Symbol Timing Recovery for SOQPSK

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OVERVIEW

- Motivation
- Continuous Phase Modulation (CPM)
- SOQPSK – A Special type of CPM
- Signal Model
- Timing Error Detectors (TED)
- Performance Bound - MCRB
- Simulation Results

MOTIVATION

- SOQPSK is very similar to OQPSK.
- Typical receiver models use suboptimal OQPSK-type detector and synchronization techniques.
- Results in a performance loss of 1-2 dB.
- Recently developed CPM based receiver models are optimal and outperform OQPSK-type detection by 1-2 dB.
- CPM-based approach can be implemented in low complexity 4-state detectors.

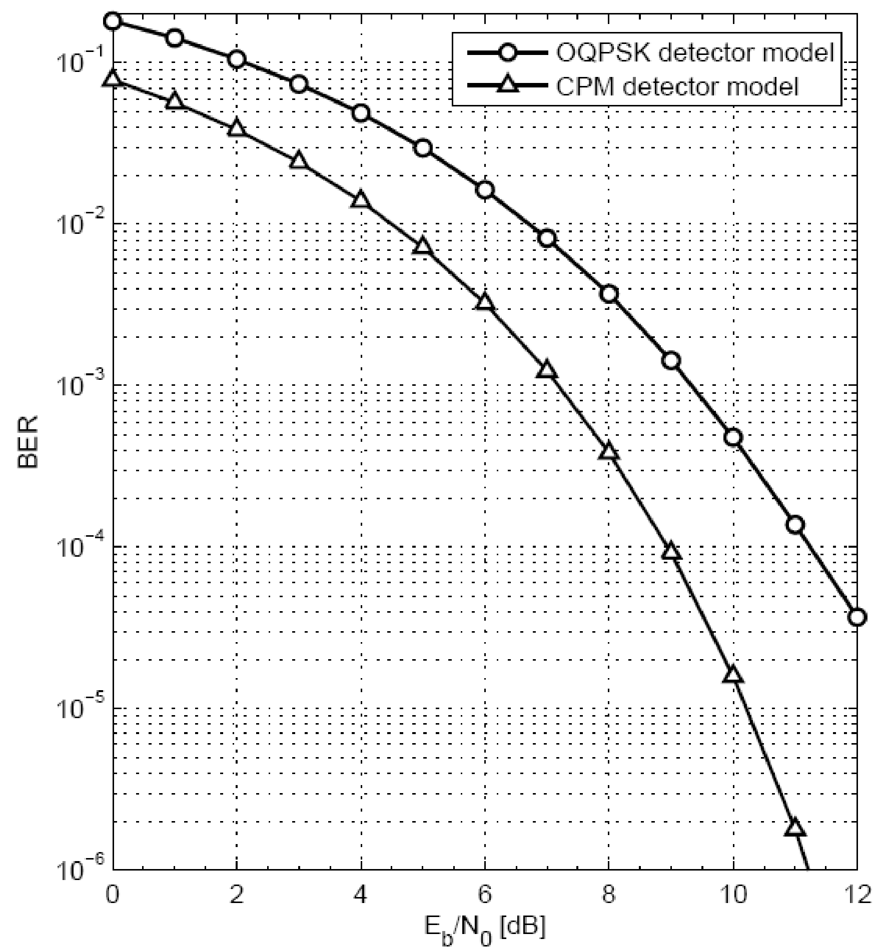
- Contributions:
  - Adapt existing CPM-based TEDs for SOQPSK, which is a constrained ternary CPM.
  - Incorporate the TEDs into the Viterbi algorithm based SOQPSK detector.
  - Analyze the performance of the TEDs when combined with the PT technique for SOQPSK-TG (512 states -> 4 states).

Publications

- Journal publications
  - P. Chandran and E. Perrins, "*Symbol Timing Recovery for CPM signals with Correlated Data Symbols,*" second revision in review, IEEE Transactions on Comm.
  - P. Chandran and E. Perrins, "*Decision Directed* Symbol *Timing Recovery for CPM signals with Correlated Data Symbols,*" in review, IEEE Transactions on Aerospace and Electronic Systems.
- Conference papers
  - P. Chandran and E. Perrins, "*Symbol Timing Recovery for SOQPSK,*" Proceedings of ITC '07. Awarded second prize in student paper competition.
  - P. Chandran and E. Perrins, "*Decision Directed Timing Recovery for SOQPSK,*" Proceedings of IEEE Milcom'07.

So What??

- BER curve for SOQPSK-TG
- OQPSK Detector
  - Symbols not perfectly matched. Hence 1-2dB loss.
- CPM model overcomes this problem.

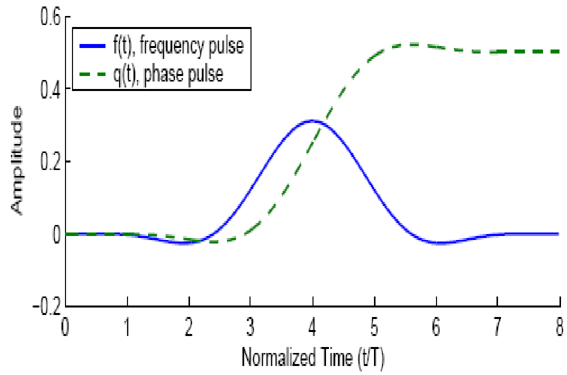


Continuous Phase Modulation

- Constant Envelope Modulation
- Power Efficient
- Good Spectral Properties
  - High Bandwidth Efficiency
  - Low Spectral Side Lobes
- Current Applications
  - Military Satellite Communication
  - Aeronautical Telemetry Standards

SOQPSK – A Special type of CPM

- Two versions of SOQPSK
  - SOQPSK – MIL (M=2, L=1, h=1/2, REC)
  - SOQPSK – TG (M=2, L=8, h=1/2)



- Precoding:

$$\alpha_i(\boldsymbol{u}) \triangleq \frac{1}{2}(-1)^{i+1}u_{i-1}(u_i - u_{i-2})$$

- u_i – original information bits which are binary
- $\boldsymbol{\alpha_i}$ **–** precoded output symbols which are ternary

- A value of +1 cannot be followed by a -1 and vice versa.

Signal Model

- The complex SOQPSK signal can be represented as

$$s(t, \boldsymbol{\alpha}) \triangleq \sqrt{\frac{E_s}{T_s}} \exp\{j\phi(t, \boldsymbol{\alpha})\}$$

where E_s is the symbol energy and T_s is the symbol duration.

$$\phi(t, \boldsymbol{\alpha}) \triangleq 2\pi h \sum_i \alpha_i q(t - iT_s)$$

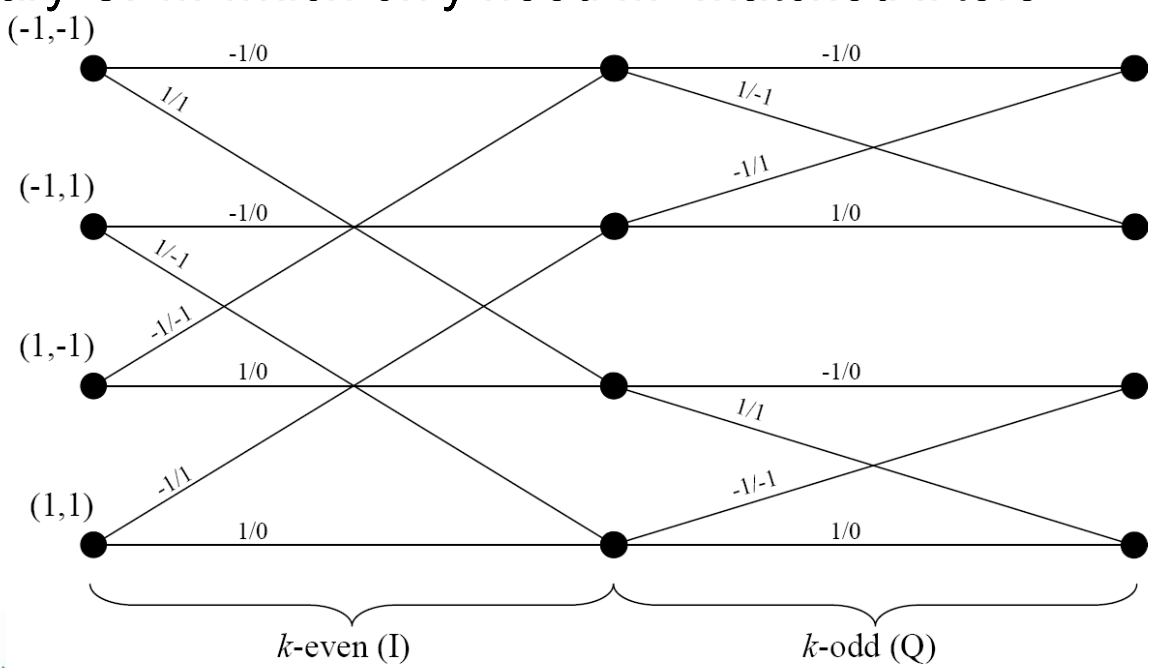
φ(t,α) is the phase which is a pulse train as shown above and 'α' is the precoded information sequence.

Timing Error Detectors

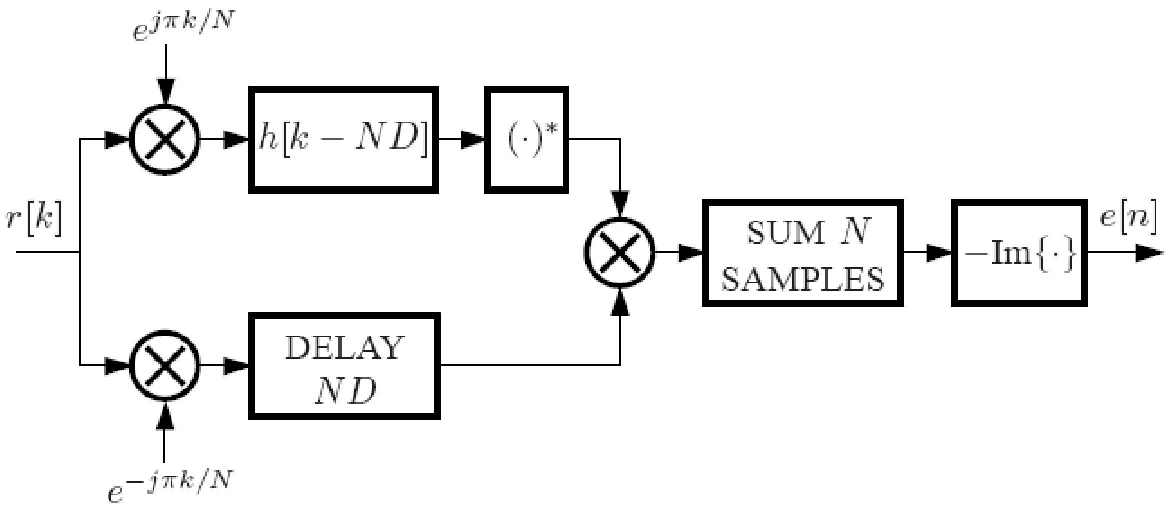
- TEDs can be broadly classified into two categories.
  - Non Data Aided or Blind TED
    - Assumes nothing is known about the actual transmitted data sequence.
  - Data Aided or Decision Directed TED
    - Uses decisions from the receiver, in this case the output of the Viterbi algorithm based detector.

Timing Error Detectors

- The TEDs used here are adaptations of existing TEDs developed for CPM by Mengali, Morelli and D'Andrea.
- They are based on maximum likelihood principles.
- SOQPSK needs a bank of three {-1, 0, +1} matched filters unlike customary CPM which only need M^L matched filters.



Non-data-aided TED



- h(k) is the impulse response of the filter derived by Mengali, Morelli and D'Andrea for CPM.
- First order PLL is used for updating the error signals.

Computing h[k] for SOQPSK

- The maximum likelihood function is defined as

$$\Lambda(r|\tilde{\alpha},\tilde{\theta},\tilde{\tau}) = e^{\frac{1}{N_0}\sqrt{\frac{E_s}{T_s}}\mathrm{Re}\left[e^{-j\tilde{\theta}}\int_0^{L_0T_s} r(t)e^{-j[2\pi\nu t+\phi(t-\tilde{\tau},\tilde{\alpha})]}dt\right]}$$

- Averaging this expression over the carrier phase results in an intermediate likelihood function on $\tilde{\alpha}$ and $\tilde{\tau}$
- On averaging this intermediate result over alpha yields

$$\Lambda(r|\tilde{\tau}) \approx \int_0^{L_0T_s}\int_0^{L_0T_s} r(t_1)r^*(t_2)e^{j2\pi\nu(t_2-t_1)}F(t_2-t_1,t_2-\tilde{\tau})dt_1dt_2$$

Computing the Expectation

$$F(\Delta t, t) \triangleq \mathrm{E}_{\tilde{\boldsymbol{\alpha}}}\left\{e^{j[\phi(t,\tilde{\boldsymbol{\alpha}})-\phi(t-\Delta t,\tilde{\boldsymbol{\alpha}})]}\right\}$$

$$F(\Delta t, t) = \mathrm{E}_{\tilde{\boldsymbol{\alpha}}}\left\{\prod_{i=-\infty}^{\infty} \exp\left[j2\pi h\tilde{\alpha}_i p(t-iT_s, \Delta t)\right]\right\}$$

$$p(t, \Delta t) \triangleq q(t) - q(t-\Delta t)$$

- The frequency pulse is non-zero only for a few values of '*i*' bounded by limits *K1* and *K2* resulting in finite number of data sequences of length $\Delta K = K2 - K1 + 1$.

$$F(\Delta t, t) = \mathrm{E}_{\tilde{\boldsymbol{\alpha}}}\left\{\prod_{i=K_1}^{K_2} \exp\left[j2\pi h\tilde{\alpha}_i p(t-iT_s, \Delta t)\right]\right\}$$

Computing the Expectation

- Now the problem of evaluating the expectation reduces to
  - Enumerating all possible data symbols of length ΔK
  - Attaching a probability distribution to these sequences. In the case of SOQPSK, it is uniform distribution.

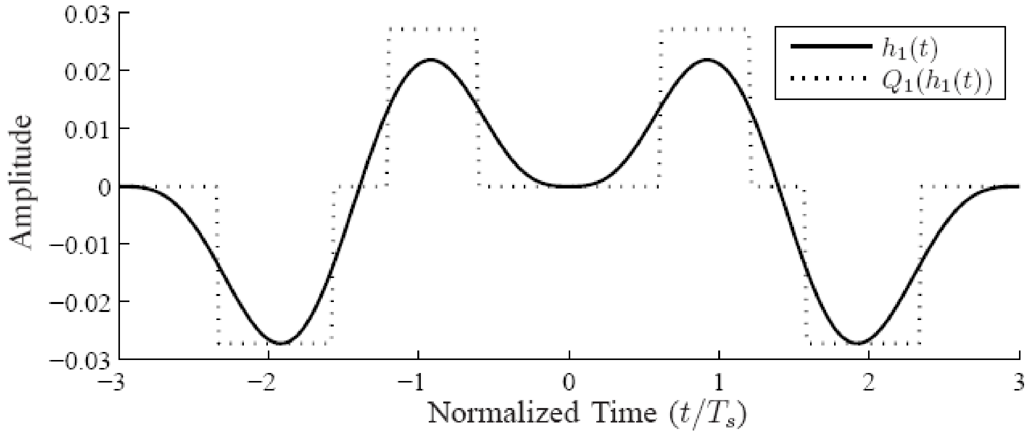
$$F(\Delta t, t) = \frac{1}{N_{\Delta K}} \sum_{(\tilde{S}, \tilde{\boldsymbol{u}})} \prod_{i=K_1}^{K_2} \exp\left[j2\pi h \tilde{\alpha}_i(\tilde{S}, \tilde{\boldsymbol{u}}) p(t - iT_s, \Delta t) \right]$$

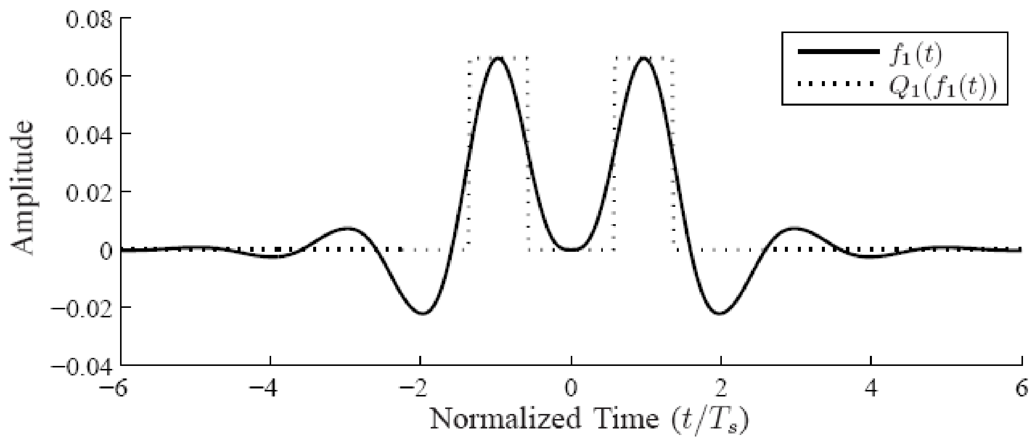
  where $N_{\Delta K} \triangleq 2^{\Delta K + 1}$.

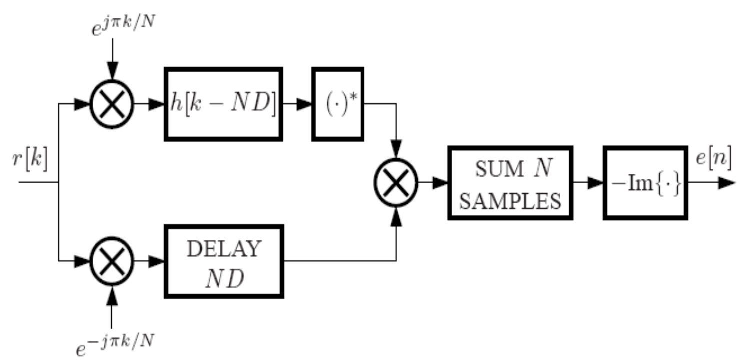
  - Using the Fourier series expansion the final impulse response is obtained as

$$h_m(t) \triangleq e^{j\pi m t/T_s} \frac{1}{T_s} \int_0^{T_s} F(-t, u) e^{j2\pi m u/T_s} du$$

Impulse response

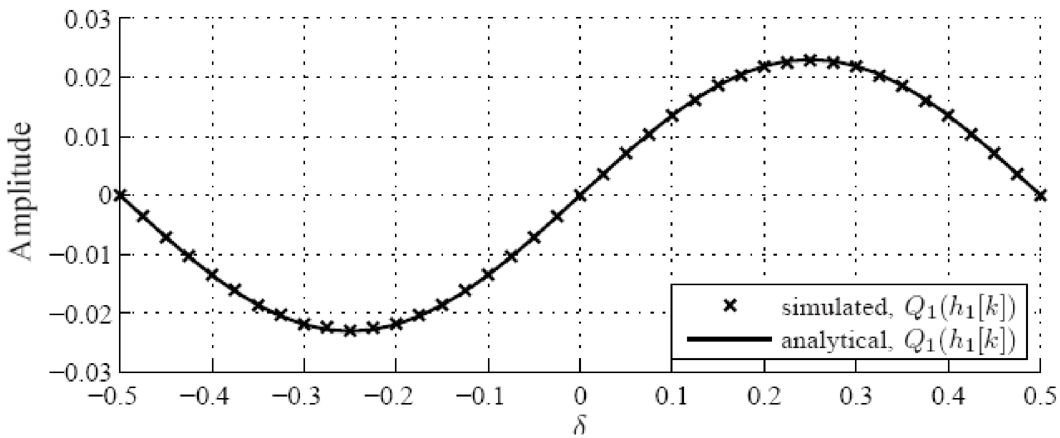




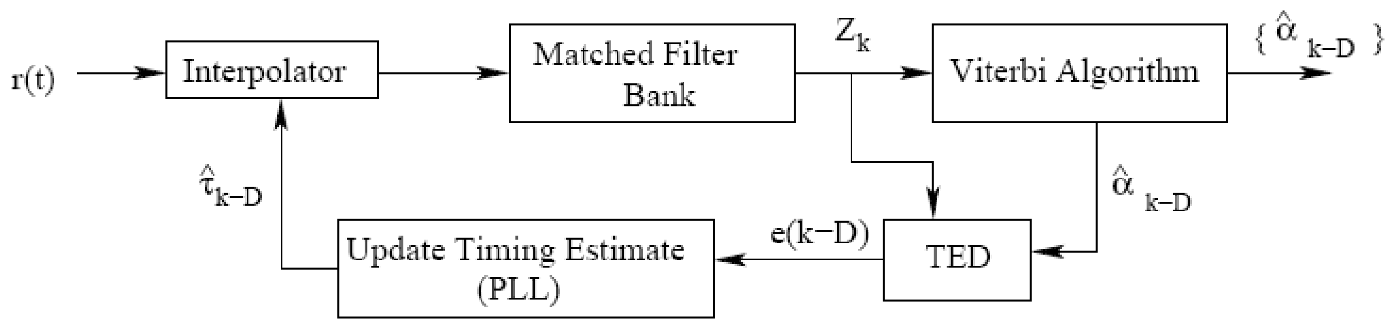


S-curve of the TED

- The S-curve identifies the stable lock points
  - These are the zero-crossing points on the curve where the slope is positive
  - We want such a point at zero error, any other such points are *false lock points*
  - The proposed TED is free of false lock points



Data-aided TED



- First order PLL is used for updating the timing estimates.
- The error signal obtained is

$$e(k-D) \triangleq \mathrm{Re}\left\{\boldsymbol{Y}_{k-D}(\boldsymbol{C}^b_{k-D}, \alpha^b_{k-D}, \hat{\tau}_{k-D})e^{-j\phi^b_{k-D}}\right\}$$

where D is the trace back time for computing the error and Y_k is the derivative of the matched filter output.

Error Signal

- Though the error signal has complicated-looking notation, it is rather simple to compute.
- The delay D in the system can be implemented by computing e(k) instantly and storing it, but using e(k-D) for updating the timing offset estimates in the PLL.
- It is worth noting that D=1 produces satisfactory results thus making the system rather simple.

S-curve of the TED

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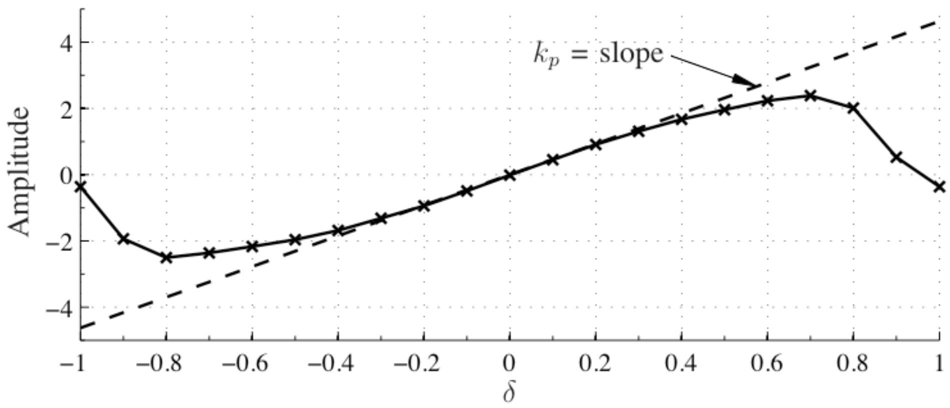


Numerical Results

- TED output refined into a stable timing estimate using a standard first-order PLL in a feedback loop.

$$\hat{\tau}[k] \triangleq \hat{\tau}[k-1] + \gamma e[k]$$

where the *step size* is

$$\gamma \triangleq \frac{4BT_s}{k_p}$$

- BT_s , the normalized loop bandwidth is an important parameter influencing the performance of the timing recovery system.

Normalized timing variance

- Accuracy of the feedback scheme measured numerically via simulation for the two versions of SOQPSK.

$$\frac{1}{T_s^2} \times \sigma_\tau^2 \triangleq \frac{1}{T_s^2} \times \mathrm{Var}\left\{\hat{\tau}[k] - \tau\right\}$$

- Simulations done for four cases:
  - SOQPSK-MIL with Loop BW $1\text{x}10^{-3}$ and $1\text{x}10^{-2}$.
  - SOQPSK-TG with Loop BW $1\text{x}10^{-3}$ and $1\text{x}10^{-2}$ along with PT.

Modified Cramer-Rao Bound

- MCRB establishes the lower bound on the accuracy of timing estimates.
- Autocorrelation of SOQPSK
  - Using the constraints imposed by the SOQPSK precoder, the autocorrelation function is found to be

$$R_{\alpha}(l) = \begin{cases} 1/2, & l = 0 \\ 1/4, & |l| = 1 \\ 0 & \text{otherwise.} \end{cases}$$

- Since $R_{\alpha}(l)=0$ for $l>1$ and $l<-1$,
  - MCRB obtained in closed-form result MIL-STD SOQPSK.
  - Computed numerically (but easily) for SOQPSK-TG.

MCRB's for the two versions

$$\frac{1}{T_s^2} \times \mathrm{MCRB}_{\mathrm{MIL}}(\tau) = \frac{4}{\pi^2 L_0} \times \frac{1}{E_s/N_0}$$

$$\frac{1}{T_s^2} \times \mathrm{MCRB}_{\mathrm{TG}}(\tau) = \frac{1}{2\pi^2 L_0 C_{\mathrm{TG}}} \times \frac{1}{E_s/N_0}$$

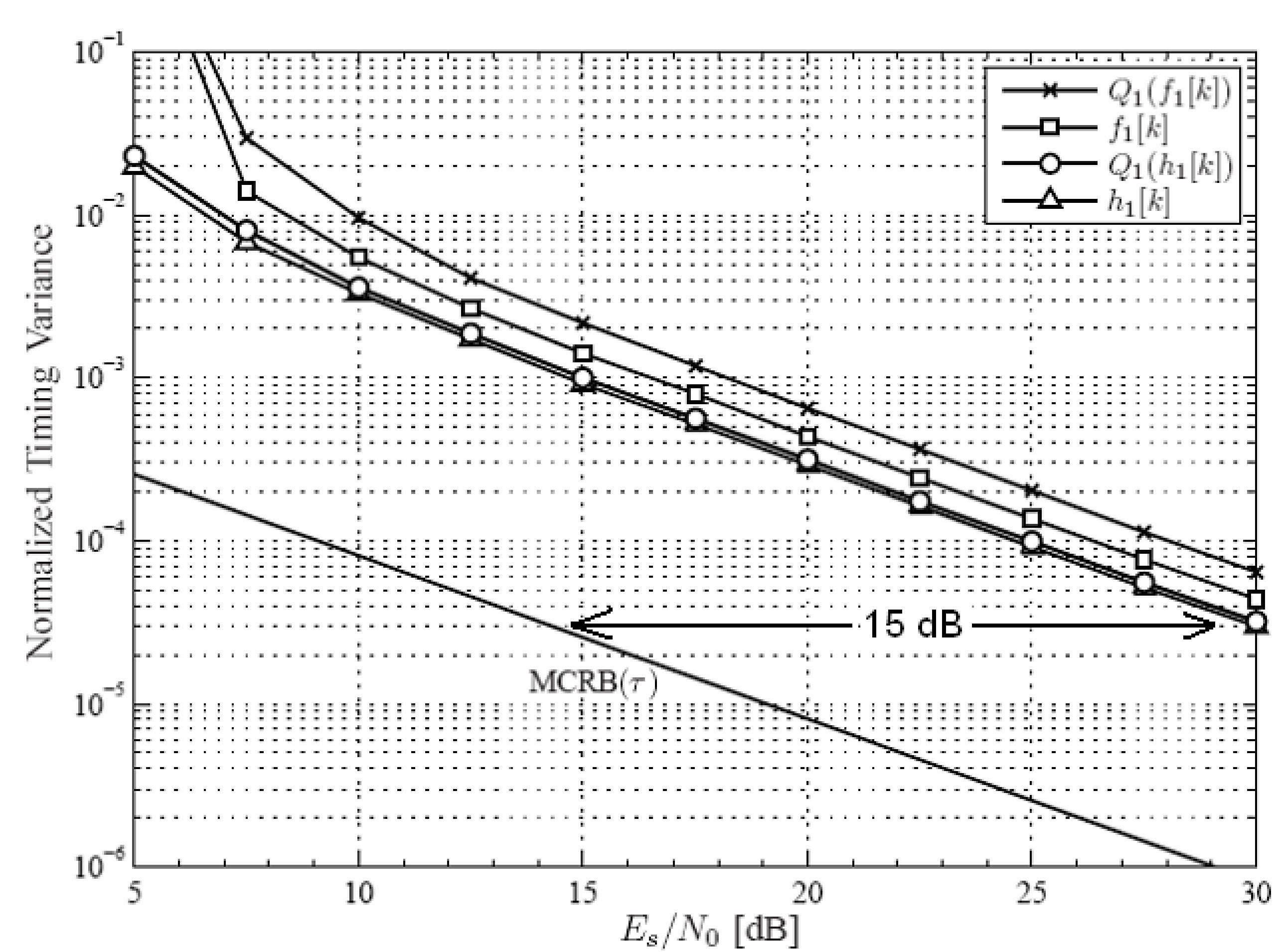
$$C_g \triangleq T_s \int_0^{T_s} G(t,\tau)\, dt$$

where $G_{\mathrm{TG}}(t,\tau)$ is given by

$$G_{\mathrm{TG}}(t,\tau) = \frac{1}{2}\sum_i g_{\mathrm{TG}}^2(t - iT_s - \tau) + \frac{1}{2}\sum_i g_{\mathrm{TG}}(t - iT_s - \tau) g_{\mathrm{TG}}(t - (i+1)T_s - \tau)$$

Simulation Results – NDA TED

- SOQPSK – MIL
- (M=2, L=1, h=1/2, REC)



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Simulation Results – NDA TED

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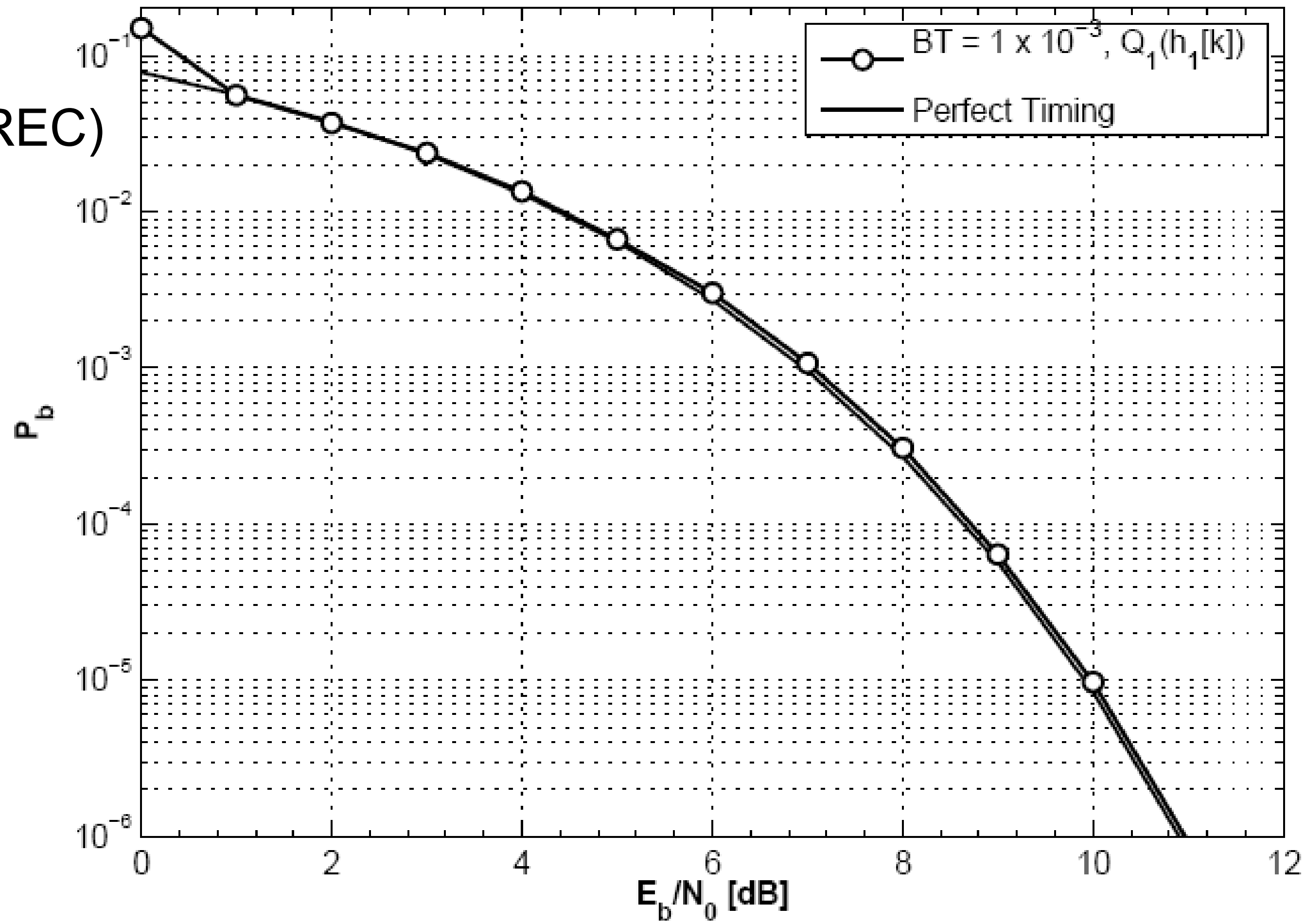
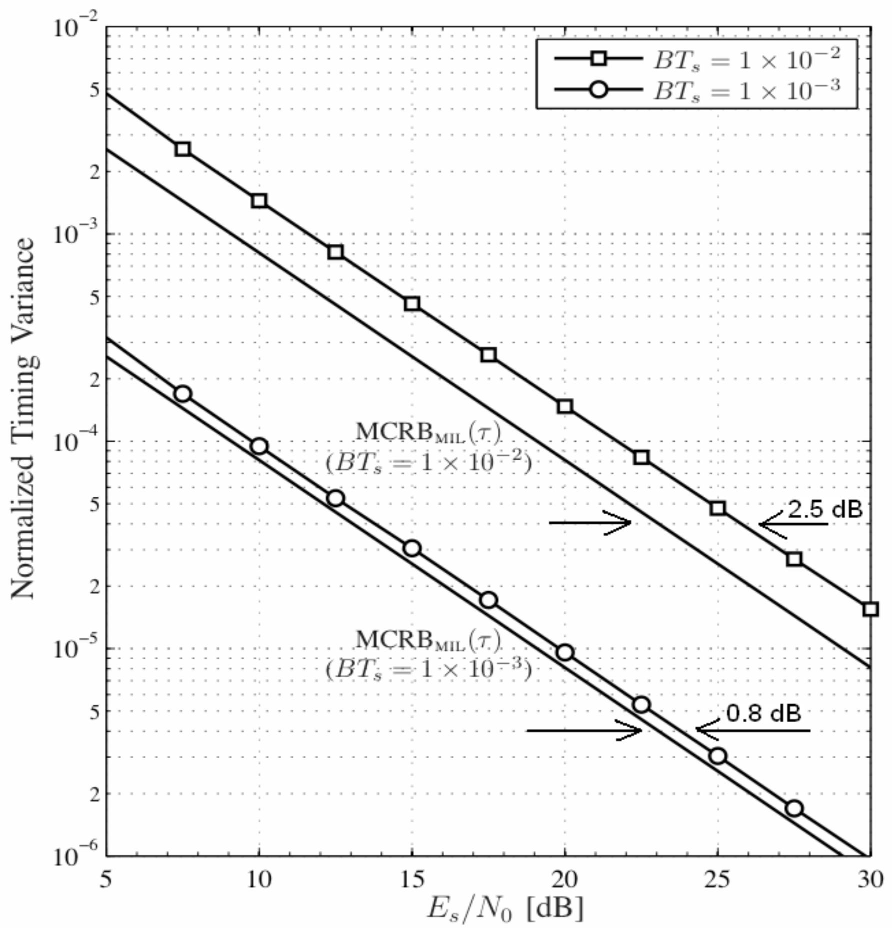
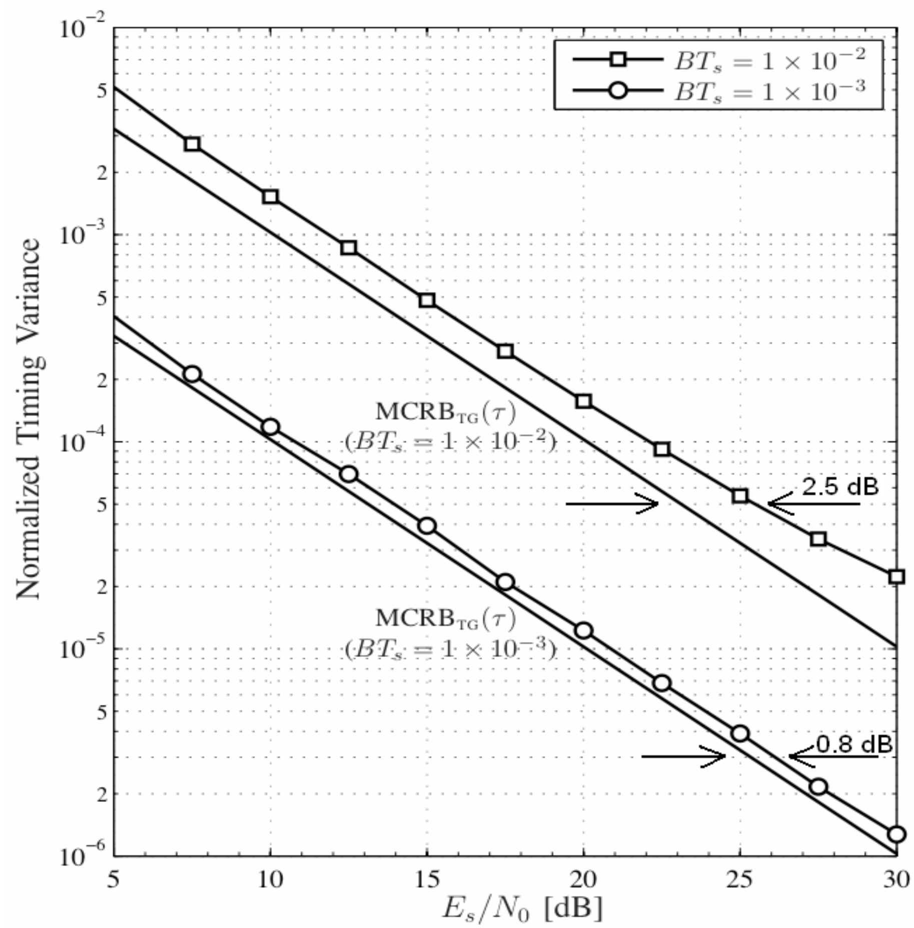


Simulation Results – DA TED

- **SOQPSK – MIL**
- **(M=2, L=1, h=1/2, REC)**



- **SOQPSK – TG**
- **(M=2, L=8, h=1/2)**



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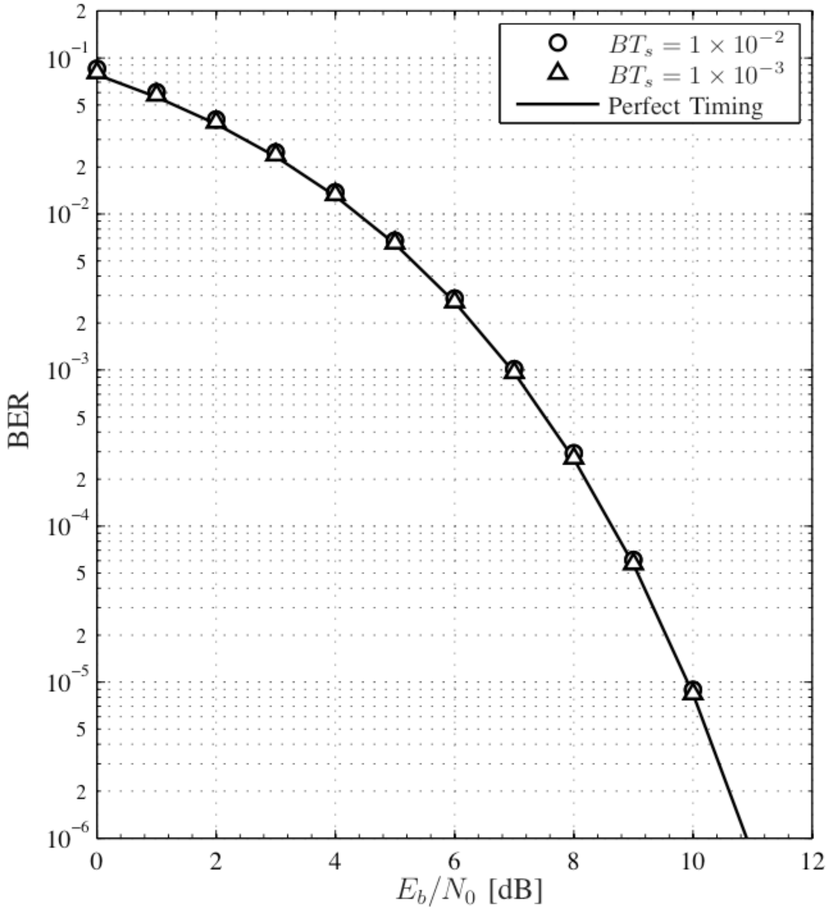
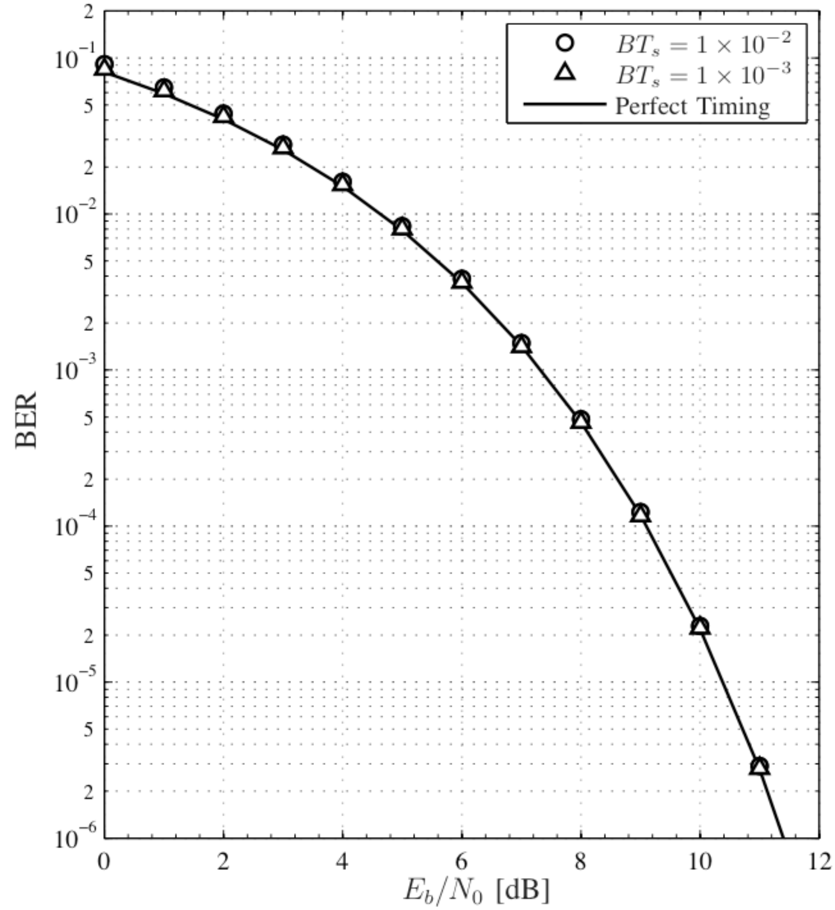
Simulation Results – DA TED

- **SOQPSK – MIL**
- **(M=2, L=1, h=1/2, REC)**



- **SOQPSK – TG**
- **(M=2, L=8, h=1/2)**



Summary of Numerical Results

- Normalized timing variance is close to the MCRB in all four cases of the data-aided TED and not so for the blind TED.
- The effectiveness of the blind TED after an extreme level of quantization is an important factor to notice though its performance in terms of normalized timing variance vs MCRB does not match that of a DA TED.
- In the case of SOQPSK-TG where PT technique used for complexity reduction, the DA TED's performance is noteworthy using suboptimal MF outputs. It allows good performance for a wider loop BW and hence rapid synchronization time.
- BER curves show that simulation points line up over the analytical curve for both the TEDs. Hence, the TEDs performs at the theoretical limit.
- To conclude we could say the decision directed TED is superior to its non-data aided counterpart in terms of normalized timing variance but is a little complicated to implement as it has to be incorporated into the Viterbi algorithm.

Conclusion

- Synchronization results validate the already proven effective CPM model for SOQPSK.
- The schemes are very advantageous due to the following reasons
  - Low complexity
  - Lack of false lock points
  - Fast acquisition time
- Results obtained in this work should provide a new outlook towards building CPM based receivers and synchronizers which help exploit the 1-2dB losses incurred because of using matched filters that are not completely matched to the transmitted symbols.

Thank You