



Technical Report

**Analytic Prediction of OFDM Performance
with a Covert Interferer**

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ITTC-FY2014-TR-71328-02

August 2013

Project Sponsor:
National Science Foundation

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Abstract— This paper addresses how to hide the transmission of information within the RF environment created by packet-based broadband wireless (infrastructure) networks, like LTE. The desire to hide the transmission of information has existed since antiquity; including concealing the very existence of transmissions, since exposing the presence of transmissions may expose the location of the sender or an increase in the frequency of transmissions may portend the occurrence of an event. To achieve high data rates current infrastructure networks typically use Orthogonal Frequency Division Multiplexing (OFDM) along with protocols that adapt to the environment, e.g., adaptive modulation and coding (AMC), Hybrid ARQ (HARQ), and opportunistic scheduling. It is expected that future systems will continue to use these methods in addition to advanced interference management procedures to accommodate heterogeneous networks (HetNets). The adaptive nature of the protocols used in packet-based broadband wireless networks makes them vulnerable to exploitation. The contribution of this work is the prediction of the bit error rate (BER) of an OFDM with a covert interferer in Nakagami- m fading; the performance of the covert interferer is also derived. The analytic model is used to show trade-offs between the OFDM system and covert link performance. These results extend our previous simulation study where fading was not considered. The results reported here provide the first step toward evaluating the impact in terms of reduced end-to-end throughput, and increased end-to-end delay and packet loss of the presence a covert link on an infrastructure network employing OFDM, AMC, HARQ and opportunistic scheduling.

Keywords—Orthogonal frequency division multiple access (OFDM), Nakagami- m fading, Interference, Covert Communication.

I. INTRODUCTION

This paper addresses how to hide the transmission of information within the RF environment created by packet-based broadband wireless (infrastructure) networks, like LTE [1]. The desire to hide the transmission of information has existed since antiquity [2], including hiding the very existence of the transmission [3] and [4]. Exposing the presence of a transmission may expose the location of the sender or an increase in the frequency of transmissions may portend the occurrence of an event. Thus encryption is not

sufficient to hide information transmission. Hiding information in content, e.g., audio or video, is usually called steganography while a covert channel [4] is usually associated with embedding information within network protocols. Or as noted in [4] the US DoD defines a covert channel as “[...] any communication channel that can be exploited by a process to transfer information in a manner that violates the system’s security policy” [5]. Using steganography or a covert channel requires the communications entities to be common system elements, e.g., hosts, on the target network, and the covert communication “rides” within normal transmissions. Low probability detection (LPD) communications systems have a long history of providing a mechanism for hiding radio transmission in the RF environment, i.e., in noise [6], [7], and [8]. The concept of embedding communications signals inside other emissions, e.g., radar backscatter, goes back to the 1940’s [9]; more recently covert communications using both cooperating and non-cooperating radar emitters has been reported [10]. In steganography or using a covert channel the sender is an element (or node) in the target network, while in traditional LPD systems the covert communicating entities are independent entities and are not usually part of the target network. The advent of packet-based broadband wireless networks provides the opportunity to create covert communications mechanisms that combine the attributes of LPD communications systems and covert channels by using knowledge of the underlying protocols to enhance the covert communications capability.

In [11] we used simulation to predict the performance of the covert link as well as the target OFDM infrastructure system. The BER of the target system as a function of the covert’s power, bandwidth and spectral location was presented. The BER of the covert link was also shown as a function of its spectral location. The contributions presented here are the analytic prediction of the covert link as well as the target OFDM infrastructure system BER performance in the presence of Nakagami- m fading and evaluation of the associated system trade-offs.

The assumptions of the analysis are presented next. In Section III the probability density function (pdf) for the instantaneous signal-to-noise ratio (SNR) with Nakagami- m fading at both the covert and target receiver is derived. Here

the SNR refers to the power of a sub-carrier in the OFDM system or covert link divided by the noise power. The modeling in Section III, i.e., treating the interfering signal as white noise, is validated using simulation in Section IV. The results are then used in Section V to characterize the BER performance of the covert and target receiver and associated system trade-offs. In Section VI conclusions and next steps are discussed to evaluate the end-to-end impact in terms of reduced end-to-end throughput, and increased end-to-end delay and packet loss in the presence a covert link on an infrastructure network operating with OFDM, AMC, HARQ and opportunistic scheduling.

II. ASSUMPTIONS

As in [11] there are two sets of transmitters/receivers; a transmitter and receiver for the target communication system and a transmitter and receiver for the covert communication link. There are four channels that must be represented in the model as shown in Fig. 1. In this paper we assume Nakagami- m fading for all the channels with a common Nakagami- m fading parameter of m . Flat fading is assumed.

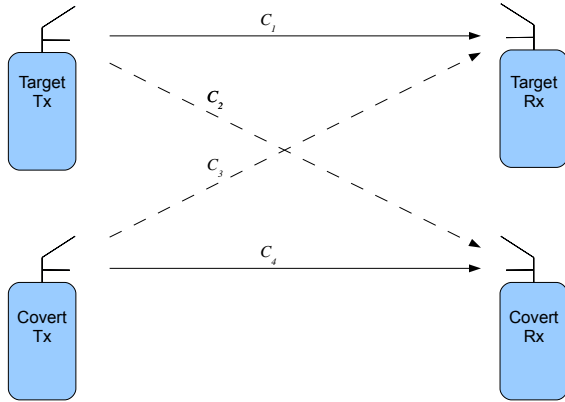


Figure 1: Target and Covert Channels.

The modulation on each subcarrier of the target OFDM signals is 16-QAM with rectangular pulse shaping. Here the covert link uses BPSK or QPSK. Un-coded transmission is assumed. Optimum receiver structures are utilized; perfect channel state information (CSI) for each transmitter/receiver pair and ideal phase and frequency synchronization is assumed. While the analysis framework developed here is more general, for this paper we assume that the covert signal is placed in frequency in the guard band adjacent to the target OFDM signal. The target OFDM and covert systems are not in synchronization; thus the covert and target signals are not orthogonal. To facilitate the analysis it is assumed that the interference generated by the undesired signal (the covert signal in the case of the target receiver and the target signal in the case of the covert receiver) can be modeled by white Gaussian noise with a noise power equal to the in-band interference power. A similar approach was used in [12] and [13]. We define in-band power as the power within the zero-to-zero crossings of the signal spectrum, here an $\sim \text{sinc}^2(f/BW)$. The impact of modeling the interfering signal

as white noise for this environment is discussed in this paper. While in the analysis presented here the covert signal is placed in the guard band of the OFDM system, the approach is more general and can be applied to other covert waveforms, e.g., time or frequency hopping where the LPD performance for the covert link can be improved.

III. PROBABILITY DENSITY FUNCTION FOR SNR AND BER CALCULATION

A. General Case

The pdf for the instantaneous SNR is used along with knowledge of specific link modulation to calculate the BER [14]. In this case the instantaneous SNR at the desired receiver (covert or target) is

$$\gamma = \frac{\alpha^2 E_s}{N_o + \beta^2 N_i}. \quad (1)$$

Here E_s is the average energy per symbol of the desired signal. Assuming that the interferer is modeled as white Gaussian noise, N_i is the in-band interference. N_o models the white noise. The channel fading amplitude for the desired signal and interfering signals are α and β respectively. The instantaneous SNR can be written as

$$\begin{aligned} \gamma &= \frac{\left(\frac{E_s}{N_o}\right) \alpha^2}{1 + \left(\frac{N_i}{N_o}\right) \beta^2} \\ &= \frac{k_1 * W}{1 + k_2 * R}, \end{aligned} \quad (2)$$

where $k_1 = \frac{E_s}{N_o}$ and $k_2 = \frac{N_i}{N_o}$ is the interference-to-noise ratio (or jammer-to-noise ratio-JNR). To calculate the BER requires calculation of $\frac{N_i}{N_o}$ using the spectral location and modulation of the desired signal and interfering signals and determination of the pdf for the instantaneous SNR, γ .

First, the pdf of γ will be found for Nakagami- m fading channels. Let $W = \alpha^2$ and $R = \beta^2$. For Nakagami- m fading channels the pdfs for W and R are well known [14],

$$f_W(w) = \frac{m^m w^{m-1} \exp(-mw)}{\Gamma(m)} \quad (3)$$

and

$$f_R(r) = \frac{m^m r^{m-1} \exp(-mr)}{\Gamma(m)}, \quad (4)$$

where $W > 0$, $R > 0$ and m is the Nakagami- m fading parameter. Now let

$$\gamma = \frac{U}{V}, \quad (5)$$

with

$$\begin{aligned} U &= k_1 W & U > 0 \\ V &= 1 + k_2 R & V > 1. \end{aligned}$$

Now

$$f_U(u) = \frac{f_W\left(\frac{u}{k_1}\right)}{k_1} \quad U > 0 \quad \text{and} \quad f_V(v) = \frac{f_R\left(\frac{v-1}{k_2}\right)}{k_2} \quad V > 1.$$

Using the standard transformation of random variables, the pdf for the instantaneous SNR at the desired receiver is found as

$$\begin{aligned} f_Y(\gamma) &= \int_1^\infty v f_V(v) f_U(v\gamma) dv \\ &= \frac{1}{2\sqrt{\pi}\gamma \Gamma(m)} m^{2m} \left(\frac{1}{k_2}\right)^m \left(\frac{\gamma}{k_1}\right)^m e^{-\frac{m(k_1+k_2\gamma)}{2k_1k_2}} \\ &\quad \left(m \left(\frac{\gamma}{k_1} + \frac{1}{k_2}\right) \right)^{\frac{1}{2}-m} \left(K_{\frac{1}{2}-m} \left(\frac{m(k_1+k_2\gamma)}{2k_1k_2} \right) \right. \\ &\quad \left. + K_{-m-\frac{1}{2}} \left(\frac{m(k_1+k_2\gamma)}{2k_1k_2} \right) \right), \end{aligned} \quad (6)$$

where K_k is the modified Bessel function of the second kind [15] (this integral was solved using Mathematica [23]). For Raleigh fading ($m = 1$) the $f_Y(\gamma)$ reduces to

$$\frac{e^{-\frac{\gamma}{k_1}(k_1k_2 + k_1 + k_2\gamma)}}{(k_1 + k_2\gamma)^2}. \quad (7)$$

The BER for M-QAM given an SNR of γ is needed to calculate the average BER. Here we use Equation 8-15 from [14] that improves the prediction for both low and high SNR regimes:

$$\begin{aligned} \text{BER}(\gamma) &= \frac{4(\sqrt{M}-1) \sum_{i=0}^{\frac{\sqrt{M}-1}{2}-1} Q\left((2i+1)\sqrt{\frac{3\gamma}{M-1}}\right)}{\sqrt{M} \log_2(M)}. \end{aligned} \quad (8)$$

Now the BER can be found from

$$\text{BER} = \int_0^\infty \text{BER}(\gamma) f_Y(\gamma) d\gamma. \quad (9)$$

While we have not found a closed form solution for the above integral, the BER can be found by numerical integration. Fig. 2 shows the BER performance of a 16-QAM link as a function of its SNR with a small covert link power ($\text{SNR}_{\text{Covert}} = -6$ dB) under three channel conditions, $m = 0.75, 1.25$ and 1.75 .

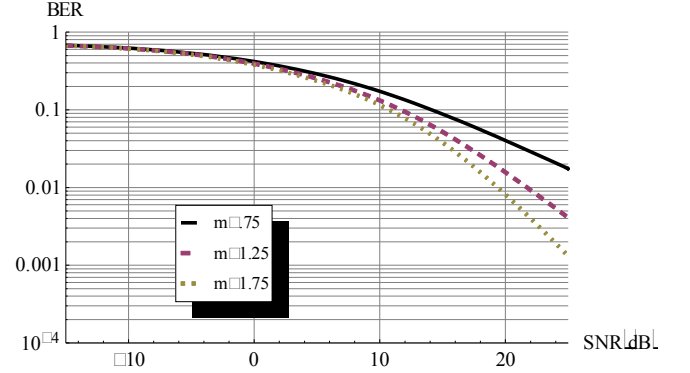


Figure 2: 16-QAM Link with Nakagami- m Fading: $m = .75, 1.25, 1.75$ and Interference: $\text{SNR}_{\text{Covert}} = -6$ dB.

The performance shown in Fig. 2 compares favorably to the BER without interference given in [16] for $m = 0.75, 1.25$ and 1.75 and 16-QAM modulation. Fig. 3 shows the performance for a 6 dB $\text{SNR}_{\text{Covert}}$. As expected, the performance improves as m increases.

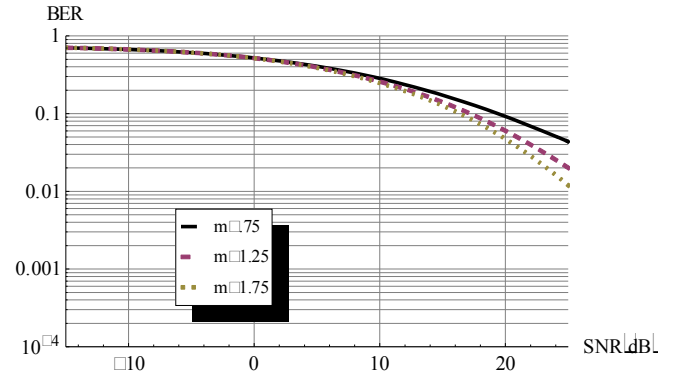


Figure 3: 16-QAM Link with Nakagami- m Fading: $m = .75, 1.25, 1.75$ and Interference: $\text{SNR}_{\text{Covert}} = 6$ dB.

B. BER of Target OFDM System with Covert Link

Consider the case shown in Fig. 4 where only three OFDM subcarriers (-1, 0, 1) are shown. Each subcarrier has a normalized power of 1 and bandwidth of 1; the covert is

operating at 1/2 of the bit rate of each sub-carrier and has a normalized power of 0.1. Here the bandwidth of the covert is 1. Note the covert signal will impact each subcarrier differently. In this figure the covert signal is 10 dB below the target, however we will show later that the covert link can operate at ~21 dB below the OFDM sub-carrier power. For the sub-carrier $k = 1$ the normalized in-band interference power is

$$\rho_1 = \int_0^2 \text{sinc}^2(\pi(f-2)) df, \quad (10)$$

and in this case $N_I = .1 \rho_1$. Then to calculate the BER in general for each subcarrier, k , a different normalization factor is needed and can be found by

$$\rho_k = \frac{1}{B_c} \int_{f_k - \frac{f_s}{2}}^{f_k + \frac{f_s}{2}} \text{sinc}^2\left(\pi \frac{f - kf_s}{B_c}\right) df, \quad (11)$$

where f_s is the subcarrier spacing (in this case $f_s = 1$) and B_c is the bandwidth of the covert signal. With a covert signal power of P_c the in-band interference for subcarrier k is $N_k = \rho_k P_c$. The BER for the k^{th} subcarrier is defined as BER_k and can be found using (9) with $N_I = N_k = \rho_k P_c$. Note each subcarrier can have a different modulation and bit rate, r_k . The total BER for the target OFDM signal is given by

$$\text{BER} = \frac{\sum_{k=-N}^N r_k \text{BER}_k}{\sum_{k=-N}^N r_k} \quad (12)$$

However for the cases of interest the covert signal will impact only a few adjacent subcarriers.

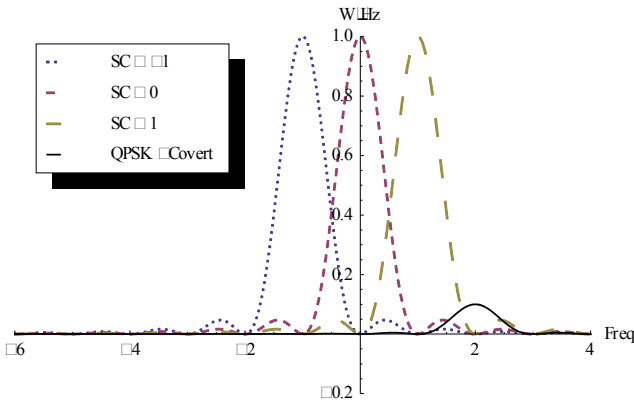


Figure 4: PSD of Target OFDM System and Covert Link.

C. BER Rate Performance of Covert Interferer with Target OFDM System

Again referring to Fig. 4, the total interference the covert receiver sees is

$$N_T = \int_{-1/4}^{1/4} (\text{sinc}[\pi * (f + 1)]^2 + \text{sinc}[\pi * f]^2 + \text{sinc}[\pi * (f - 1)]^2) df. \quad (13)$$

In general

$$N_T = \sum_{k=-N}^N \frac{P_k}{f_s} \int_{-(N+1)f_s - \frac{B_c}{2}}^{-(N+1)f_s + \frac{B_c}{2}} \text{sinc}^2\left(\pi \frac{f - kf_s}{f_s}\right) df, \quad (14)$$

where P_k is the power in the k^{th} sub-carrier. The BER of the covert link can be found utilizing (9) with $N_I = N_T$. Again, for cases of interest only the first few sub-carriers of the OFDM system will be considered in impacting the covert link.

IV. MODEL VALIDATION

This section quantifies the impact of modeling the interfering signal as white noise with a noise equivalent to the in-band interference power. Parameters are selected and the BER of the OFDM target system and covert link is calculated utilizing (7) through (12). These same parameters are utilized in simulation. The covert link is placed in the guard band of the OFDM signal. Here we only take into account the impact of the first three OFDM sub-carriers on the covert link and the impact of the covert link on the first three OFDM sub-carriers. This is due to the fact that the impact of the systems on one another is significantly reduced as the distance in the spectrum increases.

The OFDM system utilizes 16-QAM. Rayleigh fading ($m = 1$) and additive white Gaussian noise (AWGN) are assumed. The OFDM sub-carriers SNR are varied from -15 to 27 dB. The covert link utilizes QPSK signaling. Both systems are un-coded.

Fig. 5 compares the analytic results to the simulated results for a covert link SNR of 0, 10, and 20 dB. An analytic curve is included for when no covert is present as a baseline comparison. The simulated curves follow the analytic curves. It is to be noted that the analytic curves are more conservative than the simulated ones at all points. This means we can expect better performance than predicted by the analysis.

Fig. 6 compares analytic results to simulated results for the target OFDM system and covert link. Here we can see the tradeoff in performance of the OFDM system for covert system as the covert link SNR increases. The performance of both predicted by the analysis is an adequate and a conservative estimate.

The difference between the simulated and analytic results shows that white noise can be used to approximate the impact of in-band power of interference power. It also

shows that this estimate conservatively predicts the associated performance.

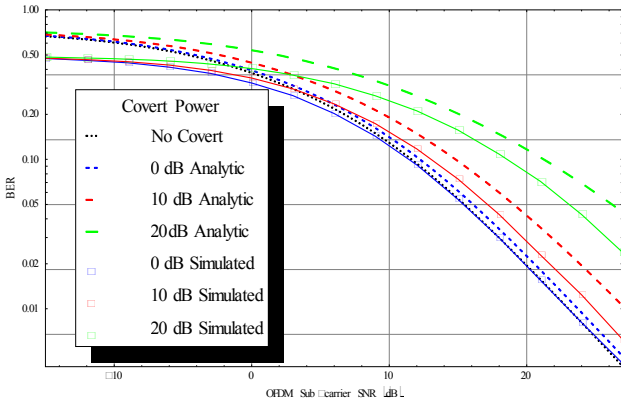


Figure 5: 16-QAM OFDM System with QPSK Covert Link.

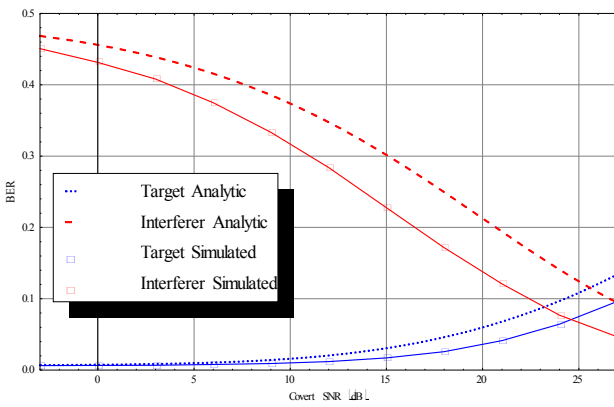


Figure 6: Performance of OFDM System and Covert Link with SNR of the OFDM link = 25 dB.

V. SYSTEM PERFORMANCE AND TRADEOFFS

As the SNR of the covert increases the negative impact on the OFDM system increases but the performance of the covert link improves; this section explores this trade-off. A goal of this trade-off analysis is to determine an operating point for the covert link, that is, its data rate, r_{bc} , and transmit power such that the impact on the OFDM system is “small”. Another dimension of this problem is the data rate of the covert link. With fixed power, as the data rate decreases the covert link performance increases. There is negligible change in impact on sub-carrier k of the OFDM system when the modulation of the covert link is changed. In these results the OFDM system uses 16-QAM on each sub-carrier and the SNR was set such that the $BER = 5 \times 10^{-3}$ in Rayleigh fading with no interference. So when the covert system uses QPSK its data rate is $\frac{1}{2}$ the data rate per sub-carrier in the OFDM system. When BPSK with the same symbol time as the 16-QAM OFDM sub-carrier is utilized the covert data rate is $\frac{1}{4}$ the rate of an OFDM sub-carrier. For a subcarrier bandwidth of 15 kHz with 16-QAM the data rate is 30 kb/s and with BPSK the covert link has a data rate of

7.5 kb/s. The data rate of the covert link can be further reduced by increasing its symbol time; this improves its BER performance while not increasing its influence on the target OFDM system. Let r_b be the OFDM per subcarrier bit rate Figure 7 and 8 shows the system trade-off for covert link bit rates, r_{bc} , ranging from $r_b/2$ to $r_b/32$.

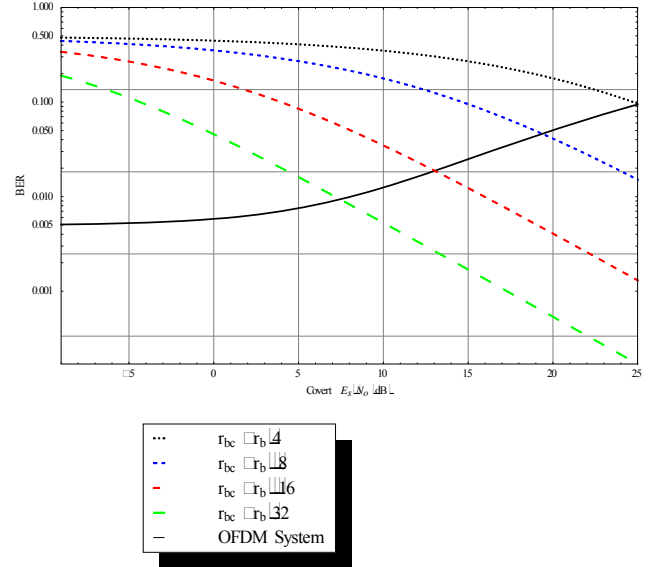


Figure 7: 16-QAM OFDM System with SNR = 26.15 dB and BPSK Covert with Variable Bit Rate.

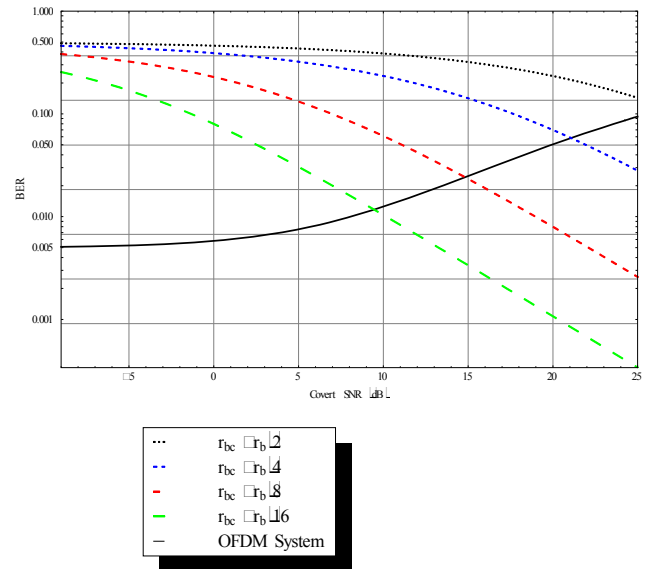


Figure 8: 16-QAM OFDM System with SNR = 26.15 dB and QPSK Covert with Variable Bit Rate.

In order to see the impact of changing the channel fading characteristics, Fig. 9 shows the trade-off between covert and OFDM system performance for different values of the fading parameter m . Measurement of channel characteristics at 900 MHz in an urban environment were reported in [17]

where the most likely values of m ranged from 0.5 to 3.5 with an average of $m=1.56$. While in [18] $m=2.38$ was reported at 870.9MHz in an urban environment. Fig. 9 shows the performance trade-off for BPSK with $r_{bc} = r_b/32$ and Nakagami- m fading for $m=0.5, 1, 1.56$ and 2.38 . The symbol rate of the covert link is $1/8^{\text{th}}$ of the target OFDM system which results in an overall bit rate for the covert link that is $1/32^{\text{nd}}$ of an OFDM sub-carrier. The BER performance is improved as the Nakagami- m fading parameter increases as expected.

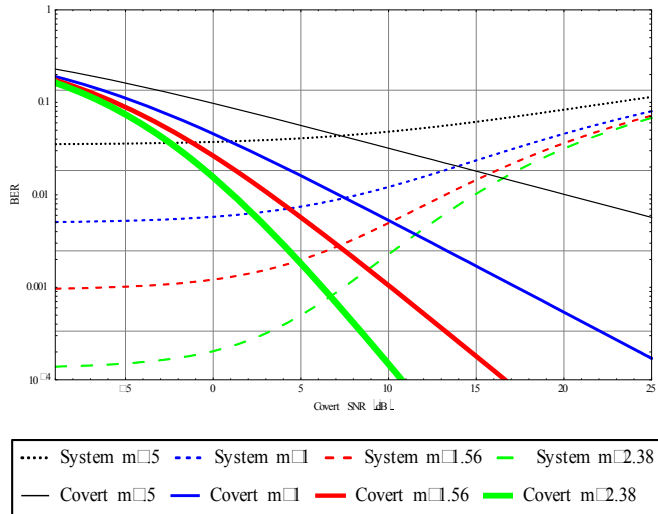


Figure 9: 16-QAM OFDM System with SNR = 26.15 and BPSK Covert with Variable m , $r_{bc} = r_b/32$.

There is a small impact on the OFDM system when the SNR of the covert link operates at approximately 5 dB. The covert link also performs well, achieving a BER $< 10^{-3}$, for $r_{bc} = r_b/32$ operating at 5 dB SNR. Here the covert signal is approximately 21 dB below the OFDM sub-carrier power. It can be seen from Fig. 8 that the increase in BER for the OFDM system could easily be attributed to fading. For example, the increase in BER for the OFDM system is greater when the m parameter decreases from 2.38 to 1.56 than when covert link is present with a SNR ratio of 5 dB.

VI. CONCLUSIONS AND FUTURE WORK

This work developed and validated an analytic model that characterizes the interaction between the target OFDM system and the covert link in Nakagami- m fading. Specifically, the BER was determined for an OFDM system with a covert interferer as well as the performance of the covert link in Nakagami- m fading. The analytic model developed here was used to illustrate trade-offs between the OFDM system and covert link performance under different channel conditions, i.e., different m . For the cases discussed here, a covert link utilizing BPSK in Nakagami- m fading is feasible. That is, low bit rate communications is possible over the covert link with the insignificant degradation caused by the covert link on the infrastructure system; this degradation likely being handled by AMC, HARQ, and opportunistic scheduling. Future work is needed to quantify and demonstrate how AMC, HARQ, and opportunistic scheduling in the target system effectively conceal this

degradation. To characterize the interactions between these systems work is needed to determine the end-user experience, thus the MAC, data link, and transport layers need to be considered. The probability density function for the instantaneous SNR derived here and given in (6) is a required first step in such an analysis and will play an important role in determining the influence of the covert transmitter. This probability density function used in conjunction with the analysis in [19-22] can determine the packet level effects. In the analysis reported here the covert transmitter is hidden in the guard band. Additional research is needed to evaluate the LPD properties of the covert transmitter. Note the covert signal could be time or frequency hopped over the RF bandwidth of broadband wireless infrastructure system to improve its LPD performance; research on the design of covert signal waveforms with good LPD properties to operate in the spectrum of broadband wireless infrastructure systems is needed.

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