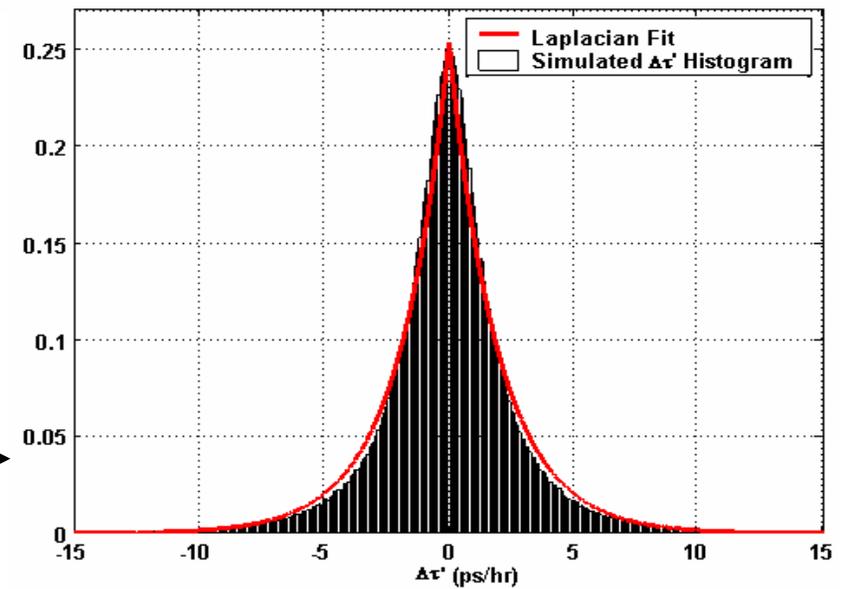
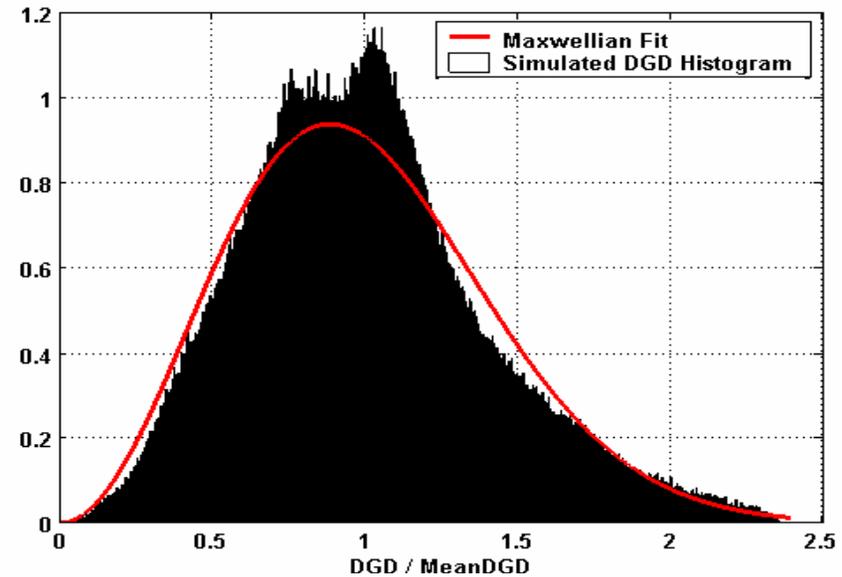
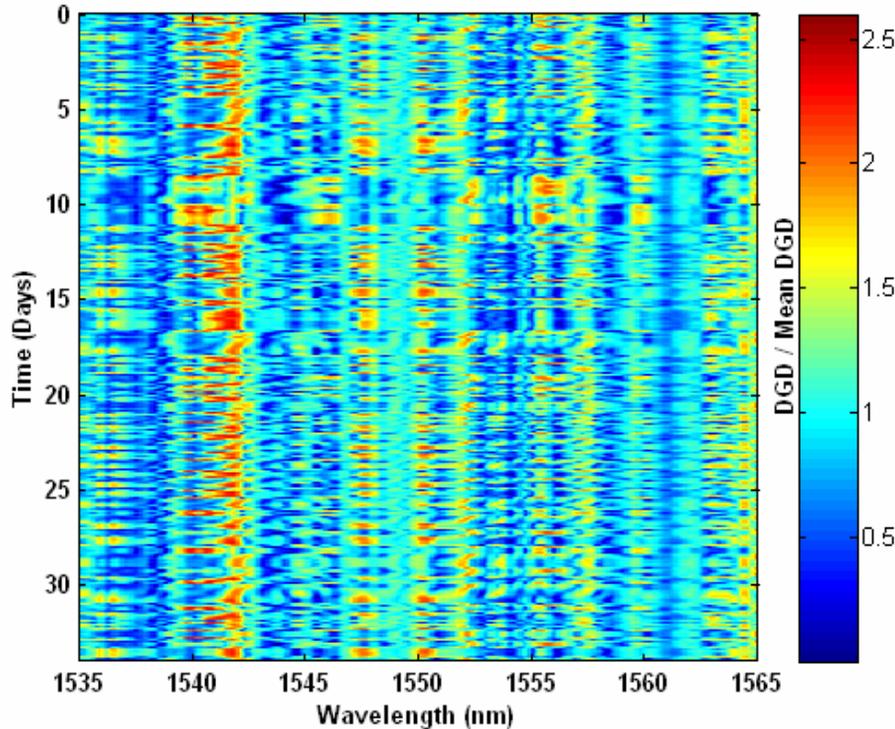
Model free parameters

Proportionality constant k (radians/°F)	Number of time-varying sections	Relative filter Bandwidth parameter	Gaussian std. deviation (radians)
0.15	4 per span	0.08	$\pi/90$ to $\pi/120$

Simulation study of first-order PMD outage rate variation with link length: Two-span link

Sampling interval: 20 min

Gaussian std. deviation: $\pi/90$ radians

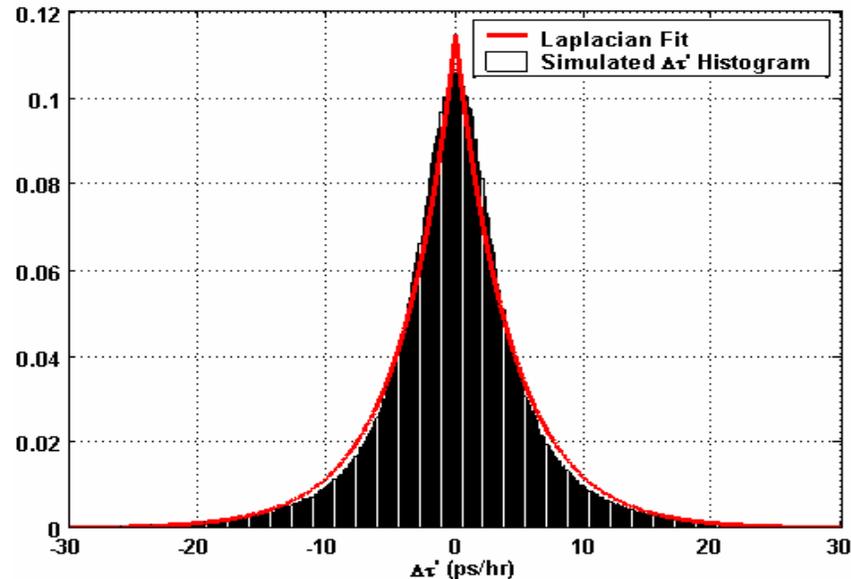
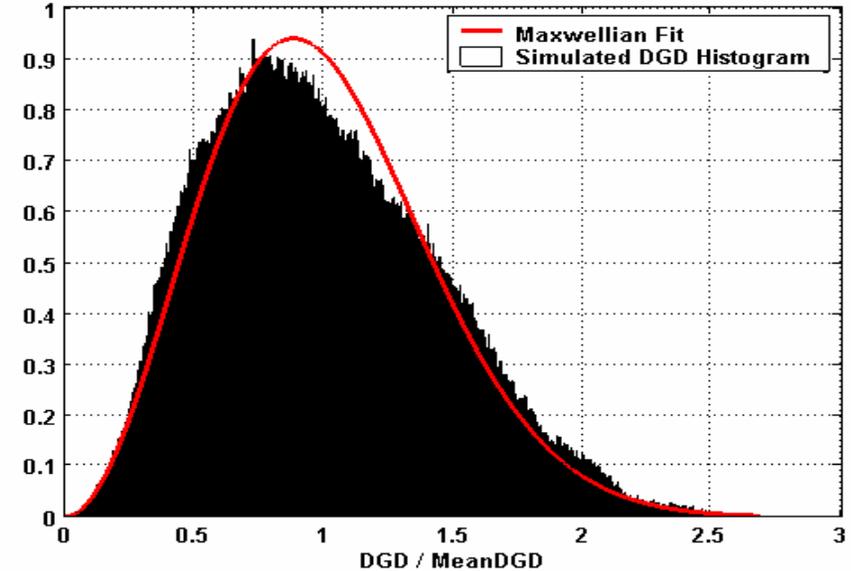
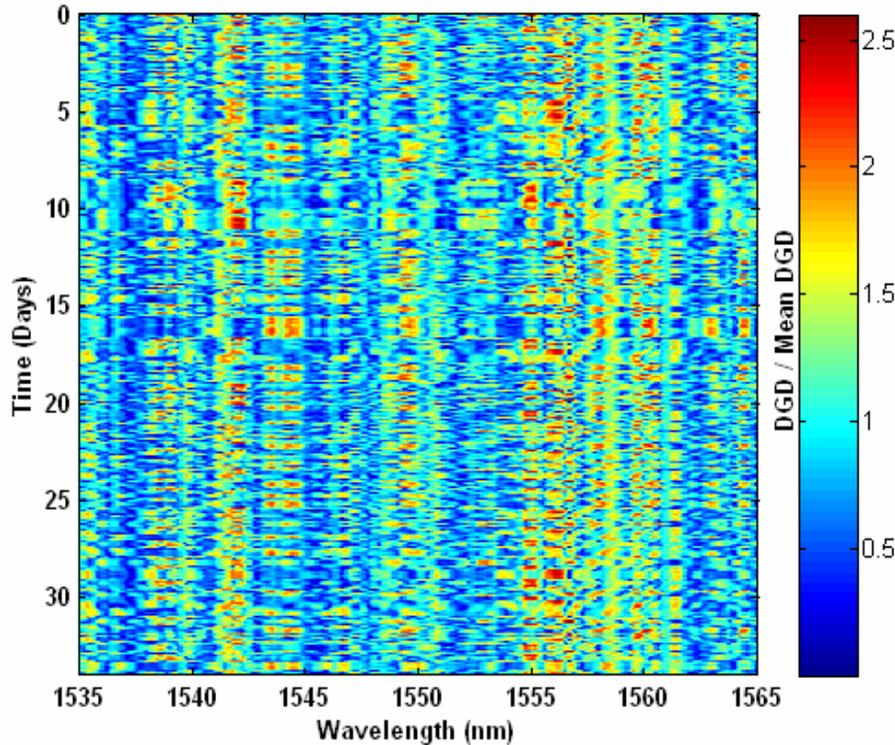


Laplacian parameter: 0.504 hr/ps →

Simulation study of first-order PMD outage rate variation with link length: Four-span link

Sampling interval: 20 min

Gaussian std. deviation: $\pi/90$ radians

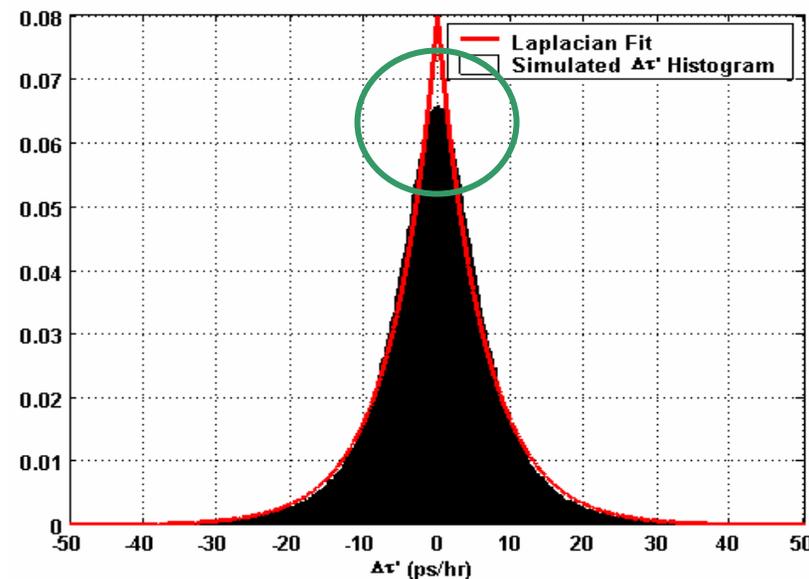
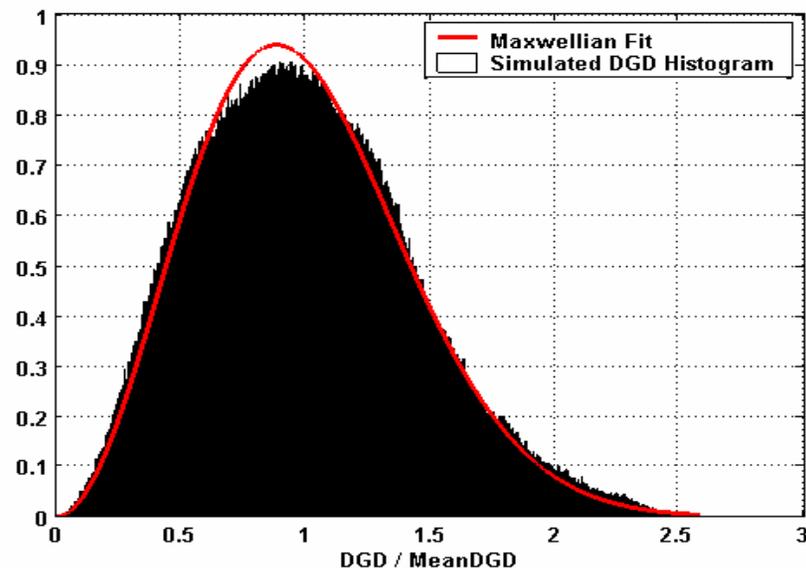
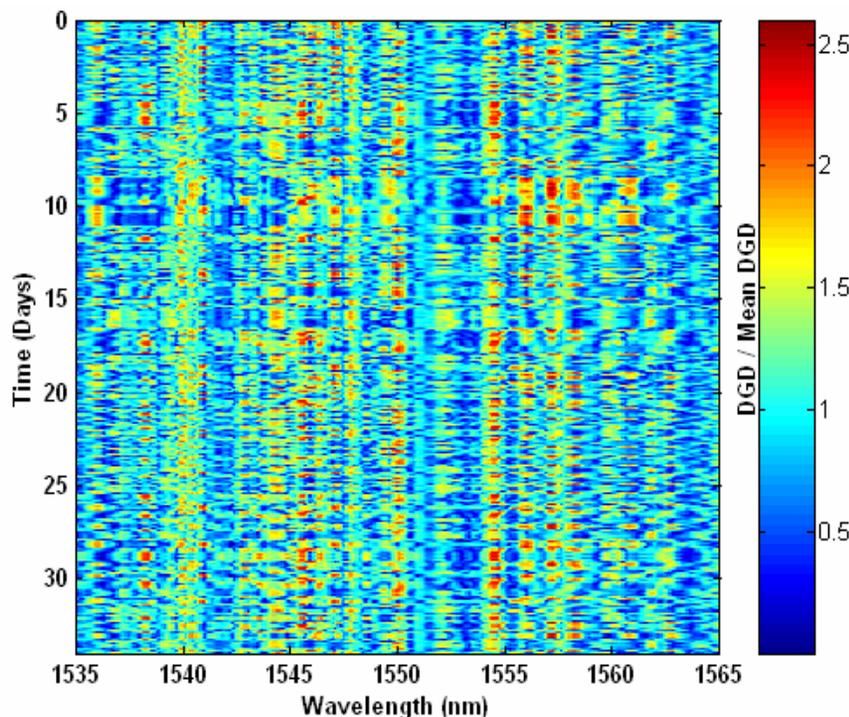


Laplacian parameter: 0.23 hr/ps →

Simulation study of first-order PMD outage rate variation with link length: Five-span link

Sampling interval: 10 min

Gaussian std. deviation: $\pi/105$ radians

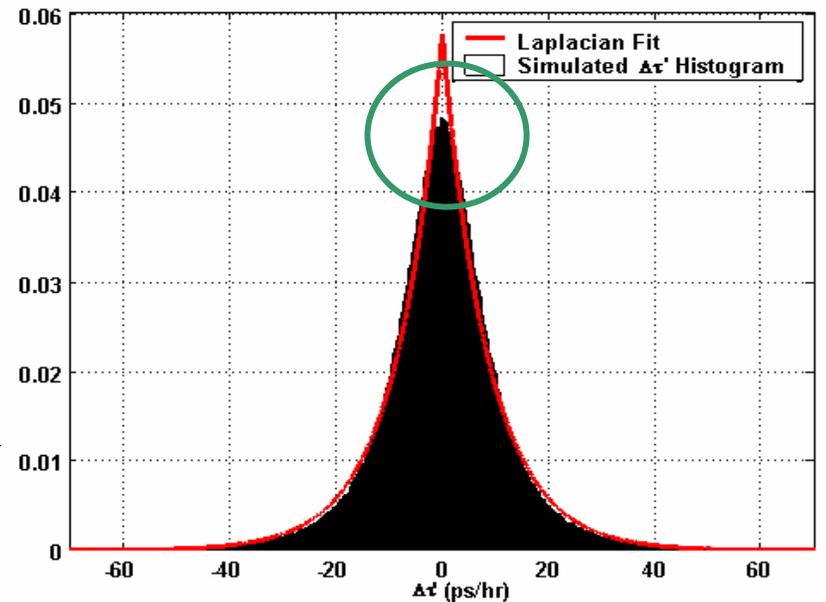
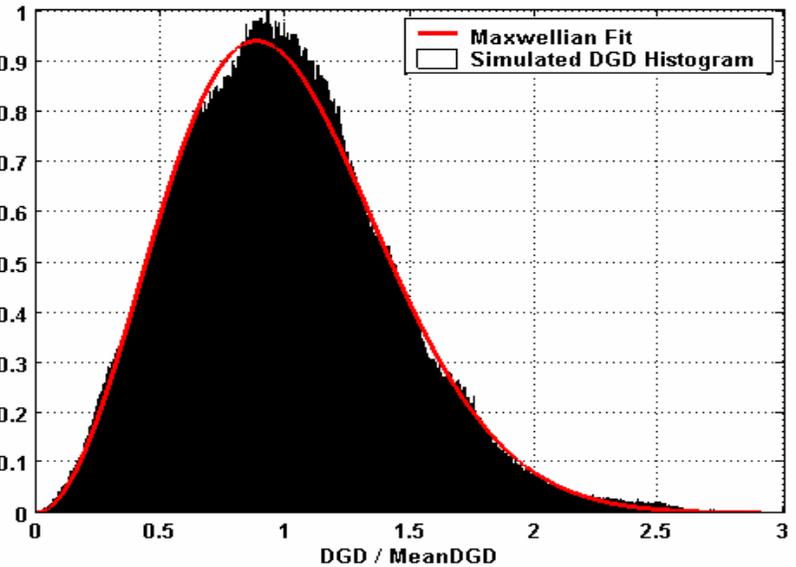
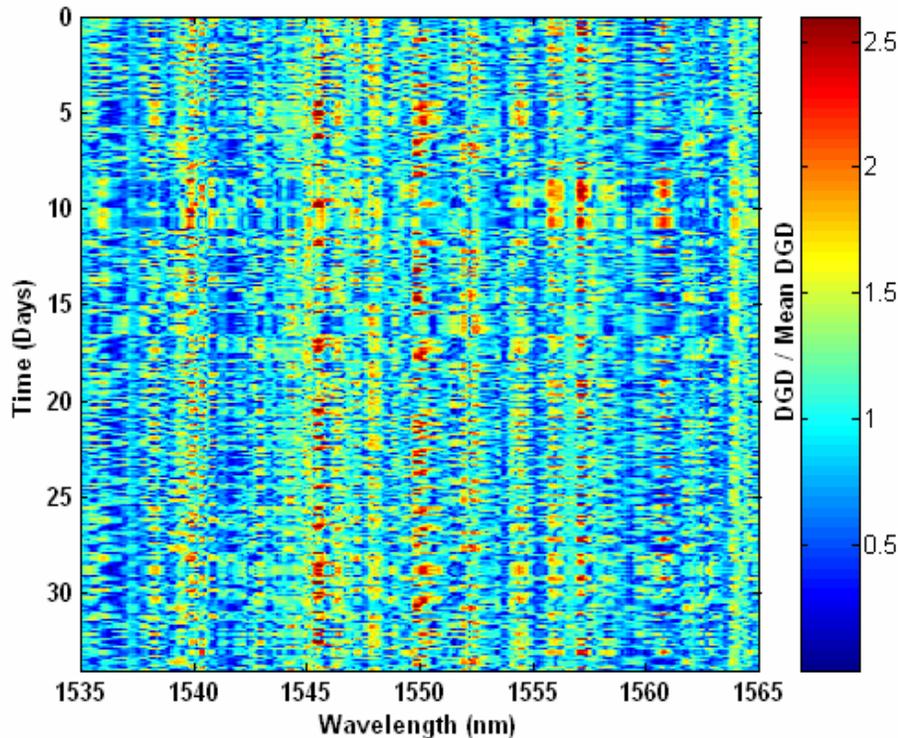


Laplacian parameter: 0.159 hr/ps \rightarrow

Simulation study of first-order PMD outage rate variation with link length: Seven-span link

Sampling interval: 10 min

Gaussian std. deviation: $\pi/105$ radians

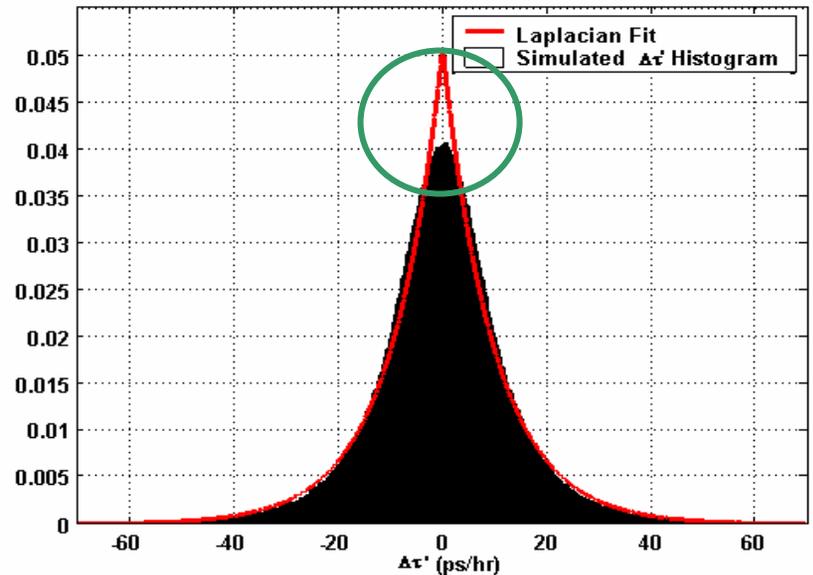
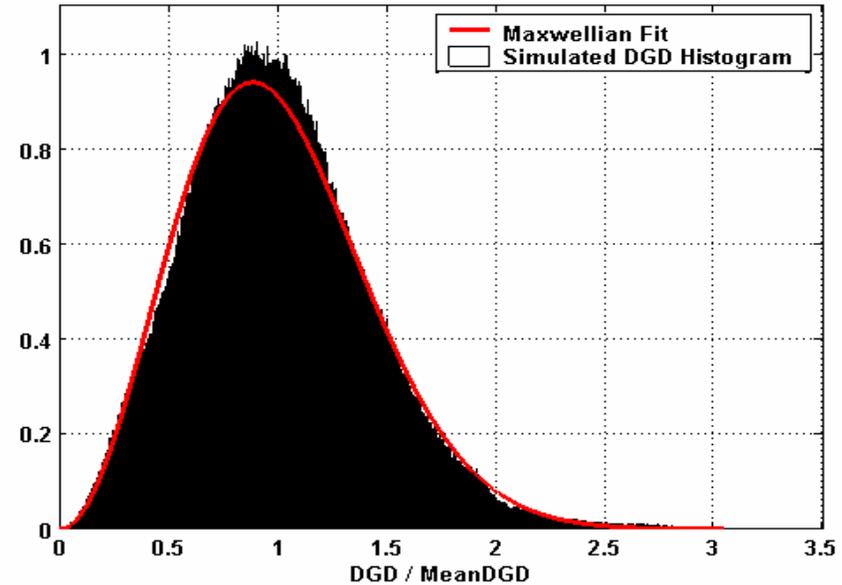
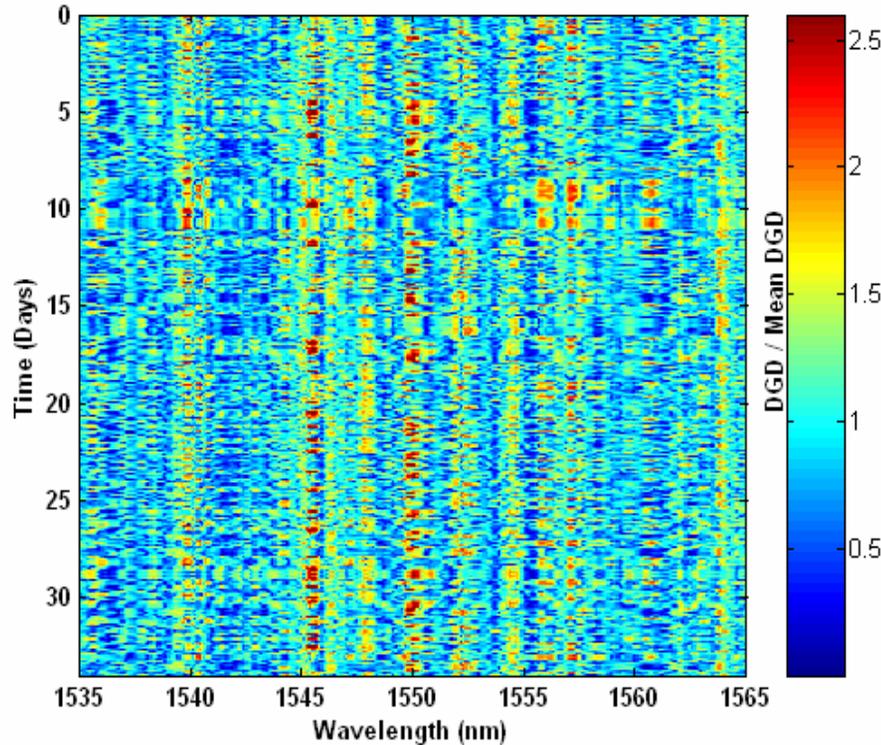


Laplacian parameter: 0.115 hr/ps →

Simulation study of first-order PMD outage rate variation with link length: Nine-span link

Sampling interval: 10 min

Gaussian std. deviation: $\pi/105$ radians

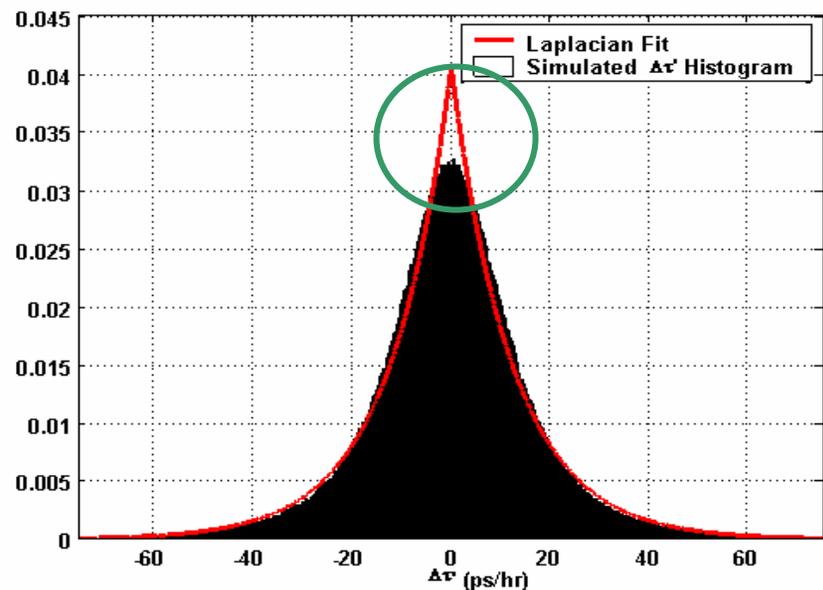
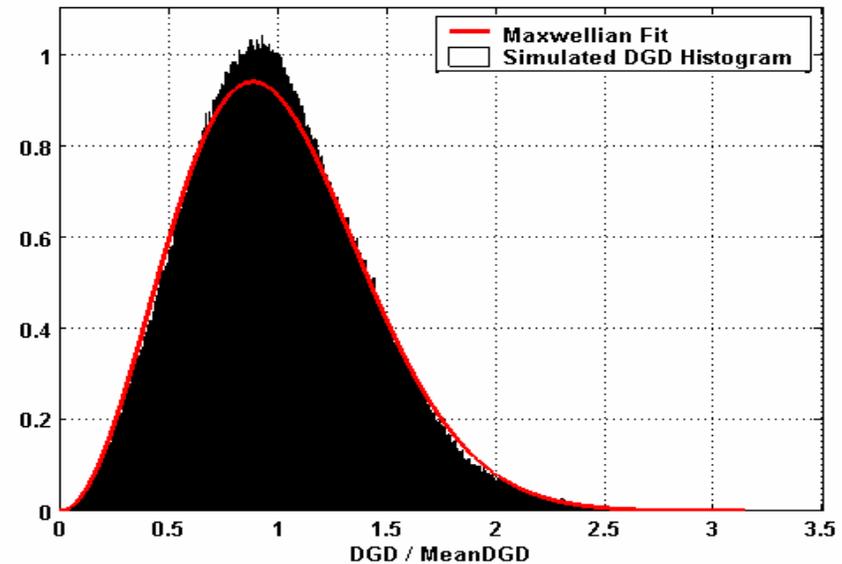
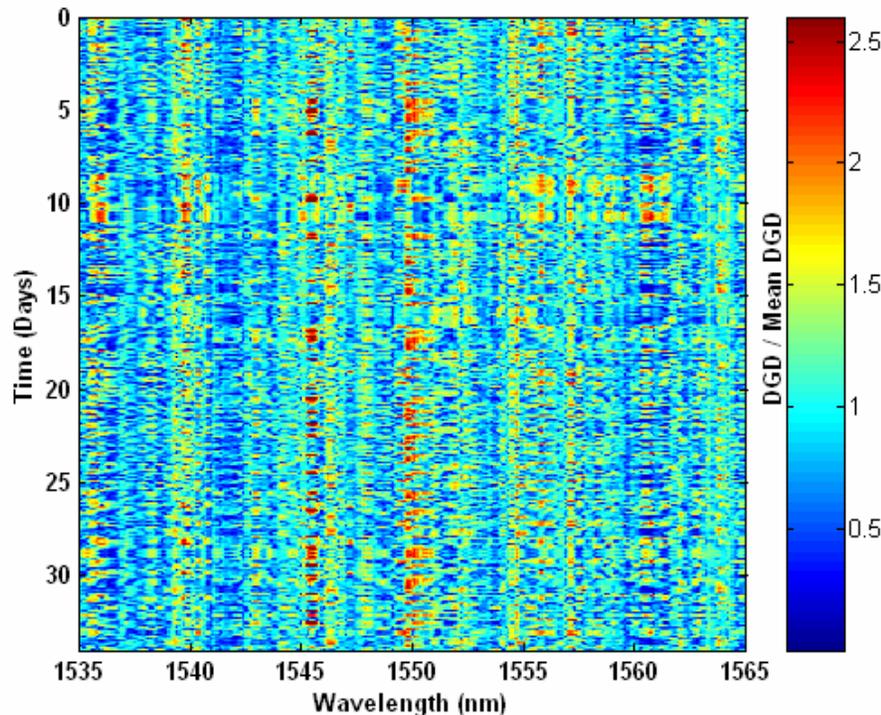


Laplacian parameter: 0.101 hr/ps →

Simulation study of first-order PMD outage rate variation with link length: Eleven-span link

Sampling interval: 10 min

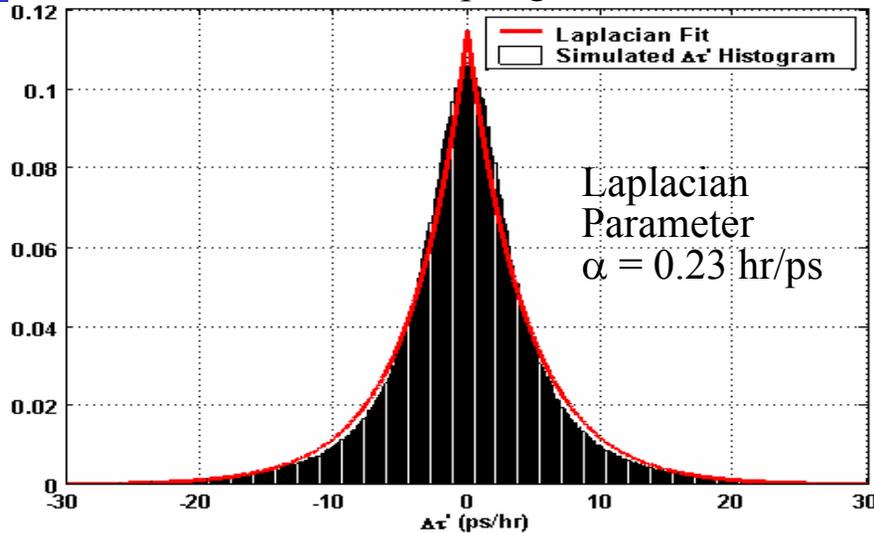
Gaussian std. deviation: $\pi/105$ radians



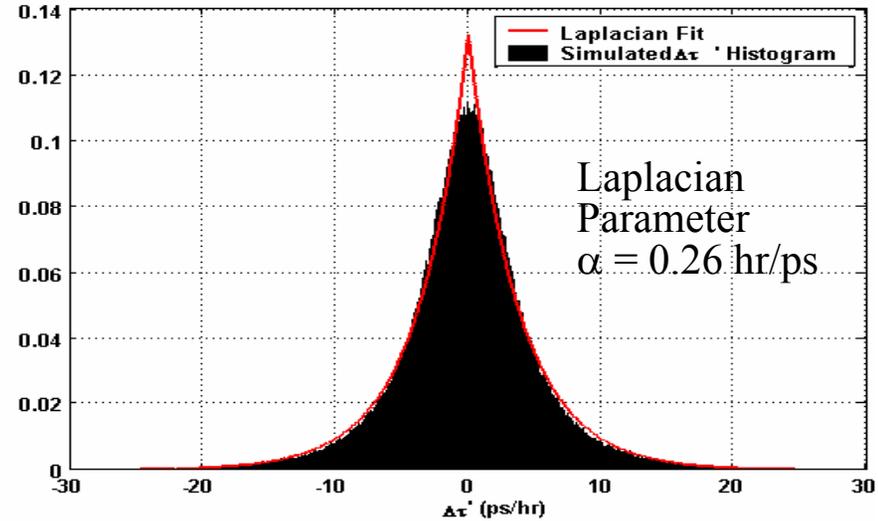
Laplacian parameter: 0.081 hr/ps →

Effect of under-sampling: Four span link

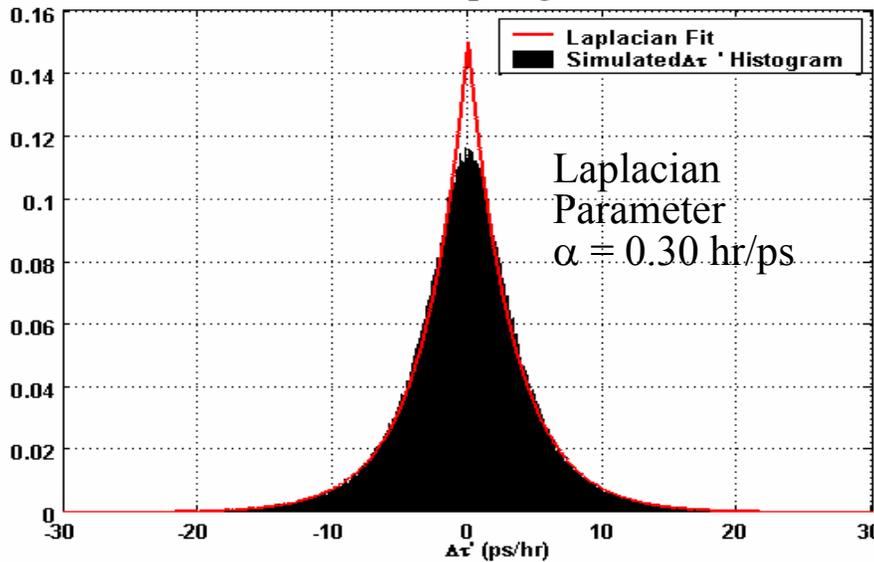
20-min sampling interval



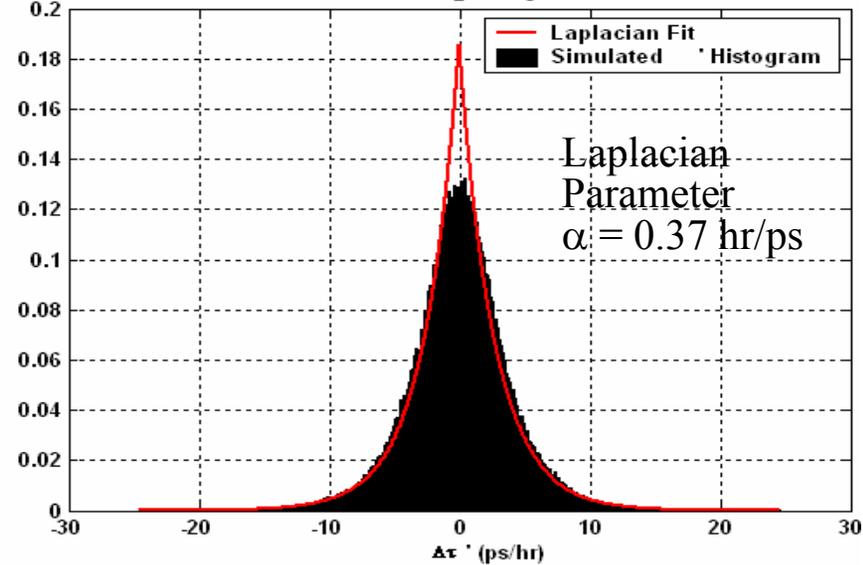
30-min sampling interval



40-min sampling interval



60-min sampling interval



Effect of under-sampling

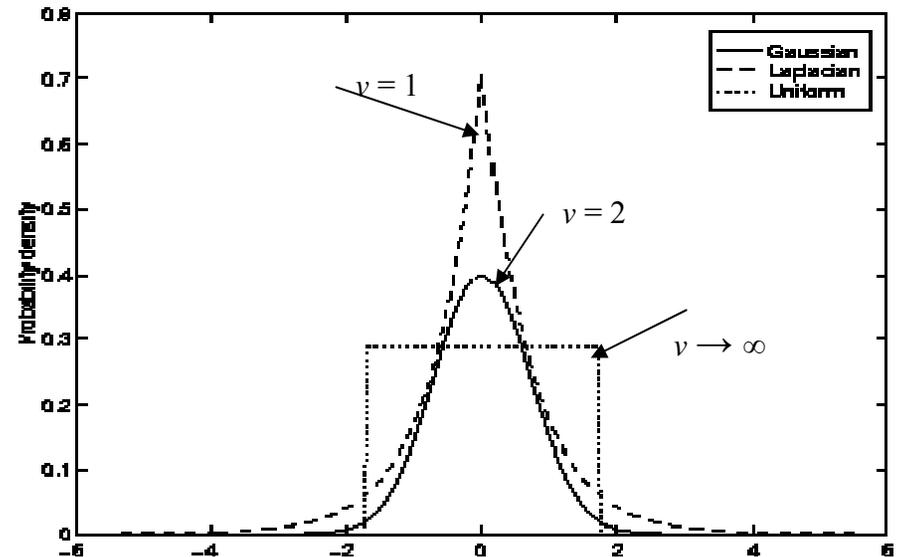
Generalized exponential PDF:

$$f_{\nu}(x) = \frac{\nu}{\sqrt{8}\sigma\Gamma(\frac{1}{\nu})} \exp\left(-\left|\frac{x-m}{\sqrt{2}\sigma}\right|^{\nu}\right)$$

$\nu = 1 \rightarrow$ Laplacian

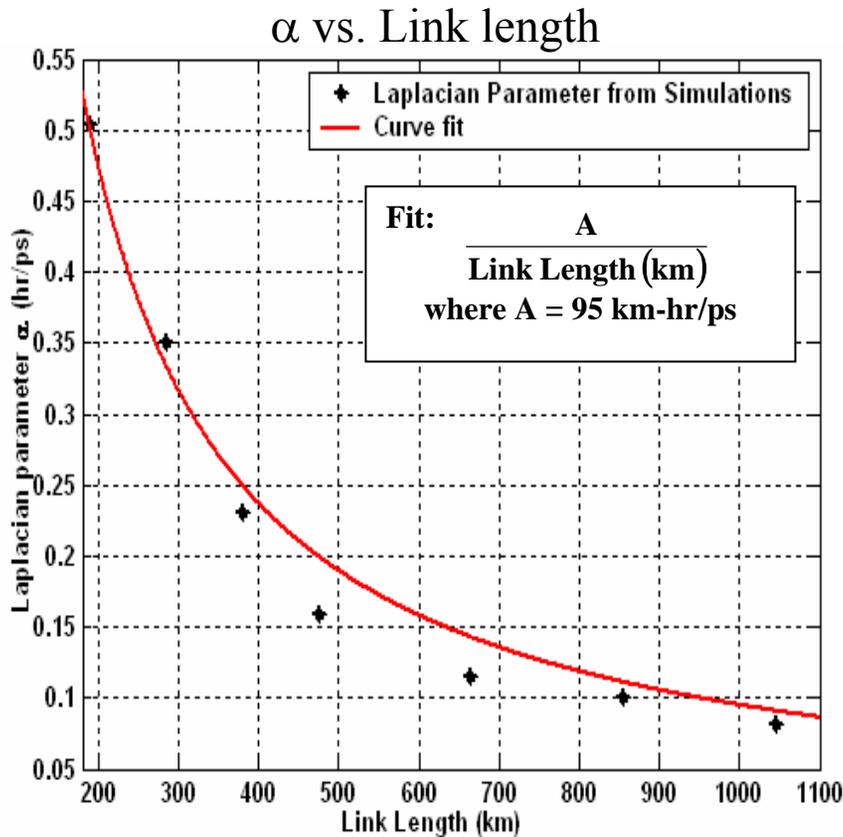
$\nu = 2 \rightarrow$ Gaussian

$\nu = \infty \rightarrow$ Uniform



- Under-sampling results in non-Laplacian PDF for DGD time derivative: first Gaussian and eventually uniform PDF
- The five-, seven-, nine- and eleven-span cases discussed before were only slightly under-sampled
- The actual α values for those cases would be slightly smaller than the ones reported

Variation of Laplacian parameter with link length



$$R_{\text{out}} = \frac{1}{2\alpha} f_{\Delta\tau}(\Delta\tau_{\text{th}})$$

Substituting

$$\alpha = \frac{A}{\text{Link Length (km)}}$$

$$R_{\text{out}} = \frac{\text{Link Length (km)}}{2A} f_{\Delta\tau}(\Delta\tau_{\text{th}})$$

From the expression of Maxwellian PDF,

$$f_{\Delta\tau}(\Delta\tau_{\text{th}}) = \frac{32}{\pi^2} \frac{\Delta\tau_{\text{th}}^2}{\langle\Delta\tau\rangle^3} e^{-\left(\frac{4\Delta\tau_{\text{th}}^2}{\pi\langle\Delta\tau\rangle^2}\right)}$$

$\langle\Delta\tau\rangle$ is the mean DGD of the link

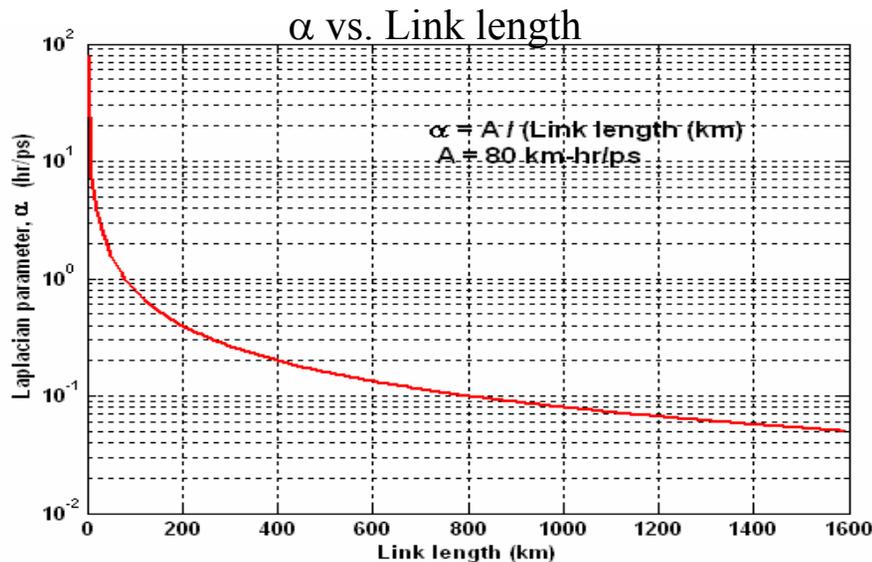
For the special case of equal span lengths,

$$R_{\text{out}} = \frac{\text{Number of spans}}{2} f_{\tau}(\Delta\tau_{\text{th}})$$

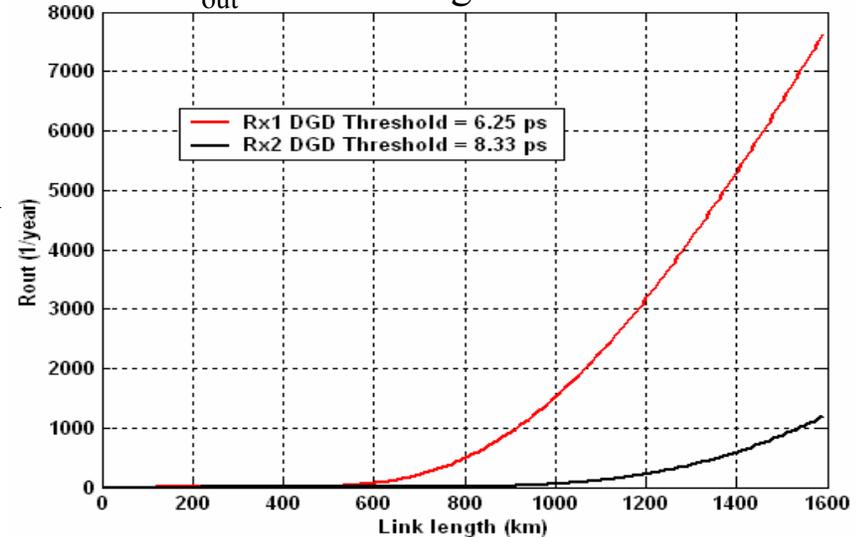
First-order PMD outage rate variation with link length: Example scenario

Consider the following scenario:

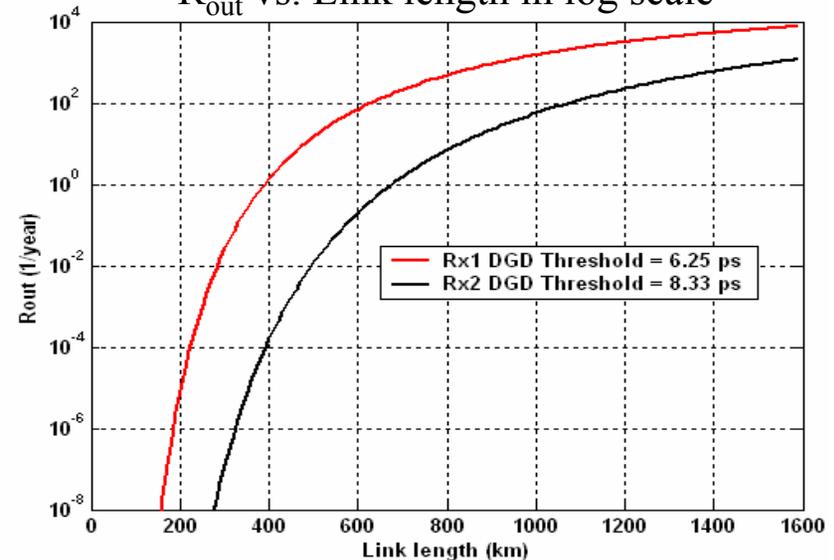
- Bit rate = 40 Gbps \Rightarrow bit period = 25 ps
- Link PMD coefficient 'b' = 0.1 ps/ $\sqrt{\text{km}}$
- Span length = 80 km; assume equal length spans; constant A = 80 km-hr/ps
- Two receivers:
 Rx1 DGD threshold: 6.25 ps
 Rx2 DGD threshold: 8.33 ps



R_{out} vs. Link length in linear scale



R_{out} vs. Link length in log scale



First-order PMD outage rate variation with link length: Example scenario

Link length (km)	Mean DGD (ps)	α (hr/ps)	Rx1 DGD threshold / Mean DGD	R_{out} for Rx1 (Outages per year)	Mean time between outages	Outage duration T_{out} (minutes)	Rx2 DGD threshold / Mean DGD	Rout for Rx2 (Outages per year)	Mean time between outages	Outage duration T_{out} (minutes)
200	1.41	0.4	4.42	7.77×10^{-6}	Few millenniums	5.91	5.89	5.7×10^{-14}	Never	7 days
400	2	0.2	3.13	1.38	9 months	6.15	4.17	1.6×10^{-4}	Few Centuries	4.4
600	2.45	0.13	2.55	71.1	5 days	6.30	3.4	0.2	5 years	4.65
800	2.83	0.1	2.21	489.15	18 hours	6.43	2.95	6.96	1 ½ months	4.66
1000	3.16	0.08	1.98	1517	6 hours	6.54	2.63	56.7	6 days	4.72
1200	3.46	0.07	1.80	3172	3 hours	6.65	2.4	225.7	1 ½ days	4.78
1400	3.74	0.06	1.67	5310	1 and ½ hours	6.76	2.23	598	15 hours	4.83
1600	4	0.05	1.56	7743	1 hour	6.86	2.08	1231	7 hours	4.88

- Special case

- If the ratio of DGD threshold and mean DGD is maintained constant as link length increases, then

$$R_{out} \propto \sqrt{\text{Link Length (km)}}$$

Conclusions

- The proposed PhD work has been successfully completed leading to some very interesting results
- The results were achieved by following a 3-step process
 - simplified the first-order PMD outage rate expression
 - enhanced the basic PMD numerical model to include the temporal component that would accurately model the PMD characteristics
 - did a simulation study using the enhanced-model and the simplified expression to predict outage rates on long-haul optical fiber links
- The study showed that the Laplacian parameter is inversely related to link length
- The first-order PMD outage rates increase monotonically with link length

Future work

- Ample scope for future work
 - Use the model to do higher-order PMD outage analysis
 - Use multiple temperature profiles corresponding to different locations along a long-haul link in the simulation and study the impact on the results reported
 - Study why the single-span links needed a much narrower filter than the multi-span links
 - If long-term access to long-haul optical fiber links is available, verify the results through measurements
 - Make the number of segments per span having a time-variant ϕ_n component a uniform or Gaussian variable and study its impact on the results reported
 - Many other variations to the model could be studied