Network Resilience Improvement and Evaluation Using Link Additions

Ph.D. Dissertation Defense

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Network Resilience Improvement
Outline

- Introduction and motivation
- Background and related work
- Graph models
- Network design and improvement
- Evaluation and improvement
- Conclusions and future work

Network Resilience Improvement
Introduction and Motivation

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Introduction and Motivation
Motivation

- Communication networks
  - e-government to provide online services to citizens
  - hospitals manages patients data records
  - e-learning an essential part of education
  - increasing number of on-line business customers
    - in 2014, business-to-consumer (B2C) sales 1.5 trillion
- The Internet topology
  - physical layer
  - logical layer

Introduction and Motivation
Challenges

- Large-scale disasters
  - earthquakes, typhoons, tornados, or hurricanes
  - cause correlated failures in physical layer
- Targeted attacks: knowledge of network topology
  - attackers target most important nodes or links in the network
  - centrality-based attacks are performed on nodes or links
  - cause significant drop in connectivity among users
- Network resilience is defined as [SHÇJRSS2010]

"the ability of the network to provide and maintain an acceptable level of service in the face of various faults and challenges to normal operation"

Introduction and Motivation
Thesis Statement

- Improvement of network resilience against attacks
  - investigate several graph robustness metrics
  - improve network resilience
    - adding a set of new links
- Thesis Statement:

Network connectivity improvement, via adding a new set of links to maximize a given graph robustness metric under cost constraints, can improve the resilience of the underlying networks against targeted attacks. Determining the best robustness metric can better improve the overall resilience.

Network Resilience Improvement
Contributions

- Investigated several graph robustness metrics
- Defined flow robustness metric for weighted graphs
- Introduced model weighted physical graph
  - via nodes' population
- Designed and implemented greedy algorithms
  - improve network given graph robustness metric
- Applied algorithms to real-world graphs
- Evaluated and compared the improvement algorithms
  - applying centrality-based attacks
  - examine their network resilience during the attacks

Network Resilience Improvement
Background and Related Work

- Introduction and motivation
- Background and related work
- Graph models
- Network design and improvement
- Evaluation and improvement
- Conclusions and future work

Background and Related Work
Network Design Problem

- Given a graph and an objective function
  - objective function: maximize robustness of the graph
  - constrained by number of links k
    - find a set of links with size k to maximize objective function
  - constrained by a budget (total cost value)
    - find a set of links with any length where
      - total cost is less than or equal to the budget
      - maximum value of objective function
- Optimal solutions using exhaustive search
  - grow exponentially with the size of the network [SSG2013]
- Many problems are considered to be NP-hard [WM2008]

Background and Related Work

Robustness Metrics

- No ideal metric that measures network resilience
- A method to measure resilience based [SHÇJRSS2010]
  - operational states
  - service states
- Graph robustness metrics
  - a large number of graph robustness metrics
    - select most promising against random or target attacks
  - study their un- and weighted versions
  - compare their algorithmic time complexity

Background and Related Work
Comparison of Graph Robustness Metrics

- Total graph diversity (TGD) [RJS2012]
  - better accuracy in predicting survivability
  - synthetic and real networks
  - compared to other graph metrics
    - clustering coefficient, average hop count, betweenness
- Algebraic connectivity (AC) [LSPM2009]
  - second smallest eigenvalue of Laplacian matrix
  - higher AC, more robust against partitioning
  - compared to average node degree
  - more informative and accurate network resilience measure

Background and Related Work
Comparison of Graph Robustness Metrics

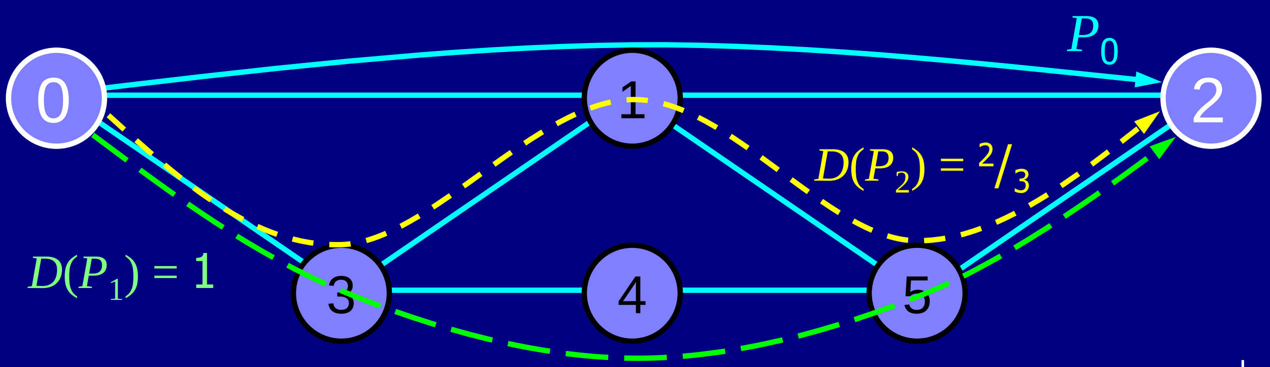
- Weighted spectral distribution (WS) [LTG2014]
  - introduced to analyze the Internet topology
  - compared to other metrics
    - geographic correlated failures
  - better measure geo. correlated vulnerable links and nodes
- Network criticality [BTG2009]
  - spectral graph robustness metric
  - smaller value indicates higher network robustness
  - compared to AC, average degree, average betweenness

Background and Related Work
Robustness Metrics: Graph Centrality

- Degree: number of links connected to a node
- Closeness: inverse average distance to other nodes
- Node betweenness
  - number of shortest paths through a node
- Link betweenness
  - number of shortest paths through a link
- Flow robustness is defined as [RJS2012]

"*the ratio of the number of reliable flows to the number of total flows in the network*"

Background and Related Work
Robustness Metrics: Path Diversity



- Path diversity
  - measure of links and nodes in common

$$D(P_k) = 1 - \frac{|P_k \cap P_0|}{|P_0|}$$

- EPD: effective path diversity [0,1)
  - normalized diversity with respect to a single shortest path
  - measure of E2E flow resilience
- TGD: total graph diversity is average of EPD
  - for all pairs: quantifies available diversity in graph

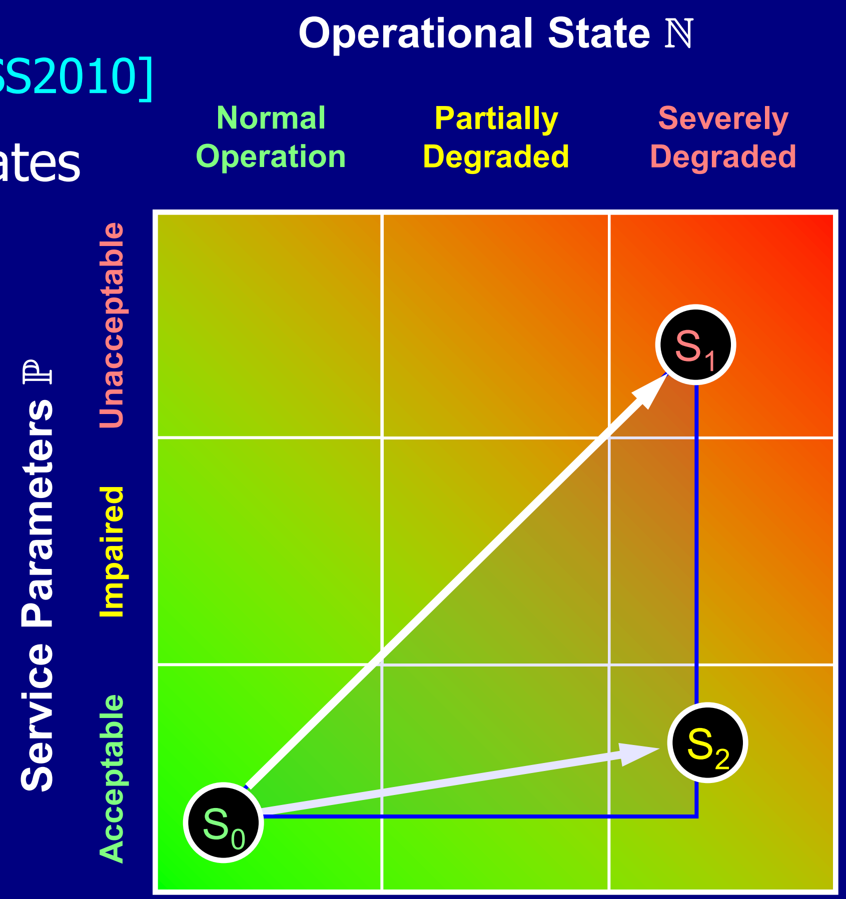
[RJS2012]

Graph Robustness Metrics
Spectral Robustness Metrics

- Algebraic connectivity (λ_2)
  - second smallest eigenvalue of Laplacian matrix
- Spectral gap ($\Delta\mu$)
  - delta of largest and second largest eigen. of adjacency matrix
- Natural connectivity ($\bar{\mu}$)
  - scaled average of eigenvalues of adjacency matrix
- Weighted spectral (WS)
- Network criticality ($\hat{\tau}$)
- Effective graph resistance (C^*)

Background and Related work
Quantifying Network Resilience

- Define [SHÇJRSS2010]
  - service and operational states
- Choose scenario
- Metrics
- Observe
  - under challenge
- Resilience
  - $\mathbb{R}$ = 1– area under curve
  - for particular scenario
  - resilience $\Re$ over all scenarios

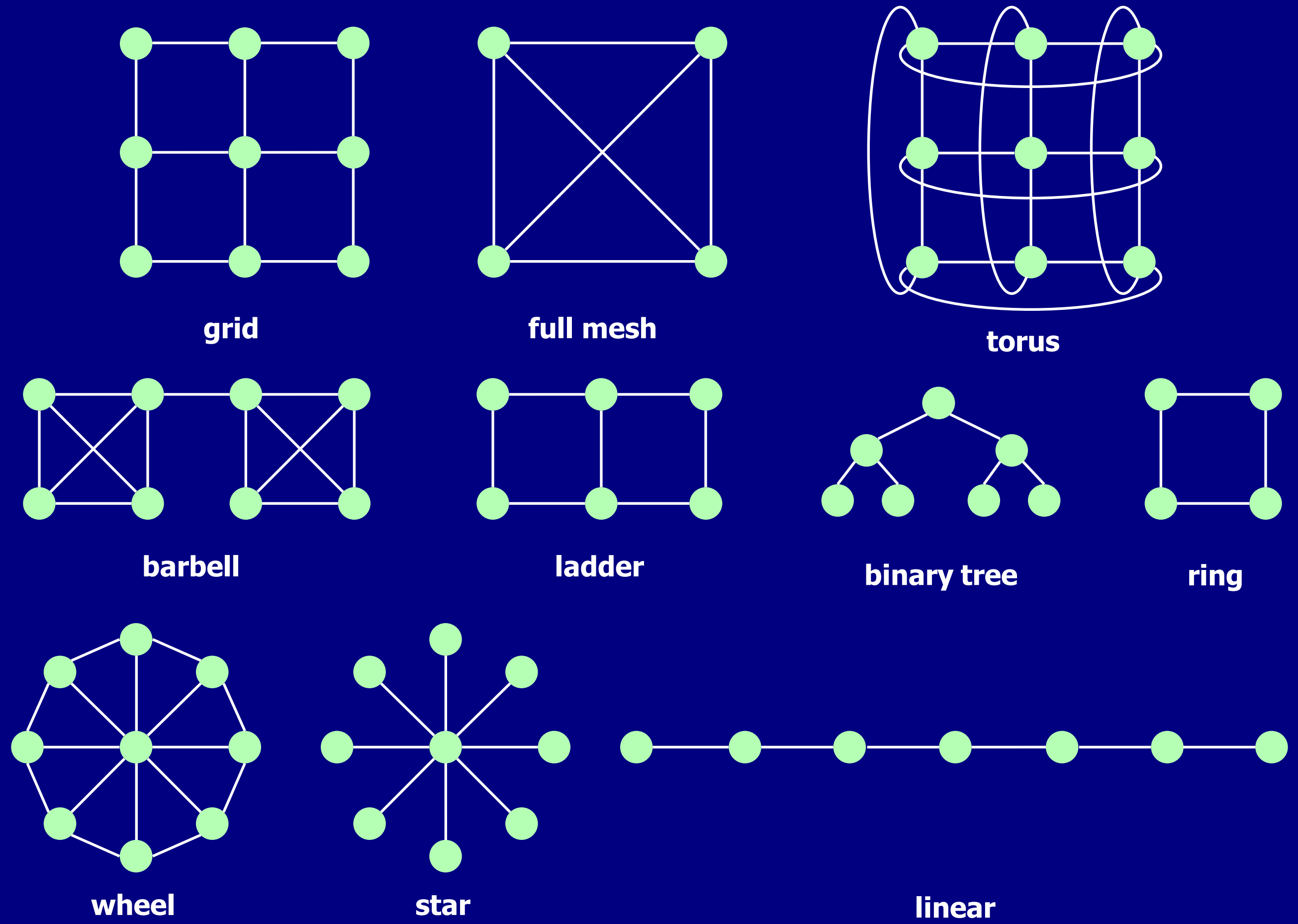


Network Resilience Improvement
Graph Models

- Introduction and motivation
- Background and related work
- Graph models
- Network design and improvement
- Evaluation and improvement
- Conclusions and future work

Dataset

Baseline Graphs



Dataset
Random Graphs

- Gilbert random graphs $\mathrm{G}(n,\ p)$
  - given n nodes; each pair connected with probability p
- Waxman random graphs:

$$P(\{u,v\}) = \beta e^{\frac{-d(u,v)}{L\alpha}}$$

L: maximum distance between two nodes

α and β: tuning parameters for long and short links

  - exhibit mesh-like properties of logical-level networks
- Gabriel random graphs
  - two nodes connected if no other nodes fall inside their circle
  - exhibit grid-like properties of physical-level networks

Dataset
Unweighted Real-World Networks

- Several US-based backbone providers
- Available in http://www.ittc.ku.edu/resilinets/maps/
- Initial graph properties

Network	Nodes	Links	Avg. Deg.	Avg. Hop.
AT&T	383	488	2.6	14.1
Level 3	99	130	2.6	7
Sprint	264	312	2.4	14.8
Internet2	57	65	2.3	6.7
CORONET	75	99	2.6	6.5

Dataset
Weighted Real-World Networks

- RENs (research and education networks)
- Capacity weighted
- Initial graph properties

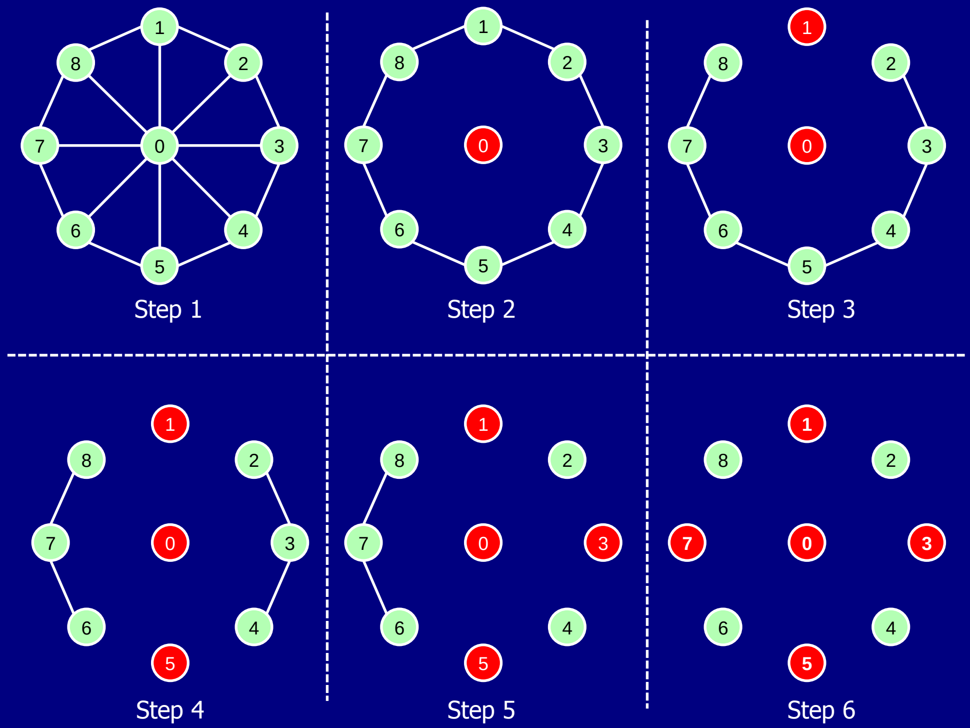
Network	Nodes	Links	Avg. Deg.	Avg. Hop.
KAREN	25	28	2.24	3.42
InternetMCI	19	33	3.47	2.39
CARNet	44	43	1.95	2.99
GEANT	37	56	3.03	3.46

Measuring Robustness
Three Robustness Measures

- Flow robustness (FR)
  - measures end to end connectivity ratio
  - always 1 for connected graphs
    - full mesh FR = 1; star FR = 1
- Three measurements based on flow robustness
  - sums of flow robustness degree attack (SFRD)
  - sums of flow robustness closeness attack (SFRC)
  - sums of flow robustness betweenness attack (SFRB)
- Captures graph robustness under stepwise attack
  - full mesh SFR* high; star SFR* low

Measuring Robustness
Example

- Measuring SFRB for 9-node wheel topology



Step	Attacked Nodes	FR	SFRB
1	{}	1.00	1.00
2	{0}	0.78	1.78
3	{0,1}	0.28	2.36
4	{0,1,5}	0.17	2.53
5	{0,1,5,3}	0.08	2.61
6	{0,1,5,3,7}	0.00	2.61

Network Resilience Improvement

Network Design and Improvement

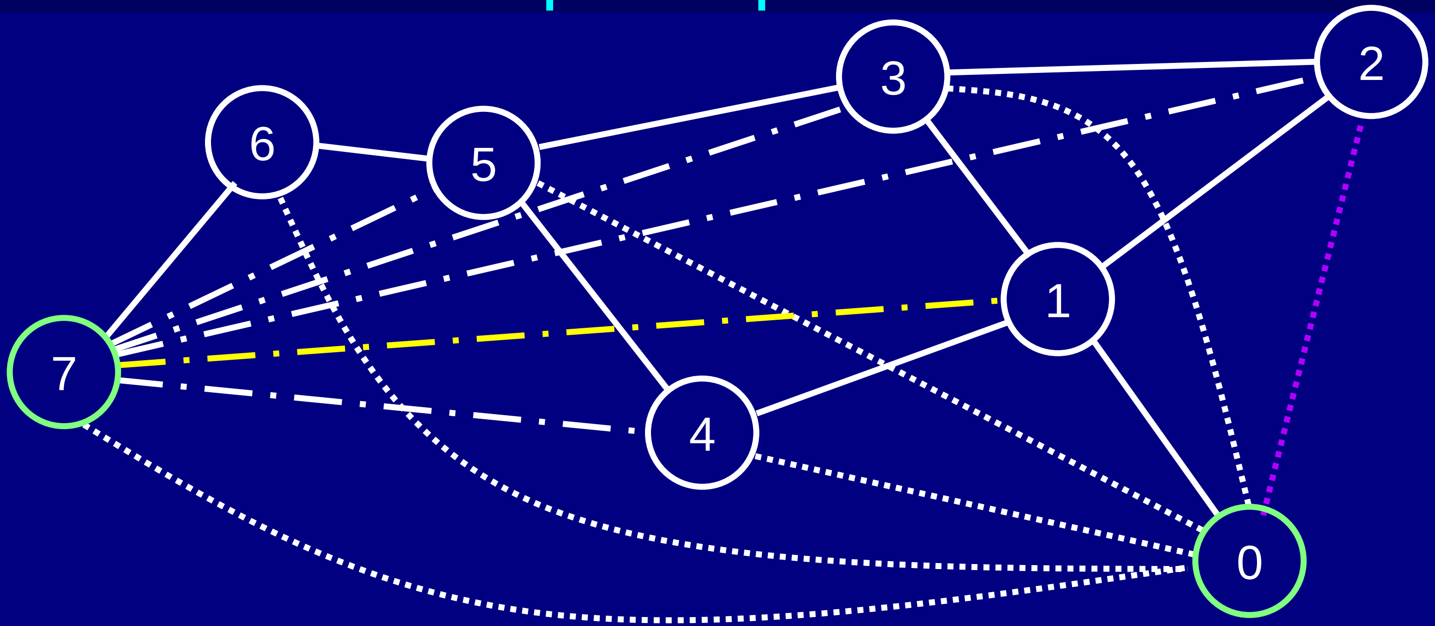
- Introduction and motivation
- Background and related work
- Graph models
- Network design and improvement
- Evaluation and improvement
- Conclusions and future work

Algebraic Connectivity Improvement
Algorithm

- Objective [AÇS2013]
  - identify the best links to be added to improve $a(G)$
  - reduce the cost by selecting the least cost links
- Candidate links
  - links connected to lowest degree nodes
  - removing long links
- Link selection based on: $a(G)$ and cost
  - rank function: $\text{rank}[L]=(1-\gamma)a(G) + \gamma(1-\text{cost}(L))$
  - tuning parameter γ

Algebraic Connectivity Improvement

Example Improvement



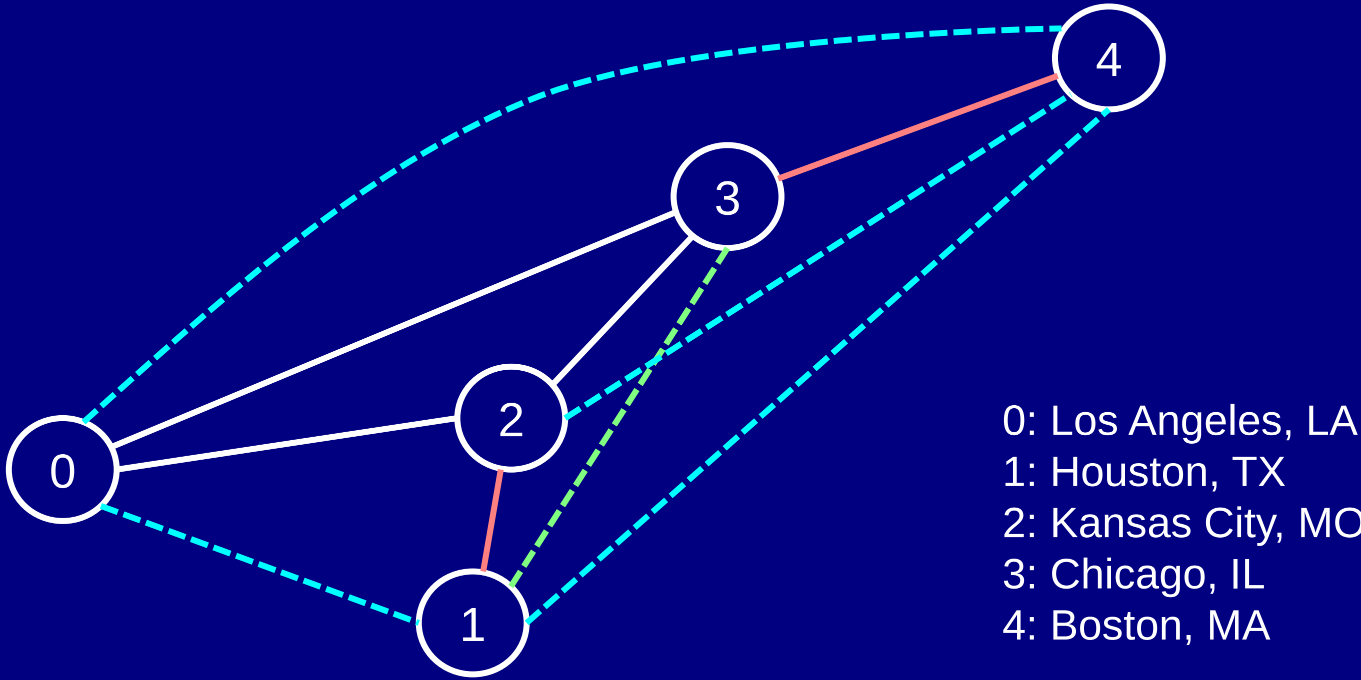
- 8 nodes and 9 links graph
  - for $\gamma = 0$, link (7,1) highest $a(G)$
  - for $\gamma = 1$, link (0,2) lowest cost

Path Diversity Improvement
Algorithm

[AÇS2014b]

- Objective
  - identify the best links to be added to improve TGD
  - reduce the cost by selecting the least cost links
- Candidate links
  - links connected to node pairs with the lowest EPD
  - removing long links
- Link selection based on: EPD and cost
  - if multiple EPD candidates, select with the least cost

Path Diversity Improvement
Algorithm



- 5 nodes, 5 links, 5 candidate links, lowest EPD pair
- Best link (1,3): the most EPD increase, least costly

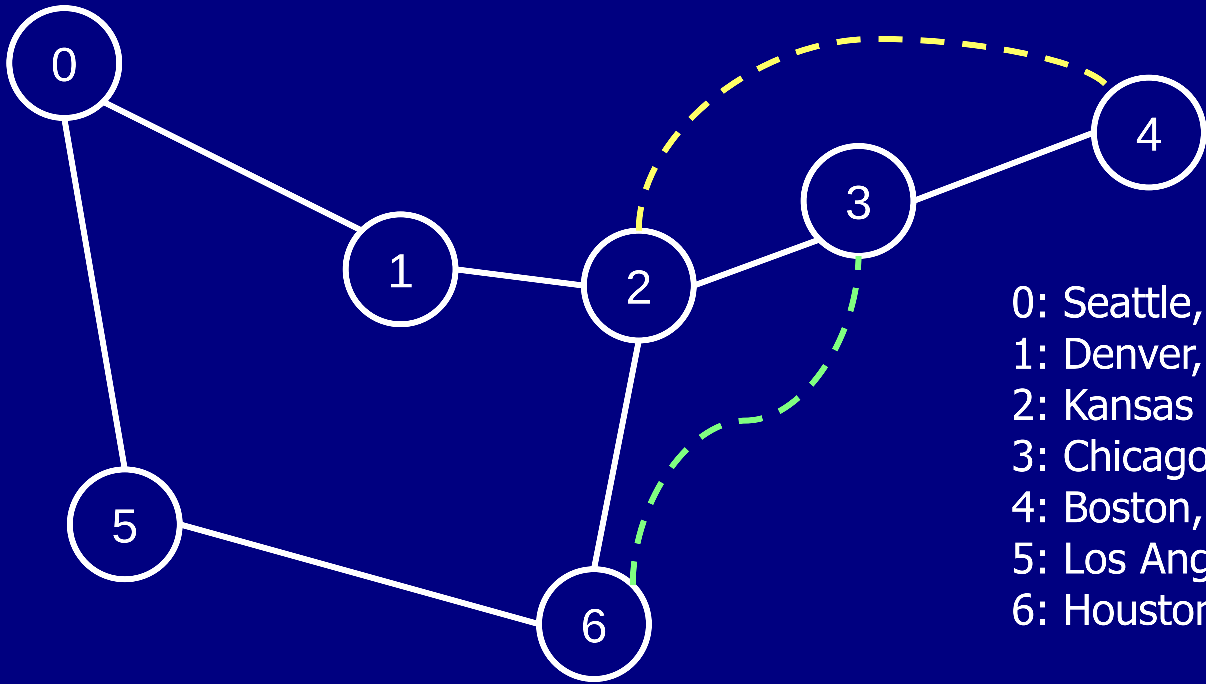
Centrality-Balanced Improvement
Algorithm

[AÇS2014c]

- Objective
  - minimize variance of a given centrality function for all nodes
  - reduce the cost by selecting the least cost links
- Candidate links
  - all links in complement graph
  - removing long links
- Link selection based on minimum variance
  - if multiple links with same variance, select least cost

Centrality-Balanced Improvement

Algorithm



0: Seattle, WA
1: Denver, CO
2: Kansas City, MO
3: Chicago, IL
4: Boston, MA
5: Los Angeles, LA
6: Houston, TX

- 7 nodes and 6 candidate links,
- betweenness and degree (3,6), for closeness (2,4)

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Graph Metrics Evaluation

Baseline Graphs

	Star	Tree	Linear	Barbell	Ring	Ladder	Grid	Wheel	Torus	Mesh
$\overline{C}_{\mathrm{D}}$	1.80	1.87	1.80	2.83	2.00	2.60	2.67	3.60	4.00	9.00
$\sigma^2_{\mathrm{C_D}}$	5.76	0.92	0.16	0.47	0.00	0.24	0.44	3.24	0.00	0.00
$\sigma^2_{\mathrm{C_C}}$	0.02	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00
$\sigma^2_{\mathrm{C}_{\mathrm{B}-n}}$	0.09	0.05	0.04	0.06	0.00	0.01	0.01	0.03	0.00	0.00
$\sigma^2_{\mathrm{C}_{\mathrm{B-l}}}$	0.00	0.02	0.02	0.04	0.00	0.00	0.00	0.00	0.00	0.00
CC	0.00	0.00	0.00	0.58	0.00	0.00	0.00	0.62	0.33	1.00
As	-1.00	-0.52	-0.12	0.13	1.00	0.28	-0.06	-0.33	1.00	1.00
R	1.00	3.00	5.00	4.00	5.00	3.00	2.00	1.00	2.00	1.00
D	2.00	6.00	9.00	7.00	5.00	5.00	4.00	2.00	2.00	1.00
$\bar{d}$	1.80	3.50	3.67	3.48	2.78	2.33	2.00	1.60	1.50	1.00
TGD	0.00	0.00	0.00	0.23	0.39	0.68	0.73	0.82	0.91	1.00
λ_2	1.00	0.10	0.10	0.09	0.38	0.38	1.00	1.47	3.00	10.00
$\Delta\mu$	3.00	0.29	0.24	0.01	0.38	0.73	1.41	2.63	3.00	10.00
$\hat{\tau}$	1.80	3.50	3.67	3.03	1.83	1.25	0.96	0.69	0.50	0.20
WS	2.00	5.46	4.37	3.02	3.75	3.04	2.44	1.48	1.27	1.00
$\bar{\lambda}$	1.49	1.18	1.09	2.19	1.19	1.61	1.67	2.95	2.87	9.66
C*	0.11	0.04	0.05	0.06	0.11	0.16	0.23	0.29	0.44	1.00
SFRD	1.00	1.61	2.11	1.97	2.56	2.62	2.72	2.91	3.14	3.67
SFRC	1.00	1.94	1.67	1.86	2.29	2.47	2.61	2.73	3.14	3.67
SFRB	1.00	1.61	1.67	1.86	2.29	2.47	2.61	2.73	3.14	3.67
$\overline{\mathrm{SFR}}$	1.00	1.72	1.82	1.90	2.38	2.52	2.64	2.79	3.14	3.67

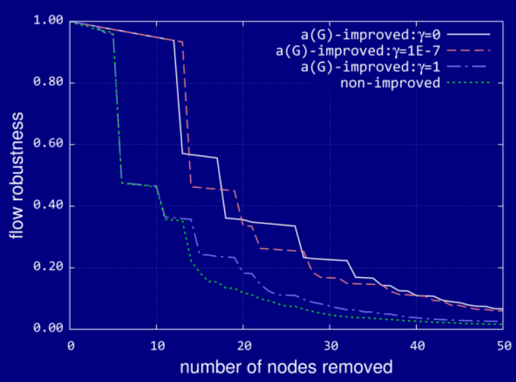
Graph Metrics Evaluation
Random Graphs

- Nonlinear correlation with SFRB measure [AÇS2015c]
- Correlation of 30,000 random graphs
  - algebraic connectivity and link betweenness best for Gilbert
  - network criticality best for the others

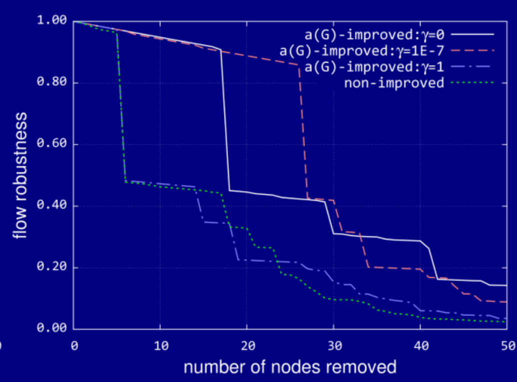
corr(X, SFRB)	$\bar{C}_D$	$\sigma^2_{C_D}$	$\sigma^2_{C_C}$	$\sigma^2_{C_{B-n}}$	$\sigma^2_{C_{B-l}}$	CC	As	R	D	$\bar{d}$	TGD	λ_2	$\Delta\mu$	$\hat{\tau}$	WS	$\bar{\lambda}$	C*
Gilbert p=0.8	0.43	-0.60	-0.41	-0.61	**-0.69**	0.32	0.25	0.15	0.00	-0.43	0.59	**0.75**	0.37	-0.49	-0.46	0.39	0.49
Gilbert p=0.5	0.49	-0.43	-0.29	-0.62	**-0.64**	0.23	0.28	0.00	-0.09	-0.50	0.42	**0.69**	0.33	-0.60	-0.46	0.40	0.60
W(0.5, 0.5)	0.76	-0.03	-0.40	-0.84	-0.81	0.22	0.15	-0.16	-0.41	-0.77	0.81	0.74	0.45	**-0.85**	-0.72	0.60	**0.85**
W(0.5, 0.8)	0.67	-0.24	-0.56	-0.78	-0.79	0.18	0.20	0.11	-0.31	-0.71	0.74	0.75	0.42	**-0.81**	-0.62	0.52	**0.81**
W(0.8, 0.5)	0.62	-0.26	-0.54	-0.73	-0.78	0.19	0.16	0.42	0.11	-0.68	0.66	0.76	0.39	**-0.78**	-0.58	0.48	**0.78**
Gabriel	0.62	0.18	0.06	-0.53	-0.68	0.17	0.10	-0.22	-0.43	-0.69	0.73	0.73	0.27	**-0.77**	-0.61	0.51	**0.77**

Algebraic Connectivity Improvement
Evaluation Results

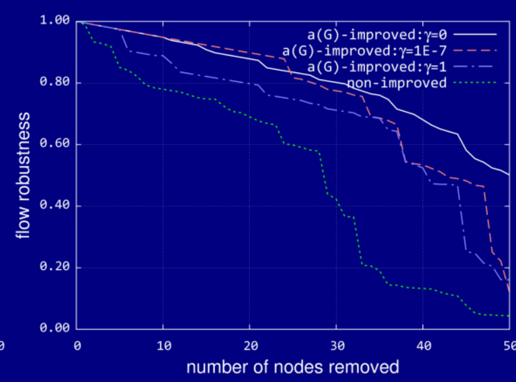
- Adding 100 links
- Betweenness attack is the most destructive
- Improved graph is more resilient γ=0



betweenness attack



closeness attack

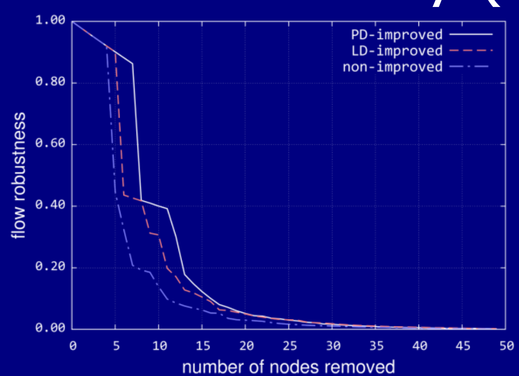


degree attack

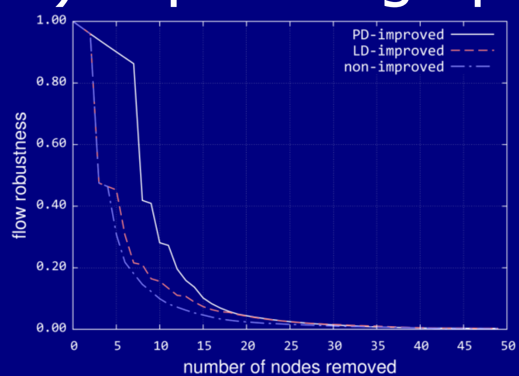
AT&T physical network

Path-Diversity Improvement
Evaluation Results

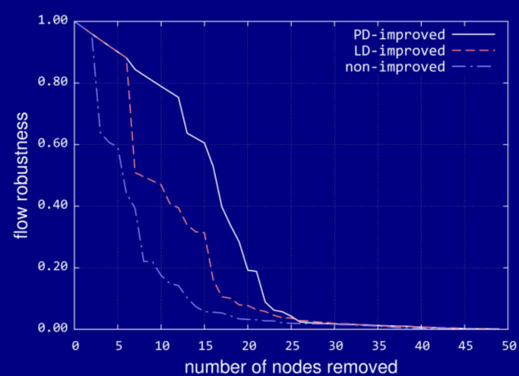
- For comparison, lowest degree (LD) improvement
  - add cost-efficient links to lowest degree nodes
- Adding 20 links
- Path-diversity (PD) improved graphs are more resilient



betweenness attack



closeness attack

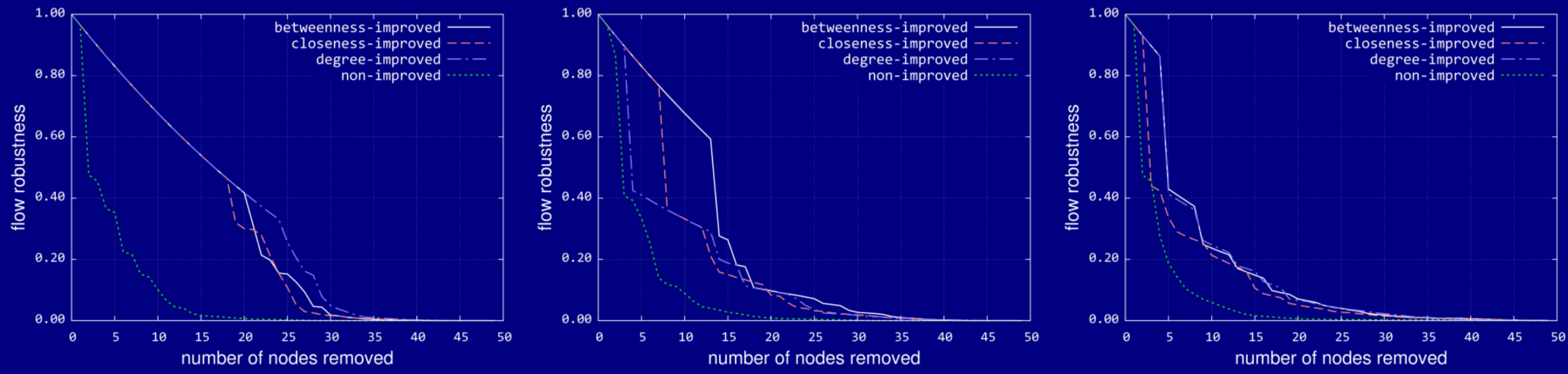


degree attack

Level 3 physical network

Centrality-Balanced Improvement
Evaluation Results

- Budget constraint
  - adding links based with 50×10^6 m total length
- Betweenness and degree based perform better
  - considering all cases



betweenness attack

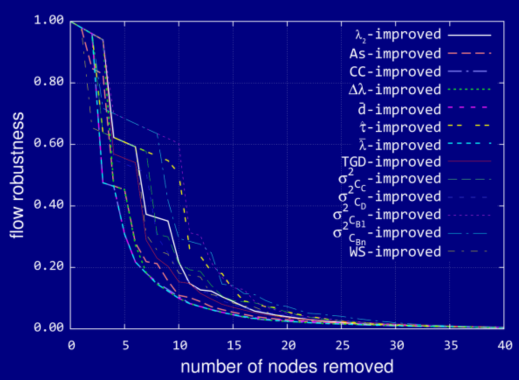
closeness attack

degree attack

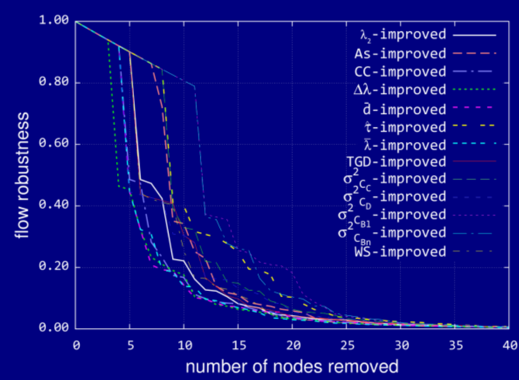
Internet2 physical network

Comprehensive Comparison
Unweighted Evaluation Results

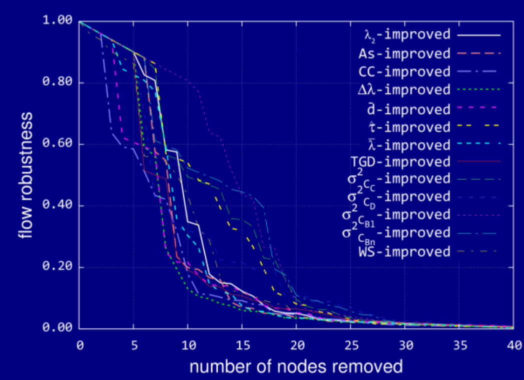
- Adding 20 links to real-world networks [AÇS2015a]
- Minimize or maximize given robustness function
- Link-betweenness balanced graphs with best results



betweenness attack



closeness attack



degree attack

Level 3 physical network

Comprehensive Comparison

Unweighted Evaluation Summary

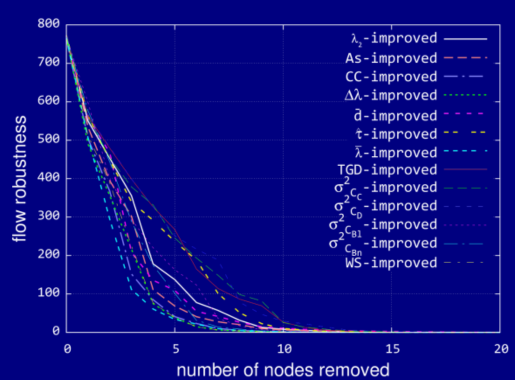
- Sum of flow robustness attacking all nodes
  - area under the curve
- Three centrality-based attacks
- Link-betweenness balanced graphs – $\sigma^2_{C_{B-l}}$
  - best results for Level 3 and the other two networks

Centrality Attack	Objective Function												
	λ_2	As	CC	$\Delta\mu$	$\bar{d}$	$\hat{\tau}$	$\bar{\lambda}$	$\sigma^2_{C_C}$	$\sigma^2_{C_D}$	$\sigma^2_{C_{B-l}}$	$\sigma^2_{C_{B-n}}$	WS	TGD
Degree	10.5	9.46	7.8	8.44	8.5	12.35	9.73	12.72	10.77	**14.62**	13.28	9.89	9.21
Closeness	8.79	10.47	7.68	6.87	7.32	11.93	7.36	11.07	8.99	**13.75**	13.38	8.9	9.22
Betweenness	8.18	6.41	5.7	6.34	5.76	9.32	5.68	8.28	7.97	**10.37**	9.79	7.28	7.56

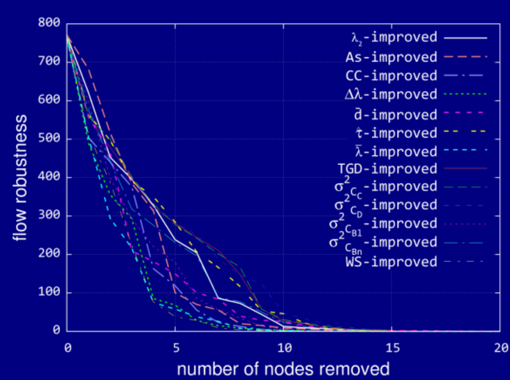
Level 3 physical network

Comprehensive Comparison
Weighted Evaluation Results

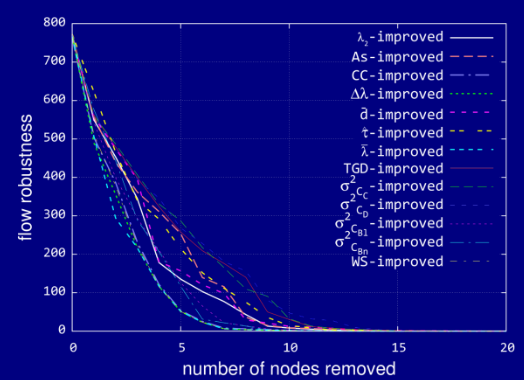
- Adding 20 links to weighted real-world networks
- Minimize or maximize given robustness function
- Degree balanced graphs with best results



betweenness attack



closeness attack



degree attack

GÉANT network

Comprehensive Comparison
Weighted Evaluation Results

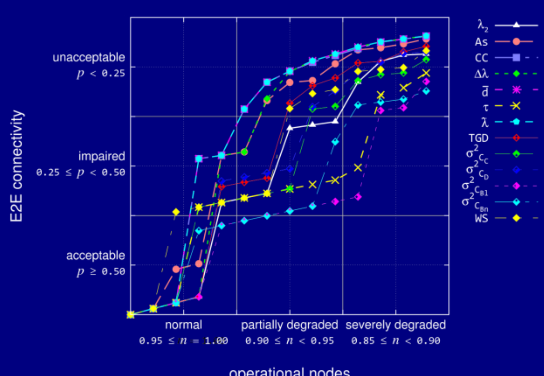
- Sum of flow robustness as attacking all nodes
- Three centrality-based attacks
- Node-betweenness balanced graphs – $\sigma^2_{C_D}$
  - best results for GÉANT and the other two networks

Centrality Attack	Objective Function												
	λ_2	TGD	$\sigma^2_{C_C}$	CC	WS	$\bar{d}$	$\sigma^2_{C_{B-n}}$	$\hat{\tau}$	$\sigma^2_{C_{B-l}}$	$\Delta\mu$	As	$\sigma^2_{C_D}$	$\bar{\lambda}$
Degree	2689.34	3430.82	3485.94	2109.32	2088.8	2840.92	2475.68	3112.7	2454.03	2035.75	3025.59	**3595.01**	1990.79
Closeness	3254.4	3547.47	3582.98	2467.93	2053.28	2650.7	3074.53	3487.71	2662.36	2123.06	2957.06	**3651.72**	1996.02
Betweenness	2645.45	3293.76	3316.64	1959.08	2036.48	2420.92	2416.86	3037.95	2633.43	1971.23	2299.24	**3407.27**	1815.88

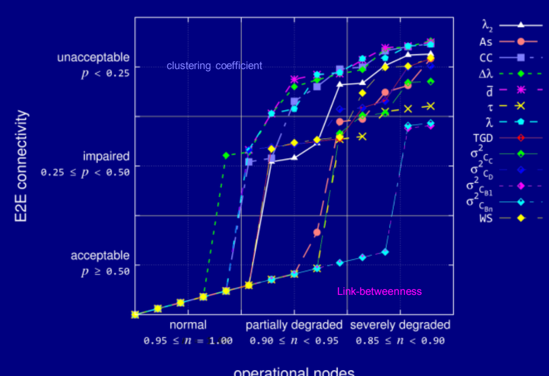
GÉANT network

Comprehensive Comparison
Resilience State-Space Analysis

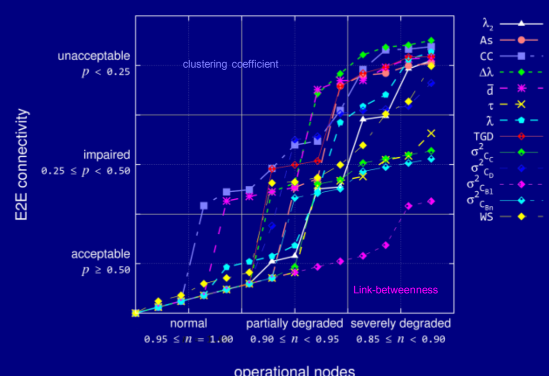
- Adding 20 links to unweighted real-world networks
- State definitions
  - service state: percentage of E2E connectivity [AÇS2015a]
  - operational state: connectivity of nodes



betweenness attack



closeness attack



degree attack

Level 3 physical network

Comprehensive Comparison
Resilience State-Space Summary

- Network resilience
  - $\mathbb{R}$ = 1– area under trajectory
- Link-betweenness balanced graphs – $\sigma^2_{C_{B-l}}$
  - best results for Level 3

Centrality Attack	Objective Function												
	λ_2	As	CC	$\Delta\mu$	$\bar{d}$	$\hat{\tau}$	$\bar{\lambda}$	$\sigma^2_{C_C}$	$\sigma^2_{C_D}$	$\sigma^2_{C_{B-l}}$	$\sigma^2_{C_{B-n}}$	WS	TGD
Degree	9.51	8.66	6.95	7.73	7.42	10.46	8.92	10.44	8.72	**11.96**	10.38	8.93	8.08
Closeness	7.94	9.48	6.92	6.21	6.58	10.02	6.72	9.77	7.75	**11.45**	11.43	8.1	8.28
Betweenness	7.4	5.81	5.19	5.84	5.17	8.28	5.17	7.32	7.12	**9.27**	8.52	6.54	6.78

Level 3 physical network

Network Resilience Improvement
Outline

- Introduction and motivation
- Background and related work
- Graph models
- Network design and improvement
- Evaluation and improvement
- Conclusions and future work

Conclusions
Graph Resilience Evaluation

- Investigated several robustness graphs
- Presented three robustness measures
  - based on sum of flow robustness during attacks
- Evaluated graph robustness metrics accuracy
  - AC and link-betweenness balanced graphs
    - consistent best results with Gilbert graphs
  - network criticality and effective graph resistance
    - for Waxman (mesh-like) and Gabriel (grid-like)
    - predicts network resilience against centrality attacks
- No ideal graph robustness metric for all graph types

Conclusions
Graph Resilience Improvement

- Several topology improvement algorithms
  - cost-efficient
  - number of links or budget constraint
- Evaluating improved graphs
  - several objective functions
  - against centrality-based attacks
- Our link- and node-betweenness balanced graphs
  - show better results for centrality-based attacks

Future Work

- Focused on adding links
  - plan to investigate adding a set of new nodes
- Multilevel evaluation and improvement
- For evaluation, we focused centrality-based attacks
  - correlated geographic failures and random failures
- Models and analysis are graph-theoretic
  - using ns-3 for application and protocol behavior
  - study other performance parameters
    - packet delivery
    - end-to-end delay

Network Resilience Improvement
References and Further Reading

- Color coding
  - my publications
  - ResiliNets publication
  - other reference

Selected Publications
References and Further Reading

1. [AS2015a] Mohammed J.F. Alenazi, James P.G. Sterbenz, "Evaluation and Improvement of Network Resilience via Multiple Graph Robustness Metrics", *IEEE/IFIP RNDM*, Oct. 2015 (to be submitted)
2. [AS2015b] Mohammed J.F. Alenazi, James P.G. Sterbenz, "Evaluation and Improvement of Network Resilience against Attacks using Graph Spectral Metrics", *3rd International Symposium on Resilient Communication Systems*, Aug. 2015 (submitted)
3. [AS2015c] Mohammed J.F. Alenazi, James P.G. Sterbenz, "Comprehensive Comparison and Accuracy of Graph Metrics in Predicting Network Resilience", *Design of Reliable Communication Networks, DRCN 2015*

Selected Publications

References and Further Reading

4. [AÇS2014a] Mohammed J.F. Alenazi, Egemen K. Çetinkaya, James P.G. Sterbenz, "Cost-Efficient Algebraic Connectivity Optimisation of Backbone Networks", *Optical Switching and Networking Journal*, 2014
5. [AÇS2014b] Mohammed J.F. Alenazi, Egemen K. Çetinkaya, James P.G. Sterbenz, "Cost-Efficient Network Improvement to Achieve Maximum Path Diversity", *IEEE/IFIP RNDM*, Oct. 2014
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- [SSG2013] A. Sydney, C. Scoglio, and D. Gruenbacher, “Optimizing algebraic connectivity by edge rewiring”, *Applied Mathematics and Computation* , Volume 219, No. 10, January 2013, pp. 5465 – 5479
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Network Resilience Improvement

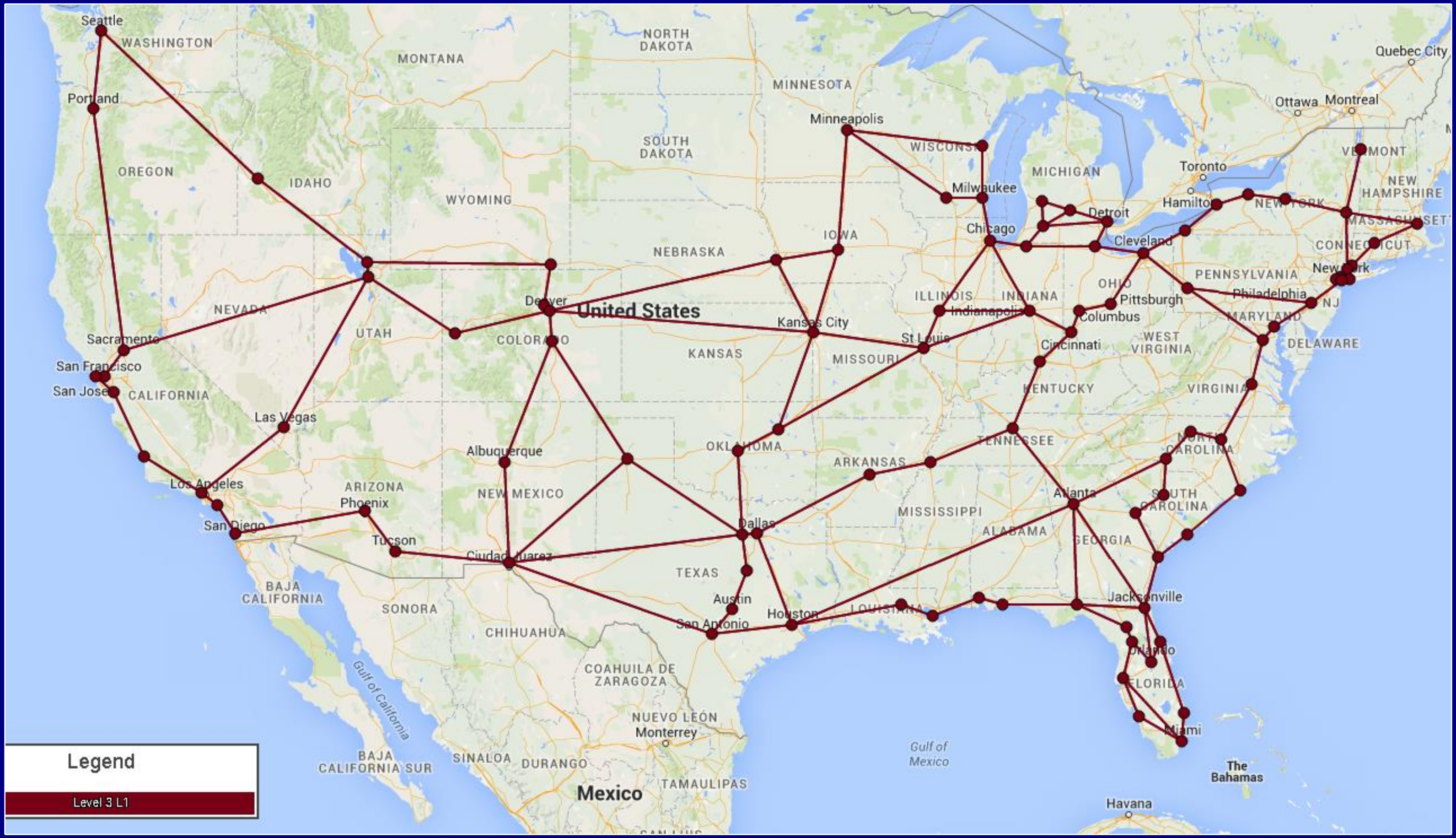
Questions?

End of Foils

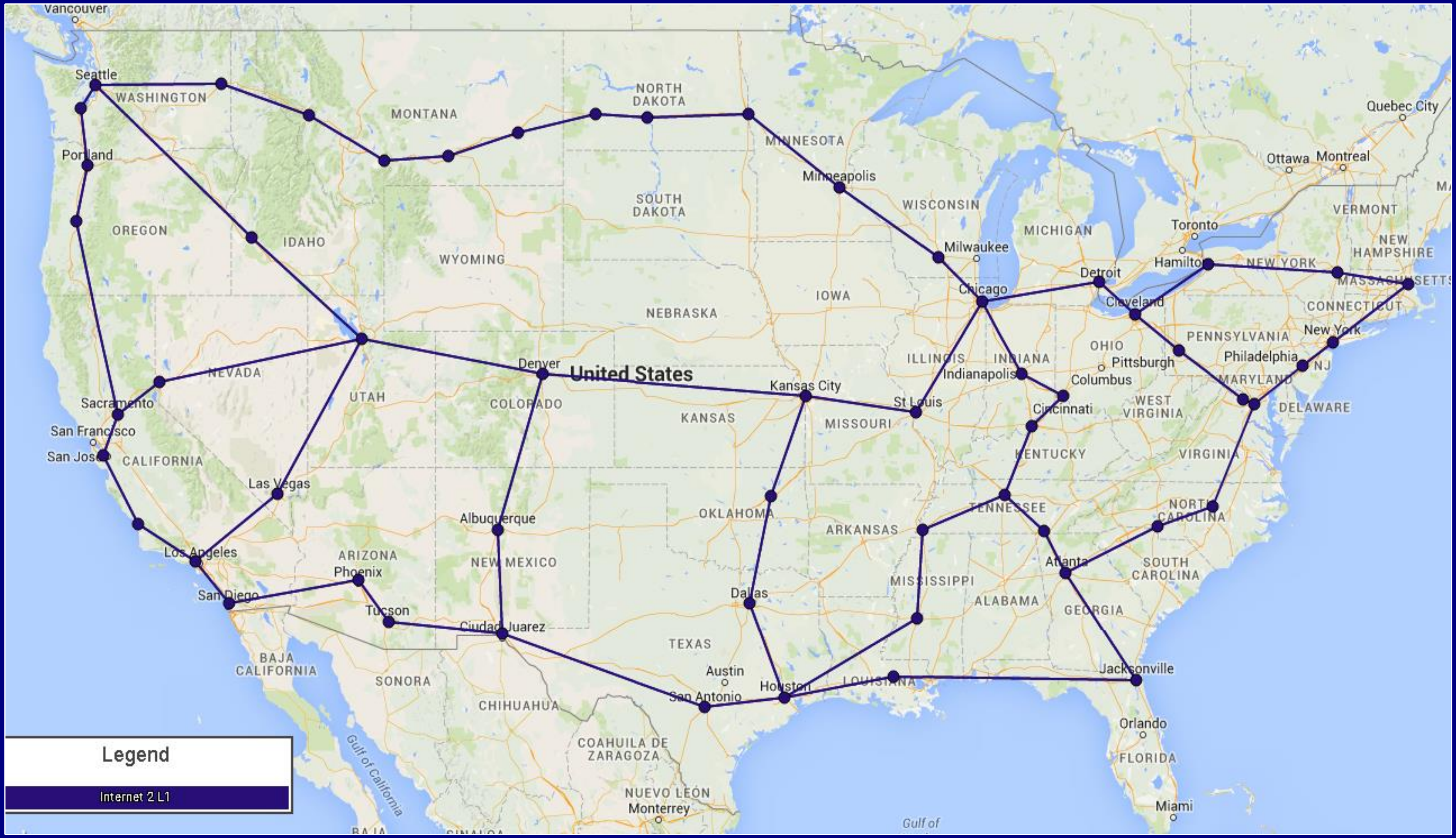
AT&T Physical

Sprint Physical

Level 3 Physical



Internet2 Physical



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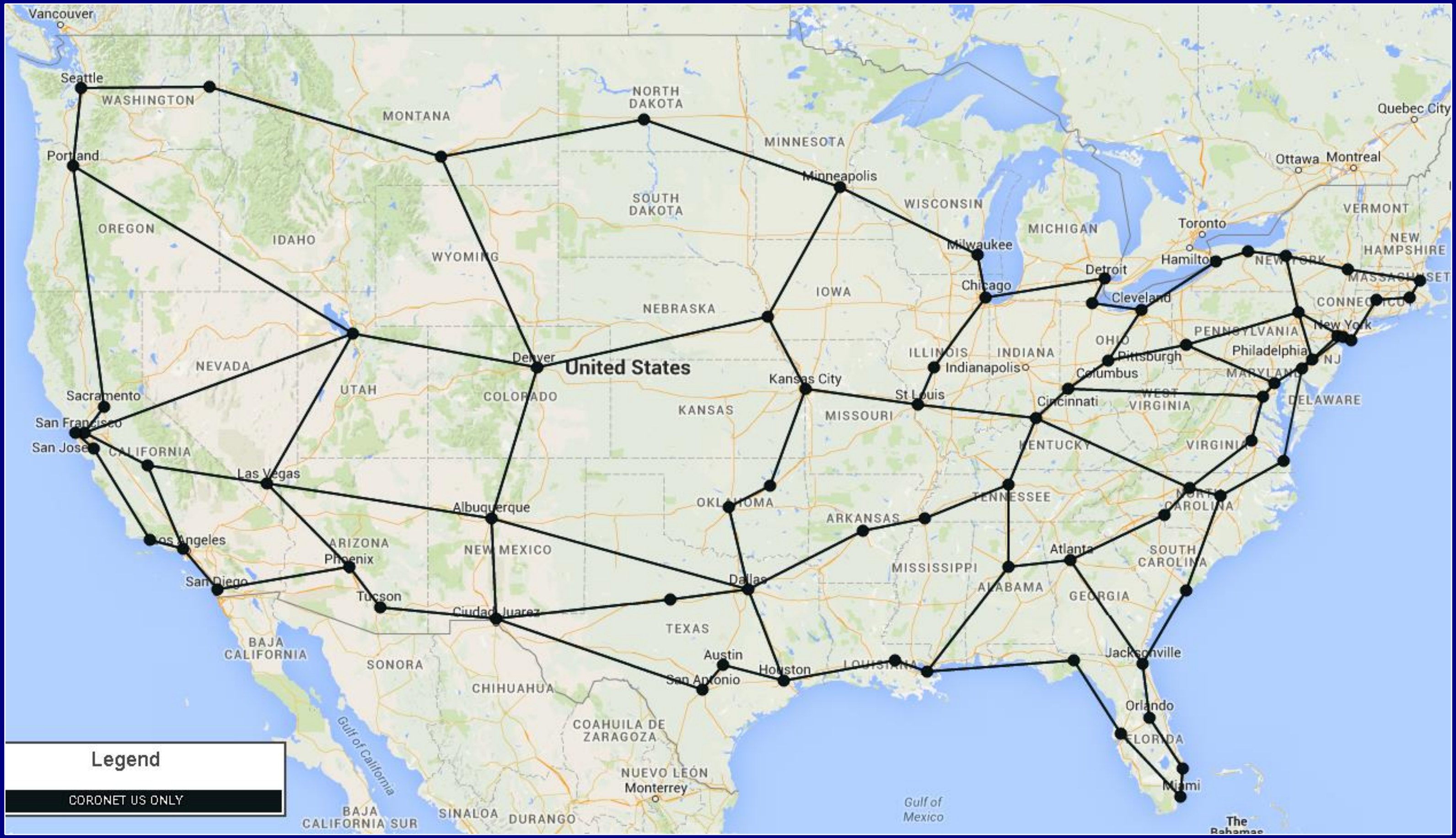
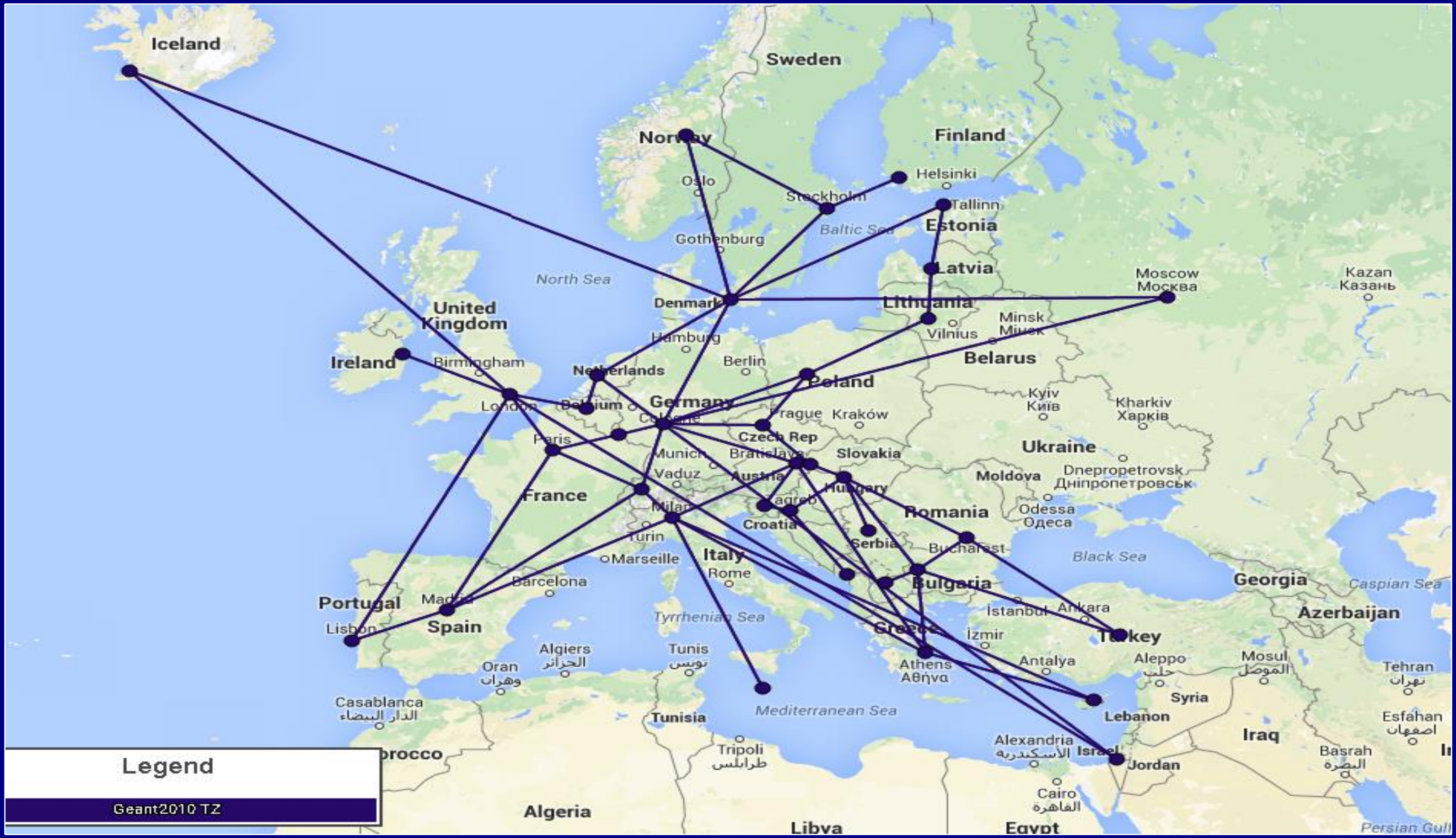
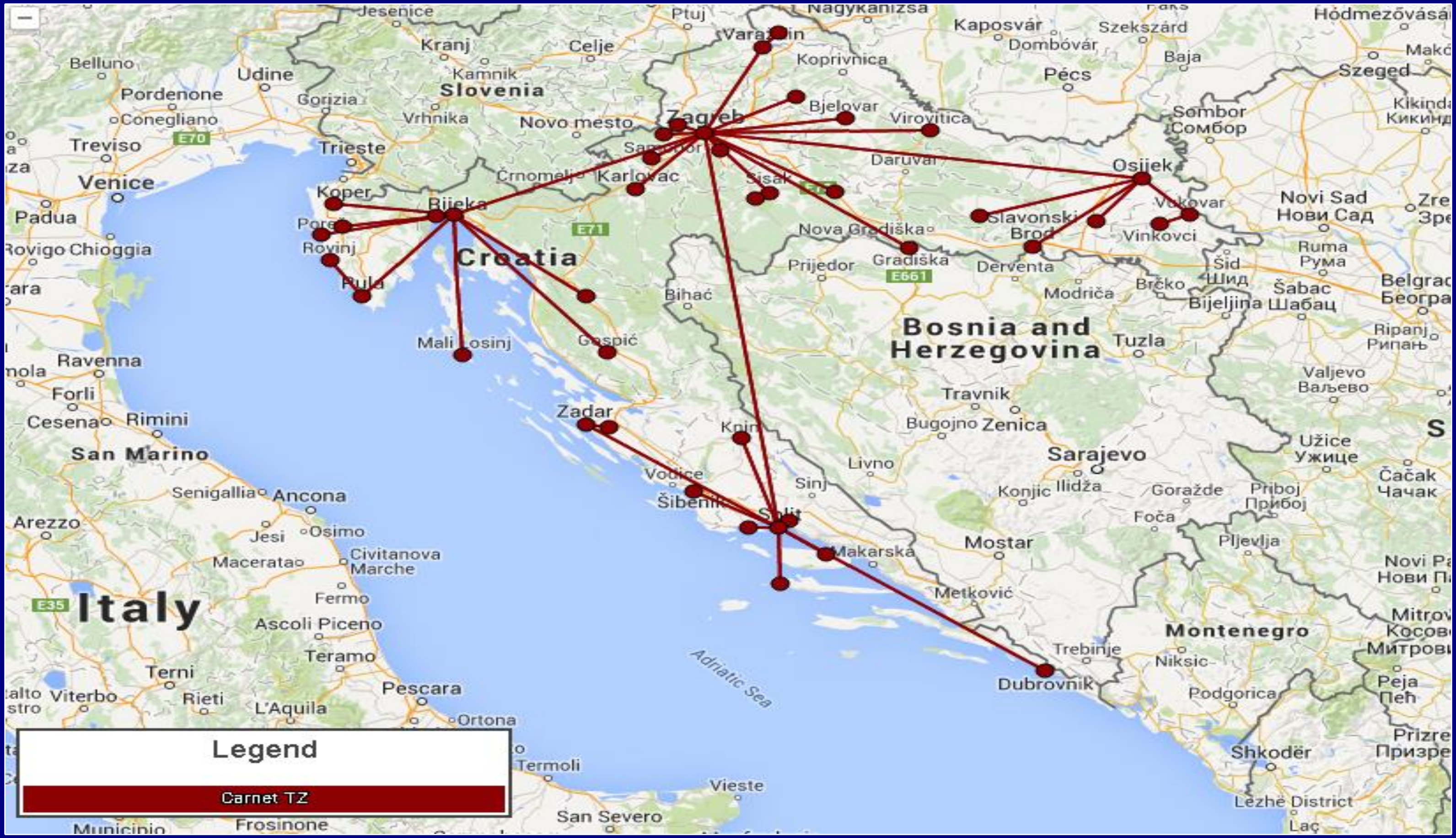


GÉANT Physical



KARen Physical

CARNet Physical



InternetMCI Physical