Maximizing the Minimum Signal-to-Interference Ratio in a Wireless Communications Network

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Abstract

This report introduces a network topology optimization problem. The signal-to-interference ratios (SIR) are selected as the criteria for optimization. Two methods that locally maximize the minimum of a set of SIRs are implemented. Additional theories for locating the global optimal solution are proposed.

1 Introduction

The origin of this problem is from the development of a high speed wireless communications network. This network consists of two basic components: user nodes and switch nodes. Switch nodes form the "backbone" or the main traficway for information flow through the network. They would be analagous to the interstates of the United States highway system. User nodes send and receive information through this highway of switches. A question that frequently arises in wireless networks is determining how to connect the various nodes of the network to optimize a specific criteria. On a highest level, the configuration between switch nodes presents the problem.[1] On a lower level, it is the connection between the user nodes and the switch nodes. This is the problem dealt with in this report.

2 Definition of the Problem

2.1 Problem Boundary

As shown in Figure 1 the problem will be limited to one switch node and M user nodes. The user nodes' positions are measured with respect to the normal of the switch node. This bearing-like coordinate system defines positive angles to be measured clockwise from the normal. Thus, each user node has associated with it a radial distance measured from the center of the switch node, and an angle measured clockwise from the normal. Since only one switch node is being considered, the term *switch* will now refer to a switch node and *node* will refer to a user node.

2.2 Electronic Beam Formation

A switches establishs connections with nodes by transmitting beams of radio waves. These waves are generated by a linearly arranged N element antenna array. Each beam is characterized by a transmit power relative to the other beam powers (Pwr) and a steering angle (SA). The steering angle defines the direction to send the main concentration of radio waves. Using the concept of destructive and constructive interference among electromagnetic waves [2], the beams are capable of being steered over a range of 180 degrees. The power propagation pattern produced by a beam over this range has been

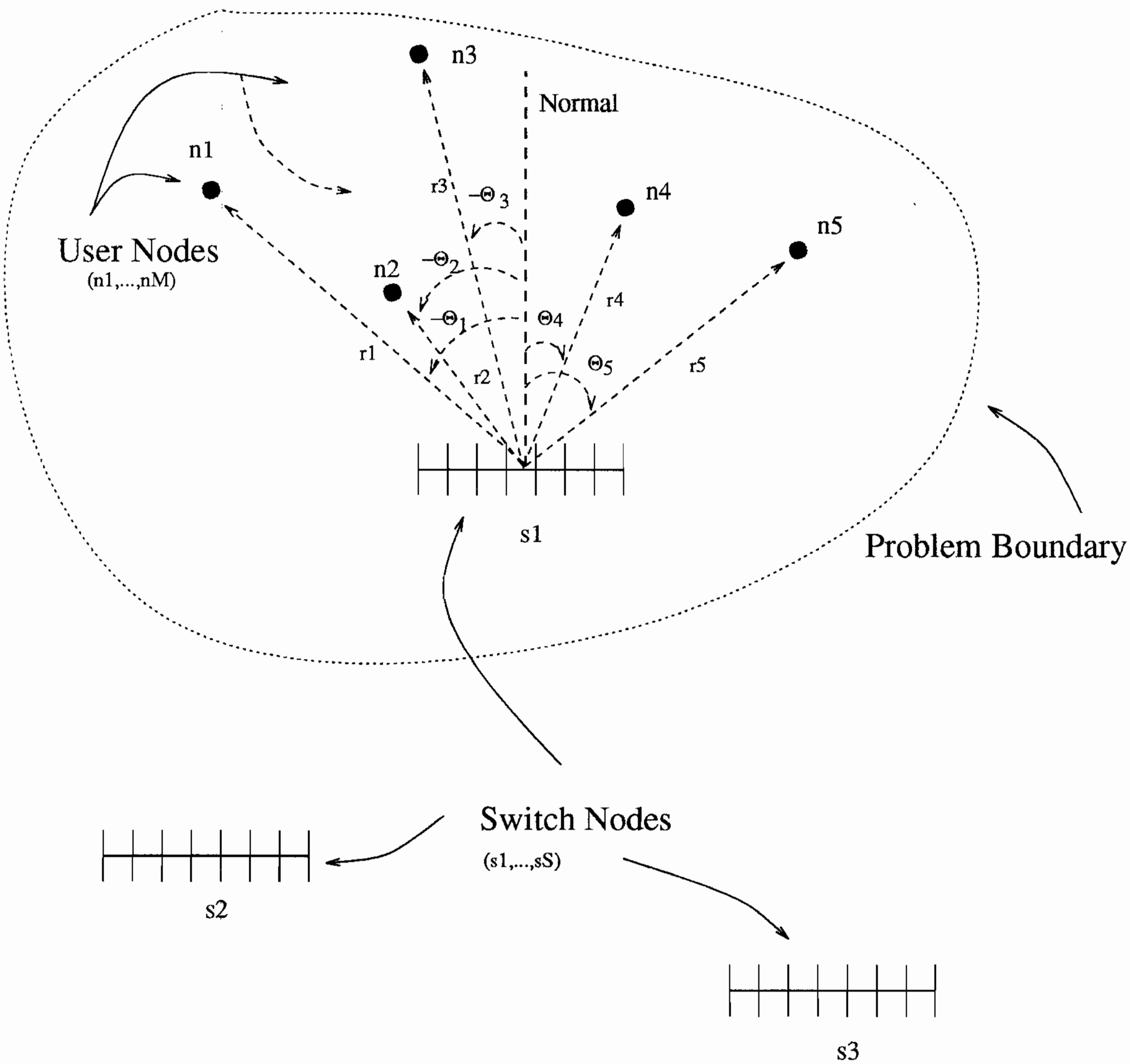

Figure 1: Diagram showing relationship between switch nodes and user nodes in the context of the problem boundary

synthesized. [3] An example of a beam with an SA of -10 degrees and a Pwr of 1 is shown in Figure 2. In addition to specifying a direction of the

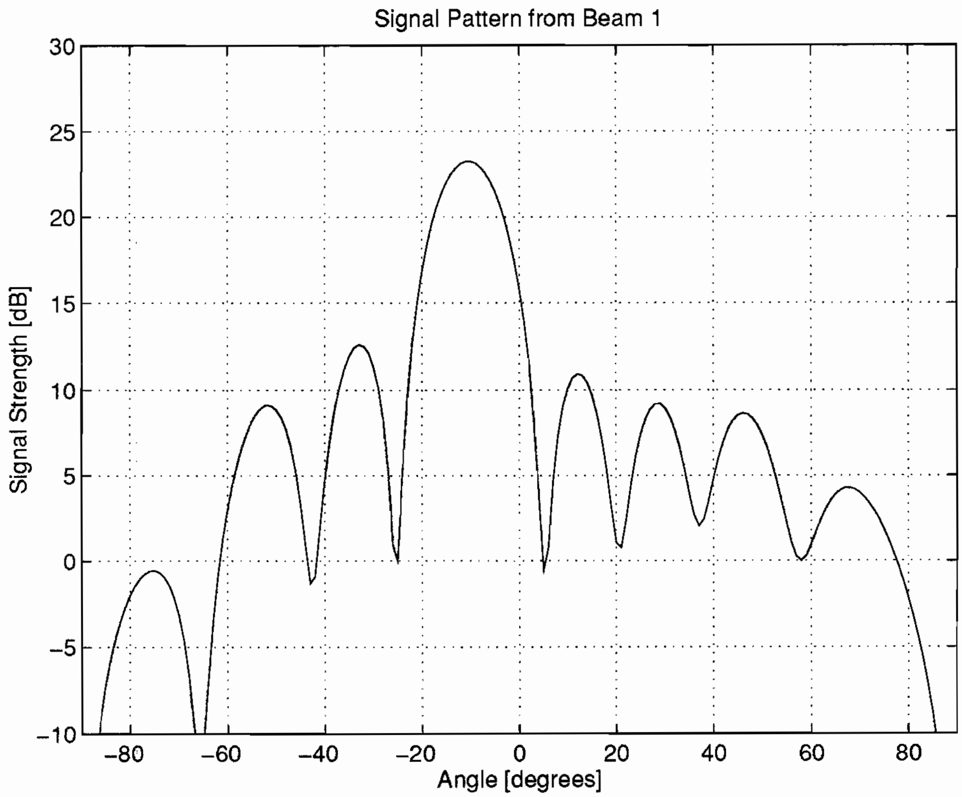

Figure 2: Example of forming a beam with a steering angle of -10 degrees and a relative power of one.

main concentration, or main lobe, of radio waves, it is possible to specify the directions for little or no electromagnetic concentration. [3] These *nulls* are useful in reducing the interference between multiple beams originating from a switch. For example, suppose two beams, B_1 and B_2, with steering angles SA_1 and SA_2 are forming at a switch. It would be advantageous when forming B_1 to place a null in the direction of SA_2 to reduce interference between the two beams. Likewise, when B_2 is being formed, a null is placed in the direction of SA_1. Thus, the propagated power or signal strength field of B_1 is not only a function of SA_1 and Pwr_1, but also that of SA_2.

The power from beam i as measured at node j can be characterized by the equation

$$S_{ij} = \frac{k_{ij} P_i}{a(r_j)} \tag{1}$$

where k_{ij} is a power gain of beam i at node j, P_i is the relative power (Pwr) of beam i, and $a(r_j)$ is a function that accounts for the attenuation in power due to distance. The power gain factor is a function of the set of steering angles for all the beams and the angular location of node j, θ_j. It basically is a factor due to the constructive and destructive interference of the waves from each antenna element.

2.3 The Signal-to-Interfernce Ratio (SIR)

The beam that will actually provide service for node j is that beam with the greatest signal strength at j.

$$s_j = \max\,[S_{ij}] \tag{2}$$

where $i = 1, \ldots, B$. Initially, this is how the *server map* is formed; however, in some instances it is necessary to have a constant or fixed server map. If the signals strengths S_{ij} are arranged in a matrix form

$$SigMat = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1M} \\ S_{21} & S_{22} & & \\ \vdots & & \ddots & \\ S_{B1} & S_{B2} & \cdots & S_{BM} \end{bmatrix} \tag{3}$$

then the serving beam for node j is the largest element of column j.

The signal-to-interference ratio (SIR) of node j is defined to be the ratio of the server's signal strength and the summation of all the other beams' signal strengths at node j.

$$SIR_j = \frac{\frac{k_{s_j j} P_{s_j}}{a(r_j)}}{\sum_{t=1}^{B} \frac{k_{tj} P_t}{a(r_j)} - \frac{k_{s_j j} P_{s_j}}{a(r_j)}}$$

Since each beam is being attenuated by the same amount, the $a(r_j)$ term cancels out of the ratio.

$$SIR_j = \frac{k_{s_j j} P_{s_j}}{\sum_{t=1}^{B} k_{tj} P_t - k_{s_j j} P_{s_j}} \tag{4}$$

Thus, only the angular component of a node's position is a parameter of its SIR as seen in Equation 4. Using the concept of the signal matrix, the set of SIRs for a group of nodes $j = 1, \ldots, M$ is

$$SIR_j = \frac{SigMat(\max[SigMat(\{1,\ldots,B\},j)],j)}{\sum[SigMat(\{1,\ldots,B\},j)] - SigMat(\max[SigMat(\{1,\ldots,B\},j)],j)}$$

2.4 Optimizing the Set of SIRs

The criteria to optimize will be the set of SIRs associated with a set of nodes. To accomplish this, the minimum SIR of the set of SIRs will be maximized.

$$maximize\{min[SIR_j]\} \qquad j = \{1, \ldots, M\}$$

Since the signal-to-interference ratio is a measure of signal quality, it is desirable to improve the worst performance.

3 Finding the Local Solution

3.1 Optimization by Adjusting Relative Signal Powers

The first method focuses on adjusting the set of relative power inputs (Pwr or P_i in Equation 4) given a fixed set of steering angles to maximize the minimum SIR. Intuitively, the optimal solution to this problem is to give infinite power to one beam and zero powers to the others. Thus, one beam would *serve* all the nodes and there would not be any interfering sources. This solution, however, defeats the reason behind using multiple beams to form the connections. In general, as a beam serves more nodes, the service to those nodes declines. To avoid this infinite power solution, the server map is held constant based on its initial value from Equation 2.

The next question to be answered is how does adjusting Pwr affect the set of SIRs. To answer this, examine Equation 4. If the relative power input to a beam is raised, then the nodes' SIRs that that beam serves will increase. Furthermore, they will all be raised by the roughly the same amount. Thus, the problem can be reduced from M nodes to at most B nodes [1] by only

[1]Due to a poor choice of steering angles it is possible for a beam not be a server of any node. In this case, that beam is just a source of interference and its Pwr can be made zero

examining those with that possess the minimum SIR in each server beam. Since these will be the limiting factors, it is valid to reduce the problem by only examining them. Now one-to-one correspondence exists between the beams and nodes. Another consequence of raising the relative power of beam, is that it also lowers the SIRs of all the nodes that it does not serve. This drop in SIR is roughly equal among those nodes.

The goal is to maximize the minimum of the SIR set. Thus, if only the minimum SIR of each serving beam are examined, the optimal solution would be to make all the elements of this narrower set of SIR [2] equal. Here is the reason why. Choose an element from $mSIR$ to be a reference (the first element works nicely). If the second SIR in $mSIR$ is greater than this reference SIR, lower the Pwr of the second SIR's beam. This will raise the reference and other SIRs and lower the second SIR of $mSIR$. (Do the opposite when the second SIR is less than the reference) When the first (reference) and second SIRs of $mSIR$ are equal, then adjust the Pwr of the third SIR's beam until it is equal to the reference and second beam. Since the first and second SIR are equal, they will both move in the same direction when the third beam is adjusted. By continuing iteration and making the refinements smaller and smaller, the minimum SIR will be raised to its maximum when all the SIRs in $mSIR$ are equal.

Examine the network of nodes in Figure 3 with the switch at the origin. Given three beams with steering angles of -60, -10, and 30 degrees, what is the relative input powers of each beam that maximizes the minimum SIR? Using the aforementioned algorithm, the Pwrs were adjusted as shown in Figure 4. Notice in Figure 5 that the lowest SIRs converge indicating that the minimum SIR cannot be raised any higher.

An alternative to the iterative approach used above, an algrebraic solution can be adopted. The optimal relative powers are found by solving a set of quadratic equations simultaneously. For simplicity, let node one have the minimum SIR in beam one, node two have the minimum SIR in beam two, etc. Then

$$\begin{aligned}
\frac{k_{11}P_1}{\sum_{t=1}^{B} k_{t1}P_t - k_{11}P_1} &= \frac{k_{22}P_2}{\sum_{t=1}^{B} k_{t2}P_t - k_{22}P_2} \\
\frac{k_{11}P_1}{\sum_{t=1}^{B} k_{t1}P_t - k_{11}P_1} &= \frac{k_{33}P_3}{\sum_{t=1}^{B} k_{t3}P_t - k_{33}P_3} \\
\vdots &= \vdots \\
\frac{k_{11}P_1}{\sum_{t=1}^{B} k_{t1}P_t - k_{11}P_1} &= \frac{k_{BB}P_B}{\sum_{t=1}^{B} k_{tB}P_t - k_{BB}P_B}
\end{aligned}$$

[2]This set will be referred to as $mSIR$.

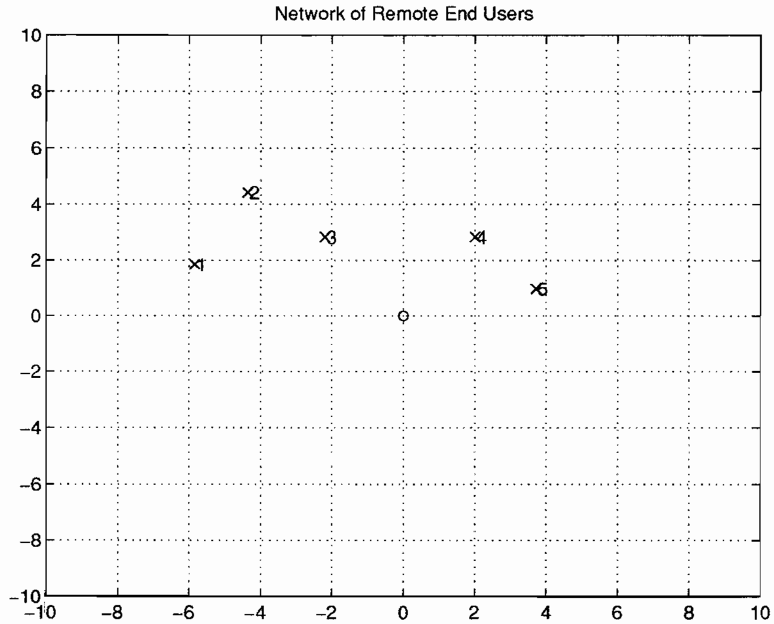

Figure 3: A plot of user nodes surrounding a switch node at the origin.

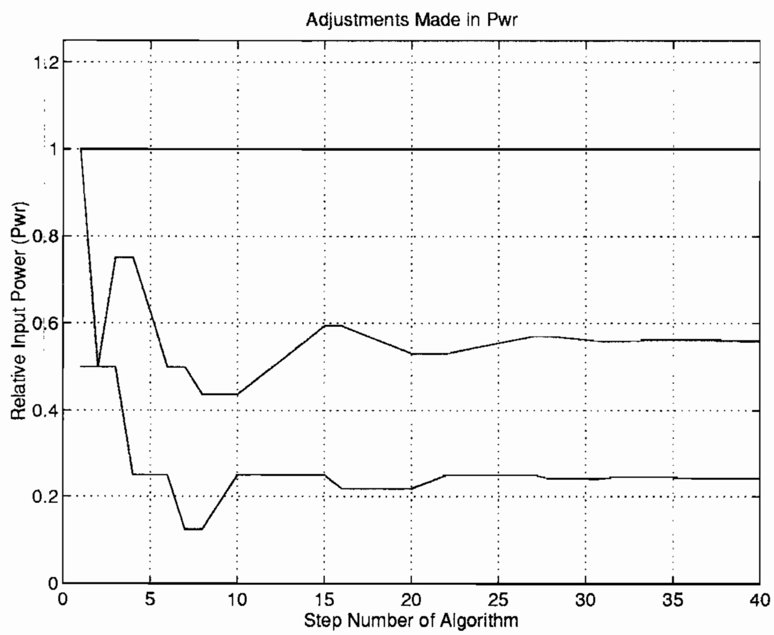

Figure 4: Adjusting the set of Pwrs to maximize the minimum SIR

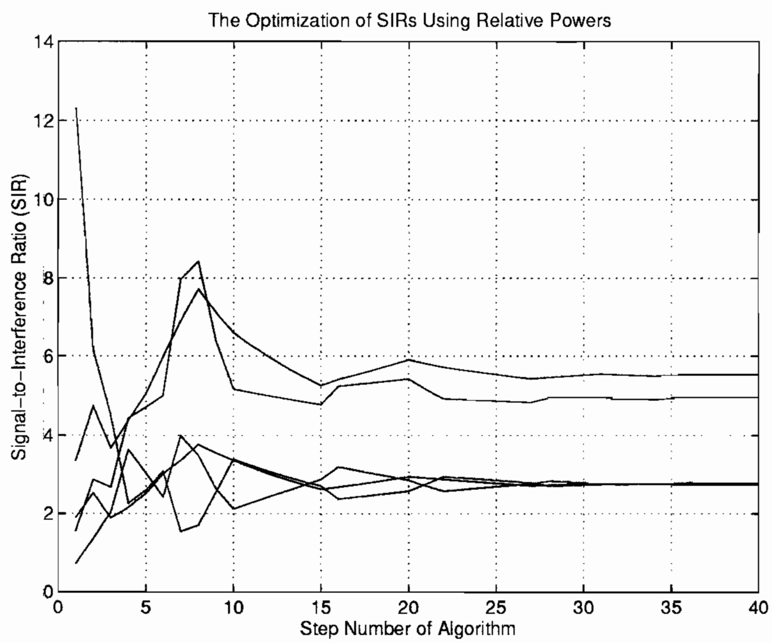

Figure 5: Signal-to-interference ratios ($SIRs$) converging to the optimal solution

The simultaneous solution to this set of equations can be rather difficult for the general case; therefore, the iterative method was developed. In addition, the iterative method demonstrates the theory in a more clear fashion than a set of equations.

3.2 Optimization by Adjusting Beam Steering Angles

The key to solving the problem using steering angles is to focus on the $min[SIR]$ at each step in the process. This is important because there are only two directions in which a steering angle can be moved, left and right. Likewise, there are only two directions in which the $min[SIR]$ can travel as well, up or down. Since each beam contributes an amount to the SIR of the $min[SIR]$, every beam must be adjusted.

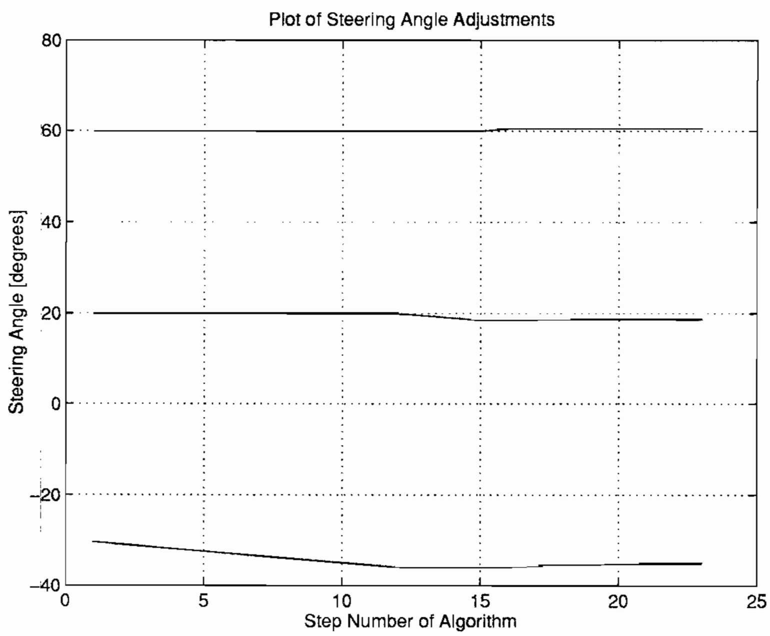

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Figure 6: Adjusting the set of steering angles (SA) to maximize the minimum SIR

The adjustment uses a "testing the water" type approach. The beam's SA is moved slightly in one direction. If that direction causes the $min[SIR]$ to increase, then the SA is continued to move in that direction until the

$min[SIR]$ begins to decrease. On the other hand, if that slight change caused the $min[SIR]$ to decrease, then the SA is moved in the opposite direction of that "test" step. This is continued until the $min[SIR]$ cannot be raised any higher by adjusting that beam. At this point, the next beam goes through the same process. In this fashion, the $min[SIR]$ continues to rise throughout the process and will reach a local maximum after adjustments have been made on the last beam. This process is shown in Figure 6 and 7.

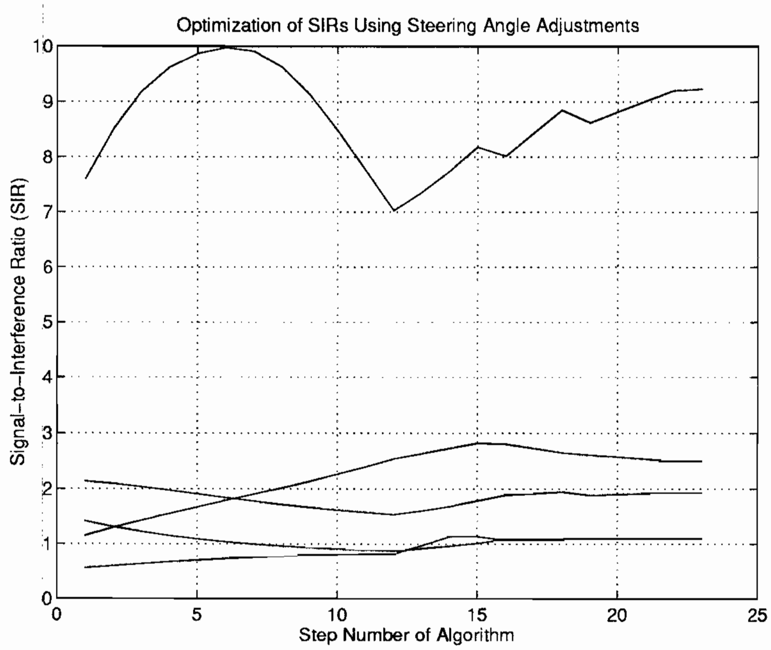

Figure 7: Maximizing the minimum SIR by adjusting the steering angles

4 Finding the Global Solution

Since the ultimate goal is to find the global solution to this optimization problem, additional search procedures must use the local algorithms on a global scale. This global search procedure will strategically choose starting points for the local searches.

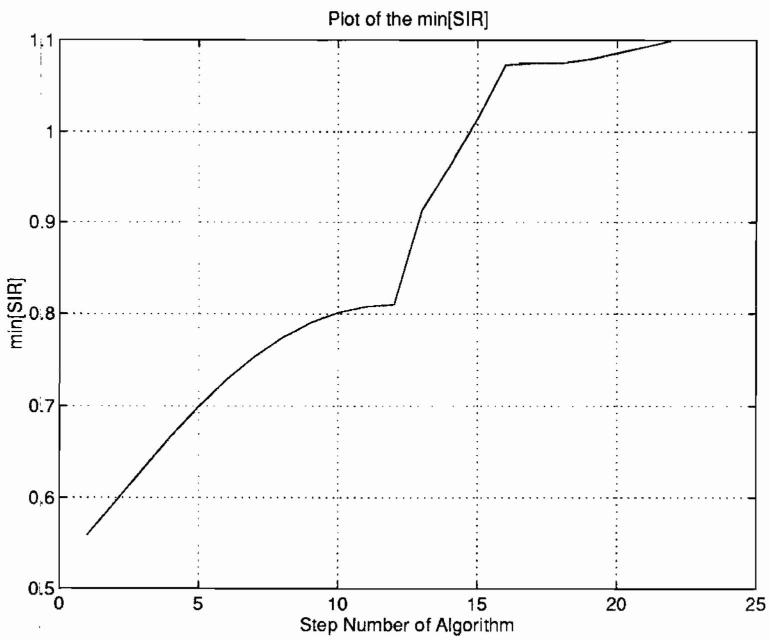

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Figure 8: Plot of the $min[SIR]$ approaching a local maximum as the result of steering angle adjustments

4.1 Tree Search

One way of choosing starting points is through a tree search. A tree search works by continually dividing the search area by half on each step. For example, in a one beam search, the search would start at zero. It would then divide the left and right regions in half and search them (ie -45 then 45). It would do the same for each one of these smaller regions (ie -67.5, -22.5, 22.5, then 67.5). This search would continue until a specified criteria is met. This could be anything from stopping when a $min[SIR]$ is found that is above some limit to stopping after it completes n cycles and return the best $min[SIR]$ its found.

Tree searches with multiple beams work similar, with the exception that every combination of possibilities must be explored for each beam. Thus, this search has an exponential operational time and can be very slow; however, it is guaranteed to find the global optimal solution, even if it does not use any local search algorithm.

4.2 Random Search

An alternative to the tree search is a search procedure that chooses random steering angles as starting points. These random variables could be uniform over B disjoint regions or take on some other distribution. In addition to its ease of implementation, this method offers the hope of finding the solution more quickly by "shooting" the problem with a shotgun instead of a rifle.

5 Conclusions

This report has shown two methods of finding local maximums in the $min[SIR]$. Given a set of steering angles and a server map, it is possible to find the optimal relative input powers for these conditions. Likewise, if a set of relative input powers are given, optimal steering angles can be found. By applying both these procedures to a set of initial conditions can significantly raise the $min[SIR]$. By using a global search procedure, the optimal solution for the entire problem may be found.

6 Acknowledgements

The coding for the algorithms to synthesize beam patterns was done through joint effort of Nathan Goodman and the author. Credit for the development of the relative power and steering angle optimization algorithms belongs to Dr. David W. Petr and the author.

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