BER Performance of 2D-Rake Receivers in DS-CDMA Over Frequency-Selective, Slow Rayleigh Fading Channels

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***Abstract*—In this letter, we extend the method that has been proposed in the literature for calculating the bit error probability of two-dimensional (2D) Rake receivers in asynchronous direct sequence code-division multiple access (DS-CDMA) over flat, slow Rayleigh fading channels to the case of frequency-selective, slow Rayleigh fading channels. We also compute and plot the lower bound on the performance that can be achieved in the above system.**

***Index Terms*—Antenna arrays, DS-CDMA, Rake, Rayleigh fading.**

I. Introduction

ONE OF THE primary limitations on the performance of cellular communication systems is multiple access interference. Hence current research activities are focused on reducing this interference, including the use of antenna arrays at the base station. Antenna arrays can be thought of as spatial filters in the sense that they can be used to form a beam toward the desired user while spatially rejecting the interferers outside the beam.

In the case of a DS-CDMA system employing Rake reception, the interference power is reduced considerably at the base station by performing temporal correlation of the signal received from all the users with the desired user's spreading code. Multipath diversity combining is also achieved using a Rake receiver [1]. In a typical mobile environment, signals from users arrive at different angles to the base station and hence antenna arrays can be used to an advantage. In the extreme case, each multipath of a user may arrive at a different angle, and this angle spread can be exploited using an antenna array [2], [3]. Thus, by combining an antenna array with Rake reception in DS-CDMA, considerable performance gain can be achieved [2].

The concept of a 2D-Rake or Space–Time Rake receiver was introduced in [3] wherein it is proposed that a beam be formed toward each multipath of the desired user using an antenna array and the resultant combined using a 1D-Rake receiver (time-only Rake). In [4] and [5], a simple method is used to arrive at closed form expressions for the average bit error probability of a 2D-Rake receiver at the base station in an asynchronous DS-CDMA system over flat, slow Rayleigh fading channel assuming the users to be uniformly distributed over the coverage area.

We briefly review the method proposed in [4]. At the base station receiver, an antenna array (using conventional beamforming techniques) is used to form a beam toward the desired user, thereby separating the interferers into two classes—in-beam and out-of-beam—based on whether their direction of arrivals lie inside or outside the beam formed toward the desired user. The attenuation provided by the antenna array to each of the out-of-beam interferers (regardless of the exact angle of arrival) is assumed to be constant. Then matched filtering of the output of the antenna arrays with the desired user's spreading sequence is done.

II. 2D-Rake Over Frequency-Selective Rayleigh Fading Channel

In this section, we extend the concepts discussed above to the frequency-selective, slow Rayleigh fading case. Let the number of resolvable multipaths of each user be L. The average E_b/N_o of each multipath is assumed to be equal, i.e., each is equal to $1/L$ times the average E_b/N_o in the flat, slow fading case. We assume that the distribution of the direction of arrival of the signals to the antenna array is uniform and that there is perfect power control in the system.

Including the multiple access interference I_o, the average total $E_b/(N_o+I_o)$ at the output of the beamformers followed by the 1D-Rake receiver is given by [4]

$$\begin{aligned}\overline{\gamma}_{\text{total}} &= \frac{\sigma_A^2}{\frac{\sigma^2}{M}+\frac{(\sigma_A^2/L)K_I}{2N}+\frac{B(2\alpha_o)(\sigma_A^2/L)(L(K-1)-K_I)}{3N}}\\ &= \frac{1}{\frac{\sigma^2}{M\sigma_A^2}+\frac{K_I}{2NL}+\frac{B(2\alpha_o)(L(K-1)-K_I)}{3NL}}\end{aligned} \tag{1}$$

where M is the number of antenna elements, $M\sigma_A^2/\sigma^2$ is the average total E_b/N_o, L is the number of multipaths, K is the number of simultaneous users, N is the length of the spreading sequence, K_I is the number of in-beam interferers, $2\theta_{\text{BW}}$ is the beamwidth, B is the number of beams formed toward the desired user, and α_o is the constant attenuation of the out-of-beam interferers.

The bit error probability for L path diversity combining using 1D-Rake is

$$P_e=\left[\frac{1}{2}(1-\mu)\right]^L\sum_{l=0}^{L-1}\binom{L-1+l}{l}\left[\frac{1}{2}(1+\mu)\right]^l \tag{2}$$

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where μ is defined as [6]

$$\mu = \sqrt{\left(\frac{\overline{\gamma}_{\text{total}}}{L + \overline{\gamma}_{\text{total}}}\right)}. \tag{3}$$

Using (1) and (2), we obtain the average bit error probability of the 2D-Rake receiver in asynchronous DS-CDMA over a frequency-selective, slow Rayleigh fading channel as

$$P(E) = \sum_{K_I=0}^{L(K-1)} \chi \eta^{K_I} \binom{L(K-1)}{K_I} P_e \tag{4}$$

where η, the probability of an in-beam interferer, and χ are defined as [4]

$$\eta = \frac{B(2\theta_{BW})}{\triangle\theta}$$
$$\chi = (1-\eta)^{L(K-1)-K_I}. \tag{5}$$

The lower bound on the performance of the 2D-Rake receiver can be found by setting $K_I = 0$. In practice, for K_I to tend to zero, the beamwidth resolution capabilities of the antenna array must be increased. This in turn requires increasing the number of antenna elements in the array at the base station since the beamwidth is inversely proportional to the number of antennas.

We consider the following possible cases that may arise depending on the angle spread introduced by the channel.

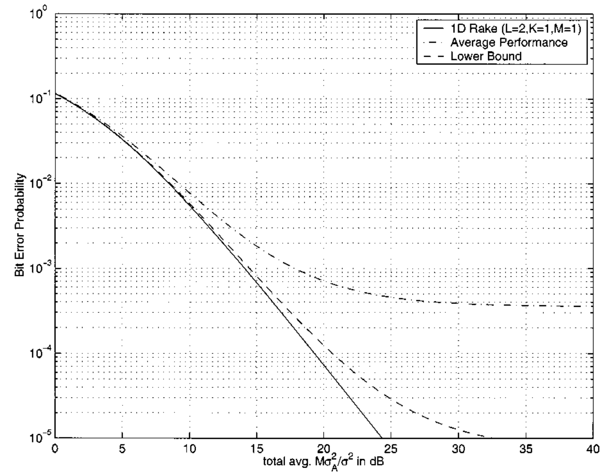
A. Channel With Small Angle Spread

Here the multipaths of the desired user arrive at the antenna array within a spatial angle of one beamwidth or less. Hence a single beam is formed toward the desired user and the output of the beamformer is fed to a 1D-Rake receiver to accomplish diversity combining and further interference rejection. In this case, the total number of interferers is $L(K-1)$, total beamwidth toward the desired user is $2\theta_{BW}$, and the attenuation of the out-of-beam interferers is α_o.

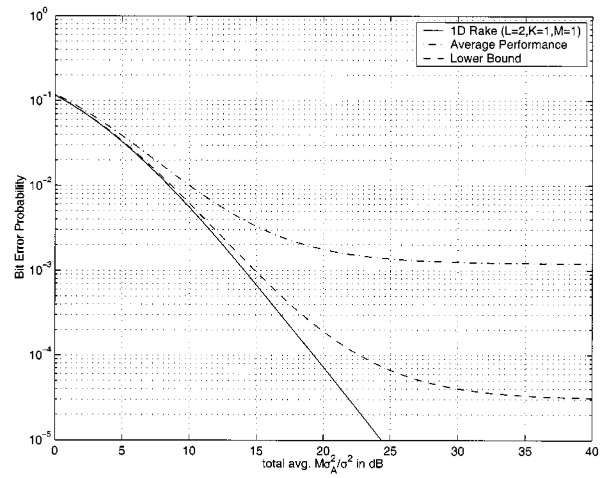
Fig. 1(a) shows the bit error rate (BER) performance of the 2D-Rake receiver for $L = 2$. The total coverage angle of the sector is $\Delta\theta = 60°$. We plot both the average performance (assuming a uniform distribution of users in the sector) and the lower bound on the performance possible with the 2D-Rake receiver. It can be seen that the average BER curve flattens out as the total E_b/N_o is increased beyond a certain point since the power of the interferers also increases by the same amount, and begins to dominate the BER.

B. Channel With Large Angle Spread

At the other extreme, each multipath of the desired user arrives at an angle that differs by more than a beamwidth, and hence a beam is formed toward each multipath. The output of the beamformers are then combined and fed to a 1D-Rake receiver which does the diversity combining and interference rejection. The total number of interferers is approximately $L(K-1)$ (by the simplifying assumption that the L separate beams can be viewed as a single beam of beamwidth L times the original beamwidth), the total beamwidth toward the multipaths of the desired user is $L(2\theta_{BW})$, and the attenuation of the out-of-beam interferers is $L\alpha_o$.



(a)



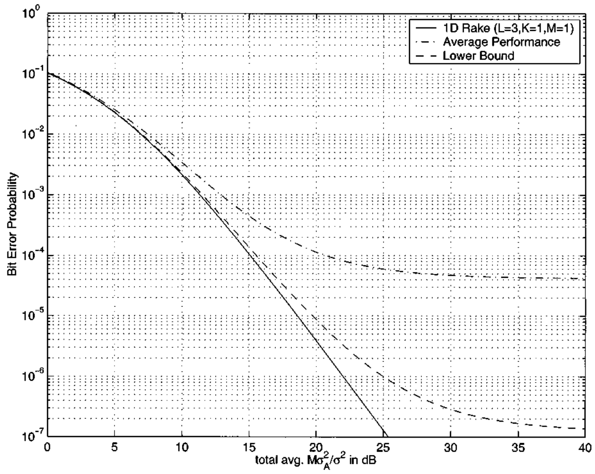
(b)

Fig. 1. Bit error probability of 2D-Rake receiver in DS-CDMA over frequency-selective, slow Rayleigh fading channel. no. of antennas $M = 8$, spreading sequence length $N = 128$, no. of simultaneous users $K = 20$, no. of multipaths of each user $L = 2$. (a) Channel with small angle spread. (b) Channel with large angle spread.

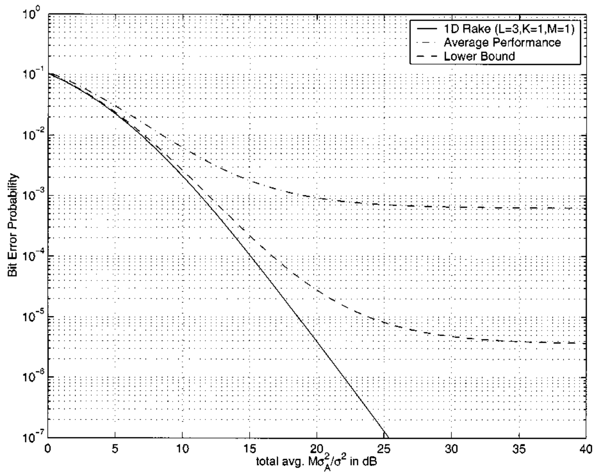
The BER performance in this case is plotted in Fig. 1(b) for $L = 2$. The total beamwidth used for capturing the desired user's multipath components is increased L times compared to that in Fig. 1(a). Thus the probability of an interferer arriving within the total beam formed for the desired user is increased, lowering the BER performance of the system. In Fig. 2, we plot the performance for $L = 3$.

C. Channel With Moderate Angle Spread

Some multipaths of the desired user may arrive within a single beam while the remaining multipaths of the same user may require a different beam. For example, say multipaths of delay 0, T_c and $2T_c$ can be captured using a single beam while multipaths of delay $3T_c$ and $4T_c$ need a separate beam. Here each beam requires a 1D-Rake receiver and the outputs of the Rake receivers are simply combined. In this case, the total number of interferers is approximately $L(K-1)$, total beamwidth toward the multipaths of the desired user is $2(2\theta_{BW})$ and the attenuation of the out-of-beam interferers is $2\alpha_o$.



(a)



(b)

Fig. 2. Bit error probability of 2D-Rake receiver in DS-CDMA over frequency-selective, slow Rayleigh fading channel. no. of antennas $M = 8$, spreading sequence length $N = 128$, no. of simultaneous users $K = 20$, no. of multipaths of each user $L = 3$. (a) Channel with small angle spread. (b) Channel with large angle spread.

In Fig. 3, the BER performance of 2D-Rake receiver for $L = 5$ is plotted. The multipaths are assumed to require 2 separate beams to be captured by the antenna array. It can be seen that the lower bound is very close to the bit error rate performance of 1D-Rake ($L = 5$, $K = 1$) till around $P_e = 10^{-7}$ and then the curve tends to flatten out.

Thus from Figs. 1–3, we see that BER performance close to that in the single user case can be obtained by combined beamforming (with a sufficiently large number of antennas) and 1D-Rake reception. As future work, we suggest simulation based performance analysis of the 2D-Rake receiver over frequency-selective, slow Rayleigh fading channels with relaxed assumptions.

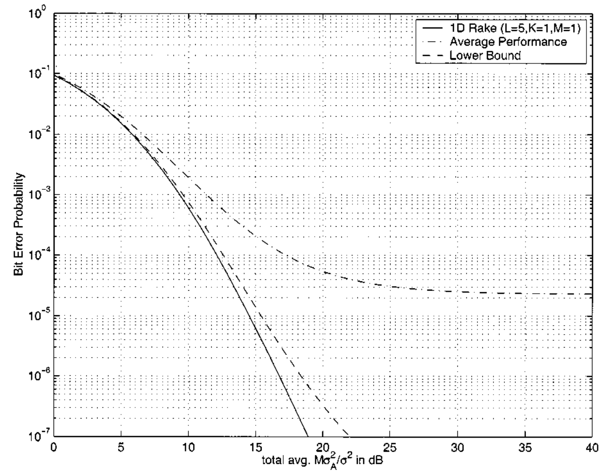


Fig. 3. Bit error probability of 2D-Rake receiver in DS-CDMA over frequency-selective, slow Rayleigh fading channel with moderate angle spread. no. of antennas $M = 8$, spreading sequence length $N = 128$, no. of simultaneous users $K = 20$, no. of multipaths of each user $L = 5$, no. of beams toward desired user $B = 2$.

III. Conclusion

In this letter, we have extended the method proposed in [4] so as to analyze the BER performance of 2D-Rake receivers in asynchronous DS-CDMA over frequency-selective, slow Rayleigh fading channels. We have also shown that the 2D-Rake receiver rejects interferers both spatially as well as temporally and achieves a BER performance that can come close to the performance of 1D-Rake receiver in the single user (no interferers) case.

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