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# Probability Density Function of SINR in Nakagami-*m* Fading with Different Channels

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***Abstract*— This letter develops probability density functions (pdfs) for the instantaneous received signal-to-interference plus noise ratio (SINR) in Nakagami-*m* fading channels where the target and interfering channels have different fading parameters. Separate pdfs are presented for integer and non-integer values of *m* and when the channels have the same fading parameter. These results are then applied to finding the average bit-error-rate (BER) for an M-QAM target link. This work shows the impact of changes in the channel parameter of the interfering signal on the BER of the target system.**

***Index Terms*— Nakagami fading, probability density function (pdf), quotient distribution, QAM performance.**

## I. Introduction

Adjacent, co-channel, and intentional interference are common problems in communication systems. The probability density function (pdf) of the instantaneous signal-to-noise ratio with interference present (SINR) combined with an expression for the bit error rate (BER) is often used in predicting system performance. The goal of this work is to develop pdfs for the SINR that take into account the presence of interference when target and interfering channels have different fading parameters, specifically, for different *m* for Nakagami-*m* fading channels. Much work has been done on fading channels [1] and interference analysis. The work in [2] addresses interference limited systems but does not account for noise. The work in [3] finds a bit-error-rate (BER) for BPSK systems in Nakagami fading utilizing a characteristic function method. The characteristic function method is also utilized in [4] to calculate the outage probability. The work in [5] utilizes convergent Fourier series method to derive analytic results for average BER in partially coherent BPSK. The work in [6] and [7] calculates the BER for MPSK and M-QAM with interference for Rayleigh, Rician, and Nakagami fading. Here closed form pdfs for the SINR of a signal in Nakagami-*m* fading with interference from a signal that is also subject to an independent Nakagami fading channel with different channel parameters is developed. The resulting pdfs are applied to determine the average BER of a 16-QAM communication system. The main approximation made in finding this solution lies in modeling the in-band interference as additional additive white Gaussian noise (AWGN) with an equivalent power. Similar assumptions are used in the analysis given in [5-9]. In [7] only a Rayleigh interference channel is considered. The next section of this letter presents the model for the SINR. In Section III we derive the pdfs. Section IV uses the derived pdf to predict system M-QAM performance and Section V is the conclusion.

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## II. Modeling

The pdf of a signal's instantaneous SNR in Nakagami-*m* fading is well known and given as [1]

$$f_\gamma(\gamma) = \frac{m^m \gamma^{m-1} e^{-\frac{m\gamma}{\bar{\gamma}}}}{\bar{\gamma}^m \Gamma(m)}. \tag{1}$$

With interference the received SINR at the receiver is modeled by

$$\gamma = \frac{\alpha^2 E_s}{N_o + \beta^2 N_I}, \tag{2}$$

where $E_s$ is the average transmitted symbol energy, $N_0$ is the noise power, and $N_I$ is the amount of power from the interfering signal in the bandwidth of the target system which is modeled as white Gaussian noise. The random variables α and β represent the channel gain due to Nakagami fading for the target and interfering signals, respectively. The power from the interfering signal that exists in the pass-band of target signal's receiver is needed in this analysis. It is assumed here that the impact of the interference on the target link can be found through modeling the interference as white Gaussian noise with an equivalent amount of interference power as was used in [7-10]. This model is general and can be applied to a wide range of interference including co-channel, adjacent channel, and intentional interference conditions. In [10] we showed that for Rayleigh fading this approximation yields a conservative estimate of the BER on an M-QAM target link in the presence of interference generated by a QPSK signal.

## III. PROBABILITY DENSITY FUNCTIONS OF SIGNAL TO INTERFERENCE PLUS NOISE RATIO

After some rearranging, (2) becomes

$$\gamma = \frac{\left(\frac{E_S}{N_O}\right)\alpha^2}{1+\left(\frac{N_I}{N_O}\right)\beta^2} = \frac{\bar{\gamma} * W}{1+\bar{\gamma}_I * R}, \quad (3)$$

where $\bar{\gamma}_I = \frac{N_I}{N_o}$ is the interference-to-noise ratio. For Nakagami-*m* fading the pdfs for *W* and *R* are given by

$$f_W(w) = \frac{m^m w^{m-1} e^{-mw}}{\Gamma(m)} \quad (4)$$

and

$$f_R(r) = \frac{m_I^{m_I} r^{m_I-1} e^{-m_I r}}{\Gamma(m_I)}, \quad (5)$$

where $W > 0$, $R > 0$ and $m$ is the Nakagami-*m* fading parameter for the target channel while $m_I$ is for the interfering channel.

Finding the pdf of $\gamma$ requires deriving the distribution of the quotient of two random variables [11]. Now let

$$\gamma = \frac{U}{V},$$

with

$$U = \bar{\gamma}\, W \quad U > 0 \;\; and \; V = 1 + \bar{\gamma}_I R \quad V > 1.$$

Now

$$f_U(u) = \frac{f_W\left(\frac{u}{\bar{\gamma}}\right)}{\bar{\gamma}} \text{ and } f_V(v) = \frac{f_R\left(\frac{v}{\bar{\gamma}_I} - \frac{1}{\bar{\gamma}_I}\right)}{\bar{\gamma}_I}.$$

Using the quotient distribution of two random variables, the pdf for the instantaneous SINR at the desired receiver is given by (6). The function ${}_1F_1(\mathrm{a}; b; z)$ is the confluent hypergeometric function. However, when b is a negative integer, ${}_1F_1(\mathrm{a}; b; z)$ is undefined (also $\Gamma(-m)$ is unbounded for integer m) [12]. Therefore, $f_\gamma(\gamma;\, \bar{\gamma}, \bar{\gamma}_I, m, m_I)$ given in equation (6) is not defined when $m$ and $m_I$ are simultaneously integers. For the integer $m$ and $m_I$ case the above integral is solved separately with the resulting pdf given by (7). Solving this integral when $m_I = m$ yields a pdf for the instantaneous SNR (as presented in [10]), where $K_k$ is the modified Bessel function of the second kind [13] and is given by (8).

$$f_\gamma(\gamma;\, \bar{\gamma}, \bar{\gamma}_I, m, m_I) = \int_1^\infty v f_V(v) f_U(v\gamma)\, dv =$$
$$\left(\frac{m^m \frac{\gamma^{m-1}}{\bar{\gamma}^m}}{\Gamma(m)} \frac{m_I{}^{m_I} e^{\frac{m_I}{\bar{\gamma}_I}} \left(\frac{1}{\bar{\gamma}_I}\right)^{m_I}}{\Gamma(m_I)}\right)\left(\Gamma(m+m_I)\left(\frac{m_I}{\bar{\gamma}_I} + \frac{\gamma m}{\bar{\gamma}}\right)^{-m_I-m} {}_1F_1\left(1-m_I; -m-m_I+1; -\frac{m\gamma}{\bar{\gamma}} - \frac{m_I}{\bar{\gamma}_I}\right) + \frac{1}{\Gamma(-m)}\Gamma(-m - m_I)\Gamma(m_I)\; {}_1F_1\left(m+1; m+m_I+1; -\frac{m\gamma}{\bar{\gamma}} - \frac{m_I}{\bar{\gamma}_I}\right)\right) \quad (6)$$

$$\left(\frac{m^m m_I{}^{m_I} e^{\frac{-m*\gamma}{\bar{\gamma}}} \gamma^{m-1} \bar{\gamma}^{m_I} \bar{\gamma}_I{}^m}{(m-1)!\,(m_I\,\bar{\gamma} + m\gamma * \bar{\gamma}_I)^{m+m_I}}\right)\left(\sum_{n=0}^{m}\left(\frac{m!}{n!\,(m-n)!}\right)\left(\frac{(m_I + m - n - 1)!}{(m_I - 1)!}\right)\left(\frac{(m_I\,\bar{\gamma} + m\gamma\bar{\gamma}_I)^n}{(\bar{\gamma}\bar{\gamma}_I)^n}\right)\right) \quad (7)$$

$$f_\gamma(\gamma;\, \bar{\gamma}, \bar{\gamma}_I, m) = \frac{1}{2\sqrt{\pi}\gamma\, \Gamma(m)} m^{2m} \left(\frac{1}{\bar{\gamma}_I}\right)^m \left(\frac{\gamma}{\bar{\gamma}}\right)^m e^{\frac{m(\bar{\gamma}-\bar{\gamma}_I\gamma)}{2\bar{\gamma}\,\bar{\gamma}_I}}$$
$$\left(m\left(\frac{\gamma}{\bar{\gamma}} + \frac{1}{\bar{\gamma}_I}\right)\right)^{\frac{1}{2}-m} \left(K_{\frac{1}{2}-m}\left(\frac{m(\bar{\gamma}+\bar{\gamma}_I\gamma)}{2\bar{\gamma}\,\bar{\gamma}_I}\right) + K_{-m-\frac{1}{2}}\left(\frac{m(\bar{\gamma}+\bar{\gamma}_I\gamma)}{2\bar{\gamma}\bar{\gamma}_I}\right)\right) \quad (8)$$

Fig. 1a shows the pdf and Fig. 1b the cumulative distribution function for the SINR. As the channel parameter *m* surpasses 1 the characteristic shape of the pdf for the SINR changes.

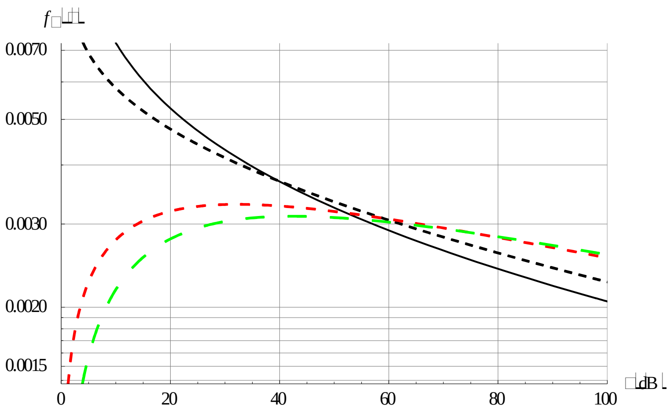


Figure 1a: PDF of SINR $\bar{\gamma}$=30 dB, $\bar{\gamma}_I$=6 dB, $m_I$ =0.8.

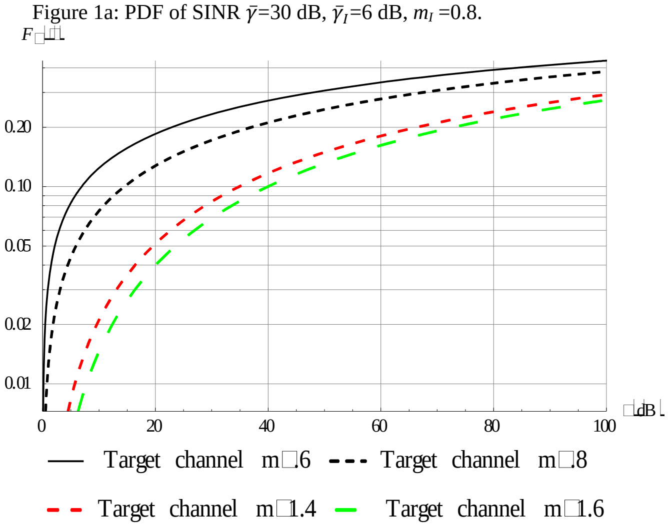


Figure 1b: CDF of SINR $\bar{\gamma}$=30 dB, $\bar{\gamma}_I$=6 dB, $m_I$ =0.8.

## IV. System Performance

Next, the BER performance of the M-QAM target link is calculated when the target and interfering channels have different Nakagami-*m* fading parameters. Here we find the BER using [1],

$$\mathrm{BER}(\gamma)=\frac{4\left(\sqrt{M}-1\right)\sum_{i=0}^{\frac{\sqrt{M}}{2}-1} Q\left((2i+1)\sqrt{\frac{3\gamma}{M-1}}\right)}{\sqrt{M}\,log_2(M)} \qquad (9)$$

and

$$\mathrm{BER}(\bar{\gamma},\bar{\gamma}_I,m,m_I)=\int_0^{\infty}\mathrm{BER}(\gamma)f_{\gamma}(\gamma;\bar{\gamma},\bar{\gamma}_I,m,m_I)\,d\gamma. \qquad (10)$$

For the Rayleigh fading case additional comparisons of BER predictions between this analysis and simulation results can be found in [10]. The results in [10] show that the above analysis provides conservative BER predictions when compared to simulation results. Fig. 2 demonstrates the impact on the BER for different channel parameters; these results are consistent with [7] when $m_I=1$. To study the effect of changing interfering channel fading parameter we define the relative BER as

$$BER_{Relative}(m_I;\bar{\gamma},\bar{\gamma}_I,m)=\frac{BER(\bar{\gamma},\bar{\gamma}_I,m,m_I)}{BER(\bar{\gamma},\bar{\gamma}_I,m,m)}. \qquad (11)$$

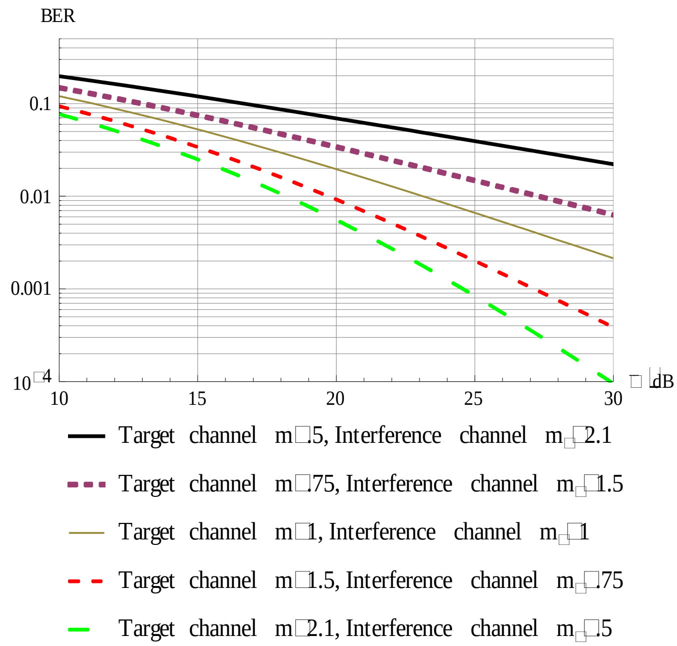


Figure 2: 16-QAM with Nakagami-*m* fading with different fading parameters $\bar{\gamma}_I$=5dB.

The relative BER is shown in Fig. 3. For fixed interference power the interfering channel fading parameter can have a modest degrading influence (here ~20%) on the target system BER. The different shape of the performance characteristics for the *m*<1 and *m*>1 cases follows from the nature of $f_{\gamma}(\gamma)$ shown in Fig. 1.

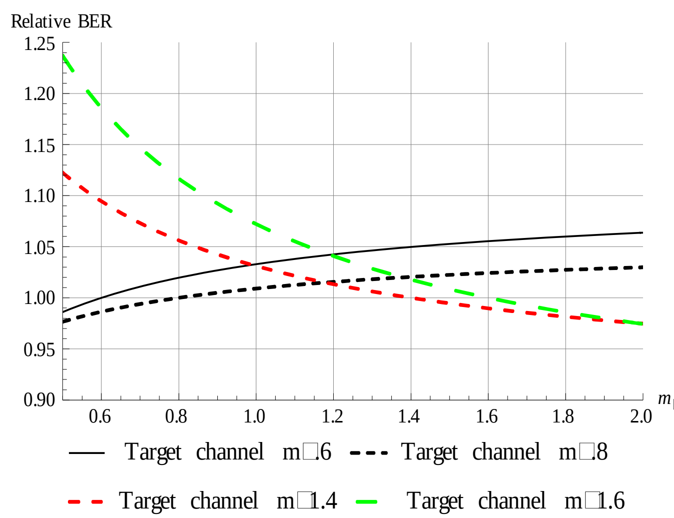


Figure 3: Relative BER for 16-QAM with Nakagami-*m* fading with different fading parameters $\bar{\gamma}$=30 dB, $\bar{\gamma}_I$=6 dB.

## V. Conclusion

Probability density functions are derived for the instantaneous received SINR when the target and interfering signals are transmitted through Nakagami-*m* channels with different fading parameters. The pdfs derived here provide the basis for conservative estimates of the BER. This work also shows that the performance of the target link is moderately influenced by changes in the interference channel characteristics. Also, the characteristic shape of the pdf of the SINR changes as *m* crosses the boundary of *m*=1.

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