

Modular Semantics for Model-Oriented Design

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WELCOME

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Problem Statement

"Different paradigms can give quite different views of the nature of computation and communication. In a large system, different subsystems can often be more naturally designed and understood using different models of computation." [Burch et al.]

- Integration of different paradigms within one specification framework dictates:
 - Common syntax (domain of discourse)
 - Formal semantics that provides notion of consistency
 - Translation of specifications
 - Composition of specifications

Proposed Solution

- Formal semantics
 - Institution
 - Relates syntax to semantics
 - Defines notion of models satisfying a specification
 - Defines a logical system, e.g. equational reasoning, firstorder logic, ...
 - Provides basis for sound and complete deduction calculus
 - Modularity in using several institutions
- Multi-model of computation framework
 - Identify unifying semantic domains (units of semantics)
 - Static
 - State-based
 - Trace-based
 - Define models of computation
 - State-based: continuous, discrete, finite-state
 - Trace-based: csp-trace

Key Contributions

- Definition of a formal semantics, giving an entailment system that allows reasoning over correctness of a heterogeneous design
- Definition of multiple unifying semantic domains and models of computations within one framework
- Definition of relations between specifications
- Demonstration of composition of specifications
- Demonstration of new heterogeneous design methodology
- Demonstration of re-use of domain-specific views

Overview

- Preliminaries
- Modular semantics
 - Static semantics
 - State-based semantics
 - Hidden algebras
 - Coalgebras
 - Trace-based semantics
- Specification in the Rosetta Language
 - Units of semantics
 - Models of computation
- Examples and Application
 - Hybrid system
- Related work and future work



PRELIMINARIES

Category Theory

- Category C
 - Collection of objects |C|
 - Collection of arrows ||C|| (with dom and cod)
 - Composition of arrows
 - Identity arrow for each object
- Examples
 - Category of algebras
 - The objects are algebras
 - The arrows are homomorphisms between algebras
 - Category of sets
 - The objects are sets
 - The arrows are functions

Concrete Example











Institution Theory

- Formalizes:
 - Truth is invariant under changes of notation
- Institution (Sign, Mod, Sen, =)
 - **Sign:** category of signatures
 - Sen: Sign → Set functor giving set of sentences for each signature
 - Mod: Sign → Cat^{op} functor giving category of models for each signature
 - $|=_{\Sigma} \subseteq |Mod(\Sigma)| \times Sen(\Sigma)$ signature-indexed family of satisfaction relations such that for $(\phi: \Sigma \to \Sigma') \in ||Sign||, e \in Sen(\Sigma), M' \in |Mod(\Sigma')|$ $M'|=_{\Sigma'} Sen(\phi)(e)$ if and only if $Mod(\phi)(M')|=_{\Sigma} e$



MODULAR SEMANTICS

Static Semantics – Programming in the small

- Notion of fixed data
- Notion of invariance
- Signature (S_{stc}, Σ_{stc})
 - S_{stc} set of sorts
 - Σ_{Stc} set of operators $S_{Stc}^* \times S_{Stc}$
- Algebra
 - $S_{\scriptscriptstyle Stc}$ -indexed family of non-empty sets, carriers $A_{\scriptscriptstyle Stc}$
 - $S_{Stc}^* \times S_{Stc}$ -indexed family of maps

 $\alpha_{u,s}: \Sigma_{Stc_{u,s}} \to [A_{Stc_u} \to A_{Stc_s}]$

- Algebra morphism from $\langle A_{stc}, \alpha \rangle \rightarrow \langle A'_{stc}, \alpha' \rangle$ is map $f: A_{stc} \rightarrow A'_{stc}$ such that $f_s(\alpha(\sigma)(a_1, \dots, a_n)) = \alpha'(\sigma)(f_{s_1}(a_1), \dots, f_{s_n}(a_n))$
- Equation $(\forall X)t1 = t2$

Static Semantics – Programming in the large

- Specification is $(S_{stc}, \Sigma_{stc}, E_{stc})$
- Algebra A_{Stc} satisfying equation e iff $a^*(t1) = a^*(t2)$ for every assignment $a: X \rightarrow |A_{Stc}|$, $A_{Stc} \models_{\Sigma_{Stc}} e$
- Institution for static algebras (equational-[Goguen]) $(Sig_{Stc}, Alg_{Stc}, Eqn_{Stc}, \models_{Stc})$
 - Sig_{Stc} category of static signatures and morphisms
 - Alg_{Stc} functor giving category of static algebras for each signature
 - Eqn_{Stc} functor giving a set of equations for each signature
 - $|=_{Stc}$ satisfaction such that $A_{Stc}^{'}|=_{\Sigma_{Stc}^{'}} \varphi(e)$ iff $A_{Stc}^{'}|_{\varphi}|=_{\Sigma_{Stc}} e$ with $\varphi:\Sigma_{Stc} \to \Sigma_{Stc}^{'}$

Static Semantics – Specification construction

- Specification extension
 - Extension satisfies no confusion and no junk constraint
 - $(S_{Stc}, \Sigma_{Stc}, E')$ extends $(S_{Stc}, \Sigma_{Stc}, E) \Rightarrow S_{Stc} \subseteq S_{Stc}, \Sigma_{Stc} \subseteq \Sigma_{Stc}, E \subseteq E'$
 - Extension is an inclusion morphism, more specifically it is an enrichment signature morphism that is conservative
- Specification parameterization and instantiation
 - Parameterization defines properties over a class of specifications
 - Instantiation reduces class to a particular specification, and involves binding signature morphism
- Specification inclusion
 - Allows information hiding that involves a signature inclusion along with an information hiding operator ()
- Specification use
 - Use packages
- Specification composition
- Pushout of two specifications syntactic composition
 9/15/2004

State-based Semantics – Programming in the small

- Notion of observing a current state and change of observations over a next transformation function
 - A state is only identified by its attributes
 - Two states that have same attributes are undistinguishable and are said to be behaviorally equivalent
- State-based signature (S_{SB}, Σ_{SB})

•
$$S_{SB} = (State, S_V)$$

- $\Sigma_{SB} = (isInit, Y, next, \Phi, \Omega, \Delta)$
 - Y set of generalized hidden constants $cst:S_{\scriptscriptstyle V_{0,\ldots,n}}\to State$
 - Φ optional set of operations $\phi: State \times S_{V_0} \longrightarrow State$
 - Ω set of attributes $\omega: State \times S_{V_0} \xrightarrow{\pi} \to S_V$
 - Δ set of data operations $\delta: S_{V_0} \to S_V$
 - Distinction between operators of Y and next

State-based Semantics – Programming in the small

- A state-based signature: hidden signature[Goguen]
 - Hidden sort = State
 - Visible data universe = (S_V, Δ, D_{SB})
 - At most one hidden sort occurs in $Y \text{ or } \Omega$
- Behavioral Satisfaction
 - A context of sort h is a visible sorted Σ-term that has a single occurrence of a new variable symbol z of sort h, e.g. x(z), x(next(z)).
 - A hidden algebra behaviorally satisfies equation e $A \models_{\Sigma} (\forall X)t = t' \quad if \quad t_1 = t_1, \dots, t_m = t_m'$

iff for each appropriate context c and assignment $\theta: X \to A$ $\theta^*(c[t]) = \theta^*(c[t'])$

whenever $\theta^*(c_j[t_j]) = \theta^*(c_j[t_j'])$ for j = 1,...,m and all appropriate c

State-based Semantics – Programming in the small

- State-based specification (S_{SB}, Σ_{SB}, E)
 - (S_{SB}, Σ_{SB}) is a state-based signature
 - $E = E_{\Delta} \oplus E_{\Omega}$ disjoint union of 2 sets of equations
 - Induces a hidden specification (*State*, Σ_{SB} , E_{Ω})
- Consistency of state-based specification
 - Consistent iff induced hidden specification has a model with non-empty carriers and all equations $E_{\rm A}$ are consistent
 - Necessary condition: E is D-safe
 - Sufficient condition: locality of equations
 - Local equation: local terms and conditions are visibly sorted and use only $\Psi\mbox{-}operations$
 - Local term: every proper subterm is a $\Psi\text{-subterm}$
 - Non-local: use rewriting and provide a model

State-based Semantics – Programming in the large

- State-based signature morphism
 - Hidden signature morphism
 - Identity over the visible data (V,Ψ)
 - Maps hidden sorts to hidden sorts $morphism \quad (S_{SB}, \Sigma_{SB}) \rightarrow (S_{SB}, \Sigma_{SB})$

signature morphism $\varphi: \Sigma_{SB} \to \Sigma_{SB}'$

if $\sigma' \in \Phi'$ or $\sigma' \in \Omega'$ then $\exists \sigma \in \Phi$ or $\sigma \in \Omega | \sigma' = \varphi(\sigma)$

- Sub-system morphism instead of enrichment morphism
- Only one State sort, use of qualified name through a renaming morphism to distinguish between State sort of different specifications

State-based Semantics – Programming in the large

- Institution for state-based algebras
 - Category of state-based signature and morphisms $Sign_{\rm SB}$
 - Functor giving a set of equations for each signature

Sen_{sb}

 Functor giving a category of hidden algebras for each signature

Mod_{SB}

Satisfaction relation

$\mid \equiv_{\Sigma_{SB}}$

Satisfaction condition

$$A'|_{\varphi}|\equiv_{\Sigma_{SB}} e \quad iff \quad A'\mid\equiv_{\Sigma_{SB}} \varphi(e)$$

State-based Semantics – Coalgebras

- Cirstea's work: Hidden algebras \rightarrow Coalgebras
- State-based signature \to destructor hidden signature (by leaving out $^{\rm Y}$ and Φ) \to abstract cosignature

$$(Set_{D_{SB}}^{S_{SB}}, F_{\Sigma_{SB}}) \quad with \quad F_{\Sigma_{SB}} : Set_{D_{SB}}^{S_{SB}} \to Set_{D_{SB}}^{S_{SB}}$$
$$(X_{S_1}, \dots, X_{S_n}, X_{State}) \to (X_{S_1}, \dots, X_{S_n}, \prod_{k \in 1, \dots, l} X_{S_k}^{X_{S_0, \dots, n}} \times X_{State}^{X_{S_0, \dots, n}})$$

- Example:
 - State-based signature State, Natural $s_0 :\rightarrow State, x : State \rightarrow Natural, next : State \rightarrow State, \Delta_{Natural}$
 - Destructor hidden subsignature $({Natural, State}, {x: State \rightarrow Natural, next: State \rightarrow State} \cup \Delta_{Natural})$
 - Associated abstract cosignature

 $(Set_N^{\{Natural,State\}},F)$ with $FX_{State} = N \times X_{State}$

A coalgebraic structure

$$\alpha: X_{\textit{State}} \to N \times X_{\textit{State}}$$

State-based Semantics – Specification construction

- Extension: similar in essence to static specification extension
 - The signature morphism is reverse $(S_{SB}, \Sigma_{SB}, E) \xrightarrow{c} (S_{SB}, \Sigma_{SB}, E')$ iff $\exists \varphi : (S_{SB}, \Sigma_{SB}) \rightarrow (S_{SB}, \Sigma_{SB})$
- Parameterization:
 - 3 parameter modes: input, output and design
- Instantiation: may involve state dependent bindings of parameters
- Translation: mapping of properties of the State sort from one specification to another
- Inclusion: similar to static inclusion, but may be supplemented by a translation relating states of specifications involved in inclusion
- Use: as for static. In this work, all packages are static

State-based Semantics – Specification composition

- Category of state-based specifications as objects and extensions as arrows
- Composition uses categorical notion of colimit
- Composition of two specifications sharing a common parent through a pushout
- Composition of two specifications on different subtrees, translation may first be needed



Trace-based semantics

- Notion of traces and operations over traces to model computation runs
- Equational signature
- Same semantics as for static
 - Institution of equational reasoning
- Enforcement of a Trace(T) sort
- Available Operations: head, tail, add, sequence, interleave, restriction, order, ...

Specification Construction across Semantic Domains

- Conservative extension from static to state-based and from static to trace-based
- Institution morphism from static to state-based is strong, persistent and additive similar to CafeOBJ's institution morphism
- Specification translation from static to state-based
 - Static represents data and invariant properties in a state-based specification
 - Minimal representation:

 $Spec_{SB} = (S_{Spec_{Stc}} \cup \{State\}, \Sigma_{Stc} \cup next, E_{Stc} \cup E_{SB})$

- Specification translation from state-based to static described by Goguen et al.
 - Translation of behavioral specification into ordinary algebraic specification

Specification Translation from State-based to Trace-based

- One-way translation $Spec_{SB} \rightarrow Spec_{TB}$
- For each input I in $Spec_{SB}$, an input set of traces of type of I in $Spec_{TB}$
- Same for output parameters
- All declarations of $Spec_{SB}$ become declarations of $Spec_{TB}$
- Add declarations of
 - A variable T_{st} :: Trace(State) representing set of traces of all reachable states
 - A variable *someTrace* representing a trace
 - A variable n of sort natural used as position of state in trace
 - All equations of $Spec_{SB}$ are included in $Spec_{TB}$
 - Add 2 new equations: $state_def$ equating State to actual, and newT stating $someTrace \in T_{st}, s \in State$ such that someTrace [n] = s and $next(s, I_0[n], \dots, I_k[n]) = someTrace [n+1]$



SPECIFICATION IN ROSETTA

The Domain organization



Static Modeling

- Semantics given by the previously defined static (equational) semantics
- Specification
 - Defines a number of types: Universal, Element, Number, Complex, Real, ..., Function, Set, Sequence, ...
 - Defines a number of operators over each sort
 - Static domain
- Static domain semantics (Boolean) $S_{stc} = \{..., Boolean, ...\}$ $\Sigma_{stc} = \{..., false : \rightarrow Boolean, true : \rightarrow Boolean, not : Boolean \rightarrow Boolean, ...$..., or : Boolean × Boolean → Boolean, ...}

Static Domain Specification

```
domain static::null is
// _____
// Boolean types
// _____
 Boolean :: type is enumeration (false, true);
// _____
// Functions for boolean type
// ------
 •••
 not__(R :: Boolean ) :: Boolean;
 or (L, R :: Boolean) :: Boolean;
 •••
begin
 •••
 not false: (not false) = true;
 not true: (not true) = false;
 true_or_true: (true or true) = true;
 true or false: (true or false) = true;
 false or true: (false or true) = true;
 false_or_false: (false or false) = false;
 ...
end domain static;
```

Initial Algebra for Static



State-based Modeling

- State-based semantics
 - Institutions of Hidden Algebras, Coalgebras
- Specification
 - State type
 - Next function that takes a state and a number of inputs and returns a new state
 - Extends static domain
- State-based domain semantics

 $S_{SB} = (State, S_{Stc})$ $\Sigma_{SB} = (isInit, Y_{SB}, next, \{\}, \{\}, \{_, \{_, _] \cup \Sigma_{Stc})$

Coalgebras

$$|A|_{State} \xrightarrow{\gamma_{next}} \{*\} \cup |A|_{State}$$
$$|A|_{State}^{R} \xrightarrow{\zeta} |A|_{State}^{R}$$

State-based Domain Specification

domain *state_based*(State::design Type) :: static is

s :: State;

next:: Function;

__@__[T::Type](lhs::<*(st::State) -> T *>; rhs::State)::T is lhs(rhs); isInit(s::State)::Boolean;

begin

// next: State x Si ... x Sn -> State with Si,...,Sn: one or more types
return_type_next: ret(next) = State;
domain_next: dom(next) = State;
end domain state_based;

The Discrete Domain Specification

domain discrete(DiscState::design Type) :: state_based(DiscState) is

isDiscrete(DiscreteSet::Type)::Boolean =

exists (fnc::<*(st::DiscreteSet)::Integer*>

forall(s1,s2::DiscreteSet|

(s1 /= s2) => (fnc(s1) /= fnc(s2)));

begin

discrete_attributes: forall (fnc::getAttributes() | isDiscrete(ran(fnc)));

end domain discrete;

The Finite-state Domain

Finite-state \Rightarrow observations are finite and discrete



```
domain finite_state(FState::design Type) :: discrete(FState) is
    isFinite(FiniteSet::Type)::Boolean is
        #FiniteSet in Natural;
begin
        fs1:forall (fnc::getAttributes() | isFinite(ran(fnc)));
end domain finite_state;
```

The Continuous Domain

Continuous observation of states \Rightarrow all observations have continuous variations with respect to a continuous observation of states

 $\frac{\Delta f}{\Delta s} = \frac{f(next(s)) - f(s)}{contAttr(next(s)) - contAttr(s)}$

```
domain continuous :: state_based is
contAttr(st::State)::Real;
variation[T::Type](fnc::<*stt::State)::T*>;st::State;next_st::State)::T is
  (f(next_st) - f(st)) / (contAttr(next_st)-contAttr(st));
```

begin

end domain continuous;

Trace-based Modeling

- Semantics
 - Static semantics (institution of equational logic)
 - As traces represent computation runs, can use coalgebras as models as well
- Specification
 - Notion of traces
 - Operations as defined in trace semantics
 - Extends static domain

Trace-based Domain Specification

```
domain trace_based()::static is
Trace(T::Type)::Type;
emptyTrace::Trace(Universal) is constant;
add[Event::Type](tr::Trace(Event);ev::Event)::Trace(Event);
head[Event::Type](tr::Trace(Event))::Event;
tail[Event::Type](tr::Trace(Event))::Trace(Event);
isEmpty[Event::Type](tr::Trace(Event))::Boolean is
tr = emptyTrace;
```

```
getEventAt[Event::Type](tr::Trace(Event);pos::Natural)::Event is
if (not isEmpty(tr))
else if (pos = 0) then head(tr)
else getEventAt(tail(tr),pos-1)
end if;
end if;
```



Examples and Applications

Example of a Stack Datatype

facet stackDT::static is

Stack::type;

emptyStack::Stack is constant;

push(stcParam::Stack; n::Natural)::Stack;

pop(stcParam::Stack)::Stack;

top(stcParam::Stack)::Natural;

val::Natural;

stcVar::Stack;

begin

pop_empty: pop(emptyStack) = emptyStack;

top_empty: top(emptyStack) = 0;

pop_push: pop(push(val,stcVar))=stcVar;

top_push: top(push(val,stcVar))=val;

$$S_{stackDT} = S_{Stc} \cup \{Stack\}$$

end facet stackDT;

 $\Sigma_{stackDT} = \Sigma_{Stc} \cup \{emptyStack, push, pop, top\}$ $E_{stackDT} = E_{Stc} \cup \{pop_empty, top_empty, pop_push, top_push\}$

Initial algebra for stackDT



Isomorphism between N_{stackDT} and N_{Stc}



Composition of State-based Parameterized Specifications

StateSet::Type;

```
memNext(st::State;val::Natural)::State;
```

```
facet memoryA(val::input Natural)
                ::discrete(StateSet) is
    memA(st::State)::Natural;
begin
    initA: isInit(s) => memA@s = 0;
    next_def: next = memNext;
    lA: memA@next(s,val) = val;
end facet memoryA;
```

```
facet memoryB(val::input Natural)
                ::discrete(StateSet) is
    memB(st::State)::Natural
begin
    initB: isInit(s) => memB@s = 0;
    next_def: next = memNext;
    lB: memB@next(s,val) = val+memB;
end facet memoryB;
```

facet twoMemory(val::input Natural)::discrete(StateSet) is

```
memoryA(val) + memoryB(val);
```

Composition of Parameterized Specifications



Pullback of Signature Morphisms

Composition of Parameterized Specifications



Pushout of Coalgebras

Trace-based MemoryA Specification

```
StateSet::Type;
```

```
memNext(st::State;val::Natural)::State;
```

```
facet traceMemA(val::input Trace(Natural))::trace_based() is
  memA(st::State)::Natural;
  StateTrace::Trace(State);
  someTrace::StateTrace;
  s::State; next::Function; ... // All declarations from domains
  pos::Natural;
  begin
```

begin

Specification of a Hybrid Automaton

- Hybrid automaton [Henzinger]
 - Variables: x, dotted x (\dot{x}) , x'
 - Control graph (V,E) of control modes and edges
 - Predicates:
 - Initial
 - Invariant
 - Flow conditions: predicate for continuous change
 - Jump conditions: predicate for each control switch
 - Events over control switches (events)

Hybrid Automaton of a Thermostat



Two states for the heater: on or off Continuous variation of the temperature: x heater on => temperature x increases at rate of 5 - 0.1x per minute heater off => temperature x decreases at rate of -0.1x per minute

The Heater Specification

```
facet heater(x::input Real; ctrl::output ControlMode):: finite_state is
mode(s::State)::ControlMode;
```

begin

end facet heater;

The Temperature Specification

```
facet temperatureVariation(ctrl::input ControlMode; x::output Real):: continuous is
  temp(s::State)::Real;
```

begin

```
initial: isInit(s) => ((temp@s = 20) and (contAttr@s = 0);
next_def: next = <*(st::State;ctrl::ControlMode)::State*>;
mono_increase: contAttr@next(s,ctrl) > contAttr@s;
output: x = temp@s;
off_cool: (ctrl = off) =>
        (variation(temp,s,next(s,ctrl)) = -0.1 * temp@s);
on_heat: (ctrl = on) =>
        (variation(temp,s,next(s,ctrl)) = 5 - 0.1 * temp@s);
next_heat: temp@next(s,ctrl) = temp@s +
        variation(temp,s,next(s,ctrl)) *
        (contAttr(next(s,ctrl)) - contAttr(s));
```

```
end facet temperatureVariation;
```

The Thermostat Specification

```
facet thermostat():: state_based is
  ctrl(st::State)::ControlMode;
```

```
x(st::State)::Real;
```

begin

next_def: next = <*(st::State)::State*>; heater_comp: heater(x@s, ctrl@s); temperature_comp: temperatureVariation(ctrl@s, x@s); inv_off: (ctrl@s = off) => (x@s >= 18); inv_on: (ctrl@s = on) => (x@s =< 22); end facet thermostat;

Analysis of the Thermostat Specification

- Two observations of the state
- The values of each observation provided by *Heater* or by *TemperatureVariation* specifications
- Models that satisfy Thermostat will have (minimal) states as pairs (controlmode, temp) with controlmode=ctrl(s) and temp=x(s)
- Controlmode: on or off
- Temp: a real number between 18 and 22
- If considering discrete Thermostat models, temp will have discretized values through "sampling"



RELATED WORK AND FUTURE WORK

- CafeOBJ http://www.ldl.jaist.ac.jp/cafeobj
- Ptolemy II Heterogeneous Concurrent Modeling and Design in Java - J. Davis, C. Hylands, B. Kienhuis, E. Lee, et al.; University of California at Berkeley
- Metropolis Overcoming Heterophobia: Modeling Concurrency in Heterogeneous Systems - J.
 Burch, R. Passerone, A. Sangiovanni-Vincentelli

- SAL An Overview of SAL J. Rushby, S. Owre, N. Shankar,
 A. Tiwari et al.
- Viewpoints Modeling Viewpoints: A Framework for Integrating Multiple Perspectives in System Development - A. Finkelstein et al.
- Feature Engineering Feature-Oriented Description, Formal Methods, and DFC - P. Zave
- Aspect-oriented -
 - Aspect-Oriented Programming G. Kiczales et al.
 - Aspect-Oriented Requirements Engineering for Component-Based Software Systems - J. Grundy

- The MultiGraph Architecture Metamodeling Rapid Design and Evolution of Domain-Specific Modeling Environments - G. Nordstrom et al.; Vanderbilt University
- GME The Generic Modeling Environment A. Ledeczi et al.; Vanderbilt University
- UML-Metamodeling Architecture An UML-metamodeling Architecture for Interoperability of Information Systems - M. Terrasse et al.

- A Framework for Multi-Notation Requirements
 Specification and Analysis N. Day and J. Joyce
- Constructing Multi-Formalism State-Space Analysis
 Tools: Using rules to specify dynamic semantics of models
 M. Pezze and M. Young
- A Multi-Formalism Specification Environment E.
 Ipser, Jr and D. Wile
- Acme: An Architecture Description Interchange Language - D. Garlan, R. Monroe and D. Wile

Conclusion

- Modular formal semantics
- Framework supporting different models of computation
- Future Work
 - Extension of semantics to order sorted institution
 - Definition of engineering domains: definition of units of measurement, definition of engineering formulas.
 - Automatic verification tool